Programme du cours MAFY 1181
Actualités des mathématiques et de la physique.
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27 mars. Approximations de fonctions. Archéologie des tables de logarithmes.

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The present file is http://perso.uclouvain.be/alphonse.magnus/Logar.pdf

!

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II. LOGARITHMES DES NOMBRES DE 1 A 100 000. — 42e TABLE

N.	0	1	2	3	4	5	6	7	8	9
3050	484 2998	3141	3283	3426	3568	3710	3853	3995	4137	4280
1	4422	4564	4707	4849	4991	5134	5276	5418	5561	5703
2	5845	5988	6130	6272	6414	6557	6699	6841	6984	7126
3	7268	7410	7553	7695	7837	7979	8121	8264	8406	8548
4	8690	8833	8975	9117	9259	9401	9543	9686	9828	9970
5	485 0112	0254	0396	0539	0681	0823	0965	1107	1249	1391
6	1533	1676	1818	1960	2102	2244	2386	2528	2670	2812
7	2954	3096	3239	3381	3523	3665	3807	3949	4091	4233
8	4375	4517	4659	4801	4943	5085	5227	5369	5511	5653
9	5795	5937	6079	6221	6363	6505	6647	6788	6930	7072
3060	7214	7356	7498	7640	7782	7924	8066	8208	8350	8491
1	8633	8775	8917	9059	9201	9343	9484	9626	9768	9910
2	486 0052	0194	0336	0477	0619	0761	0903	1045	1186	1328

Ce sont des chiffres!!

3050	484 2998	3141	3283	3
1	4422	4564	4707	4
2	5845	5988	6130	6
3	7268	7410	7553	7
4	8690	8833	8975	9
5	485 0112	0254	0396	0

John Napier 1550 - 1617 http://www-history.mcs.st-and.ac.uk/Mathematicians/Napier.htm
Henry Briggs 1561 - 1630 http://www-history.mcs.st-and.ac.uk/Mathematicians/Briggs.ht
Jost Bürgi 1552 - 1632 http://www-history.mcs.st-and.ac.uk/Mathematicians/Burgi.html

Simone TROMPLER, L'histoire des logarithmes

Votre recherche : Consortium - Titre - Phrase: tables de logarithmes Notices 1 10 sur 15 Liens Occurrences Résultats

2 NOUVELLES TABLES DE LOGARITHMES A 7 DECIMALES AVEC LE CALCUL DES PARTIES PROPORTIONNELLES DES DIFFERENCES POUR LES NOMBRES DE 1 A 10000

1 NOUVELLES TABLES DE LOGARITHMES A CINQ DECIMALES POUR LES LIGNES TRIGONOMETRIQUES DANS LES DEUX SYSTEME DE LA DIVISION CENTESIMALE B ET DE LA DIVISION SEXAGESINALES DU QUADRANI ET POUR LES NOMBRES DE 1 A 12000 SUIVIES DES MEMES TABLES A QUATRE DECIMALES ET DE...

1 NOUVELLES TABLES DE LOGARITHMES A CINQ DECIMALES POUR LES LIGNES TRIGONOMETRIQUES DANS LES DEUX SYSTEMES DE LA DIVISION CENTESIMALEB3 ET DE LA DIVISION SEXAGESIMALE DU QUADRANT ET POUR

LES NOMBRES DE 1 A 12000

1 Nouvelles tables de logarithmes cinq décimales : table numérique - tables trigonométriques : division centésimale, division sexagésimale / C. Bouvart ; A. Ratinet

1 Petites tables de logarithmes à sept décimales pour les nombre et les lignes trigonométriques d'après Véga et Callet / par Ad. Lebegue

1 TABLES DE LOGARITHMES A 7 DECIMALES POUR LES NOMBRES DEPUIS 1 JUSQU'A 108000 ET POUR LES FONCTIONS TRIGONOMETRIQUES DE 10 EN 10 SECONDES

6 Tables de logarithmes a cinq décimales et autres tables / par N.-J. Schons

3 Tables de logarithmes a cinq décimales et autres tables / [Par N.-J. Schons]

1 Tables de logarithmes à cinq décimales pour les nombres de 1 10.000 et pour les fonctions trigonométriques de minute en minute

1 Tables de logarithmes à sept décimales pour les nombres depuis 1 jusqu'à 108000

et pour les fonctions trigonométriques de dix en dix secondes / par L. Schön ; précédées d'une introduction française par J. Hoüel

1 Tables de logarithmes d'intérêts composés et d'escomptes composés

1 TABLES DE LOGARITHMES,ETENDUES A SEPT DECIMALES.

0 Tables de logarithmes pour les nombres et pour les sinus

1 Tables financières des opérations à long terme, précédées d'une Table de logarithmes 7 décimales, suivies des Tables de logarithmes de Scott à dix décimales

http://www-groups.dcs.st-and.ac.uk/~history/Miscellaneous/Briggs/ Briggs' ARITHMETICA LOGARITHMICA translated and annotated by Ian Bruce, University of Adelaide, Australia

Chapter six <Chapters/Ch6.pdf> The Logarithms are formed from continued means: in which the repeated square root of 10 is taken to establish eventually a proportionality between the fractional part of the root and the index. Some non-fatal but time wasting errors are uncovered by the translator.

Chapter Six §6.1. Synopsis: Chapter Six.

In the present chapter, the real business of determining the (base 10) logarithms gets under way. Briggs constructs a sequence of rational logarithms formed from successive square roots of 10, in which a given root is the absolute number (evaluated with much labour), and the corresponding index is the logarithm of this absolute number: what happens if you persist with this sequence of continued mean numbers to ever decreasing values?

By persisting with the evaluation of successive square roots of 10, a well-defined pattern begins to emerge, and finally does so at the 54 th root extraction, labeled P in Table 6-2, to the 34 significant figures of the calculation.

	D		E
	Numbers from Continued Means between Ten & One.		Rational Logarithms
	10		1,000
1	31622,77660,16837,93319,98893,54		0,50
2	17782,79410,03892,28011,97304,13		0,25
3	13335,21432,16332,40256,65389,308		0,125
4	11547,81984,68945,81796,61918,213		0,0625
5	10746,07828,32131,74972,13817,6538		0,03125
	. . .		. . .
51	10000,00000,00000,10225531945602592 1	L	0,00000,00000,00000,44408,92098,50062,61
52	10000,00000,00000,05112765972801294 7	M	0,00000,00000,00000,22204,46049,25031,30
53	10000,00000,00000,02556382986400647 0	N	0,00000,00000,00000,11102,23024,62515,65
54	10000,00000,00000,01278191493200323 5	P	0,00000,00000,00000,05551,11512,31257,82

[Table 6-2]

At this stage, the number of zeros of the continued mean, after the initial 1, is the same as the number of significant places after the zeros [note: 0(15) below represents a string of

15 zeros, etc], and that the remaining digits are in proportion to the index or logarithm of the mean. Reversing the procedure, all the means labeled 54 - 1 in the table are continued squares of the mean P [Table 6-3], and those immediately adjoining P labeled N, M, and L show proportionality between themselves and also their logarithms. Thus, for level P, in modern notation: $10^{1/2^{54}} = r = 1 + \Delta$,where $\Delta = 0.0(15)12781914932003 23442$; with the corresponding logarithm $\ell = 1/2^{54} = 0.0(16)55511151231257827 0212$. For the next level N: $r^2 \sim 1 + 2\Delta$; with logarithm 2ℓ; and subsequently for M: $r^4 \sim 1 + 4\Delta$; with logarithm 4ℓ; and finally L: $r^8 \sim 1 + 8\Delta$, with logarithm 8ℓ; . The intermediate values $r^3, 3\ell; r^5, 5\ell$; etc, are also readily found. Hence, the numbers Δ and ℓ define a proportionality between the fractional part of the continued mean and the corresponding logarithm, between the values of the roots designated from P to L, and the logarithm of any other small number of the form $1 + \delta$ in this region can be found by proportionality to be $\delta\ell/\Delta$. All of this has been established arithmetically by Briggs: the links to modern analysis are investigated a little in the notes to the chapter. However, we may note at this stage that from the proportionality, Δ/ℓ can be written in the form $(10^{1/2^{54}} - 1).2^{54} \sim \ln(10)$, the natural logarithm of 10.

$$A^{1/N} = \exp\left(\frac{\ln A}{N}\right) = 1 + \frac{K}{N} + \underbrace{\frac{K^2}{2N^2} + \cdots}_{\text{négligeable si } N \text{ assez grand}}, \; K = \ln A \text{ (ici, } A = 10).$$

Chapter seven <Chapters/Ch7.pdf> The Logarithm of 2 is found by the Radix Method, and subsequently the logs of 5 and 3.

§7.1. Synopsis: Chapter Seven.

The region L and onwards in the series of continued means has been established in Chapter 6, in which there is finally proportionality between the fractional parts of the means and their known logarithms. Briggs shows how the logarithms of 2 and 3 can be found using a method based on this proportionality, upon forming a sequence of continued mean numbers based on the repeated square root extraction of a number derived from 2 or 3, that eventually lie in the above region. Initially, 2 is converted to $2^{10}/10^3 = 1.024$ prior to forming the mean sequence: a process that reduces the number of means evaluated; in the region L, the fractional part of the 47th continued mean of 1.024 is compared with a continued mean of 10, as we have indicated in the extended note in the last chapter, from which the logarithm of 2 is determined. The logarithm of 5, and multiples of 2, 5, and 10 can then be found. Similarly, in the case of 3, the logarithm of 6 is found from $6^9/10^7$ or 1.0077696, again reducing the number of operations slightly.

	Number of successive means	*Logarithms*
	1024 -	0,01029,99566,39811,95265,27744
47	10119,28851,25388,13862,397	0,00514,99783,19905,97632,63872
46	10059,46743,74634,83266,5424	0,00257,49891,,59952,98816,31936
45	10029,68064,49807,87373,6268	0,00128,74945,79976,49408,15968
	...	...
11	10000,00000,00017,25604,42423,25943,477	0,00000,00000,00074,94204,79391,98911,283
	...	...
1	10000,00000,00000,01685,16057,05394,977	0,00000,00000,00000,07318,55936,90623,9368

En effet, $a^{1/N} = \exp\left(\frac{\ln a}{N}\right) = 1 + \frac{K \log_A(a)}{N} + \cdots$

RADIX TABLE OF NATURAL LOGARITHMS **Table 4.3**

x	n	$\ln(1+x10^{-n})$	$-\ln(1-x10^{-n})$
1	5	0.00000 99999 50000 33333 08334	0.00001 00000 50000 33333 58334
2	5	0.00001 99998 00002 66662 66673	0.00002 00002 00002 66670 66673
3	5	0.00002 99995 50008 99979 75049	0.00003 00004 50009 00020 25049
4	5	0.00003 99992 00021 33269 33538	0.00004 00008 00021 33397 33538
5	5	0.00004 99987 50041 66510 42292	0.00005 00012 50041 66822 92292
6	5	0.00005 99982 00071 99676 01555	0.00006 00018 00072 00324 01555
7	5	0.00006 99975 50114 32733 11695	0.00007 00024 50114 33933 61695
8	5	0.00007 99968 00170 65642 73220	0.00008 00032 00170 67690 73221
9	5	0.00008 99959 50242 98359 86809	0.00009 00040 50243 01640 36811
1	4	0.00009 99950 00333 30833 53332	0.00010 00050 00333 35833 53335

...

§12.1. Synopsis: Chapter Twelve.

Briggs demonstrates an improved method of subtabulation, far superior to the 'proportional parts' method of the previous chapter. The new scheme involves the use of finite differences, and can be applied when first and second order differences only need be considered, the latter being constant, or nearly so, (and negative, though the negative signs are suppressed); Briggs uses this method extensively to fill in the blank spaces in his Chiliads of logarithms: the missing Chiliades from the 31st to the 89th represents the region where this simple scheme does not work. In modern terms, if A and B are the logarithms of two successive numbers in a table of logarithms, with first and second order differences a and b, where b is constant or almost so, in some region, then the logarithms of 9 equally spaced numbers z_i between the successive numbers are generated by the formula $z_i = A + xa + [x(x-1)/2]b$, where $x = i/10$, and $1 \leqslant i \leqslant 9$. There is not of course an algebraic representation in the *Arithmetica* of this formula, or any other. Initially, Briggs uses a numerical scheme to calculate and add on successive difference to previously found intermediate logarithm as he proceeds; he then demonstrates the formula shown above numerically, by means of which any intermediate logarithm can be found, without the requirement of evaluating all the others up to that stage; finally, he shows how the method greatly improves on finding the square root of a number with a known logarithm. This scheme is identical with that later formulated by Newton, and is now known as Newton's Forward Difference Method. In the notes, we give a little historical digression concerning the possible origins of this method.

§12.2. Chapter Twelve. [p.24.]

For two given nearby numbers, together with the logarithms of these: to place nine other numbers equidistant between them; and to find the logarithms of these. If the second differences of the given logarithms are almost equal, it will not be difficult to perform this: otherwise, if third differences are summoned, here the method is somewhat deficient.

Two nearby numbers A are taken, and the logarithms of these B, together with the first and second differences of these, C and D. [Table 12-1.] If the second differences are equal, one of these is multiplied by the numbers in the Table E, [12-2], with the ten numbers adjoined to the first [difference]; ...

							E	
		47602,0016	C					
91235	A	4,96016,14763,8639	B	5217	D	1.	45	
		47601,4799	C			2.	35	pro-
91236	A	4,96016,62365,3438	B	5217	D	3.	25	duct
		47600,9582	C			4.	15	added
						5.	5	
		[Table 12-1.]				6.	5	
						7.	15	pro-
						8.	25	duct
						9.	35	subtracted
						10.	45	
								[Table 12-2.]

Donc, soit f connue en $x = a$ et $x = a \pm h$: avec $D = [f(b = a+h) - f(a)]/h = f'(a) + hf''(a)/2 + \cdots$,

$$f(x) - f(a) - D(x-a) = ((x-a)^2 - h(x-a))f''(a)/2 + \cdots$$
$$= (x-a)(x-b)f''(a)/2 + O(h^3).$$

VS FORTRAN Application Programming: Library Reference, IBM Program Product SC26-3989, 1981[1],

Pour l'exponentielle, ce document propose

$$2^{-x} \approx 1 - \frac{2x}{0.034657359x^2 + x + 9.9545948 - \dfrac{617.97227}{x^2 + 87.417497}}$$

avec une erreur $< 10^{-8}$ quand on s'est ramené à $0 \leqslant x \leqslant 1$.

Des formules de même type sont utilisées pour $\sin$, $\ln$, etc.

Cf. aussi J.F. Hart & *al.*, *Computer Approximations*, Wiley, 1968.

`http://sci.tech-archive.net/Archive/sci.math.num-analysis/2007-01/msg00301.html`

[1] Grand merci à M. J.L. Marrion qui a fourni cette documentation.

Re: polynomial approx. of natural logarithm

From: israel@xxxxxxxxxxx Date: 26 Jan 2007 10:23:19 -0800

$>>>$ log$(y+1) = a_0/2 + \sum_{i=1}^{\infty} a_k T_k(2*y-1)$ where T_k are the Chebyshev polynomials of the first kind and a_k the corresponding coefficients, e.g. in Clenshaws tables published by N.I.ST. 16 digits precision requires degree 18 here. Using the Clenshaw trick to compute such chebyshev developments you need then 18 multiplications only. There are better (max-error optimal) approximations in the book of Hart, Cheney et al on "computer approximations" (Wiley interscience) hence you need additions and multiplications only as desired.

$>>$ According to Maple's "minimax", the following expression approximates $\ln(x)$ on the interval [1,2] with maximum error less than 3.08e-16:

.405465108108164479766848218041+(.666666666666677733176708586037
+(-.222222222222292615563999977470 +(.987654320961263302457746148661e-1
+(-.493827160409480212568624479861e-1 +(.263374487439984912536137644541e-1
+(-.146319162610832072489328323277e-1 +(.836108894173642764386658876471e-2

+(-.487729587180873427291731663486e-2 +(.289035485018365468463367659448e-2
+(-.173428236905831034242311471786e-2 +(.105000376893099721663304663901e-2
+(-.641197490972256849186454645772e-3 +(.401222535445756813288920496419e-3
+(-.250241046792760884723756142417e-3 +(.131375663779446965248604221296e-3
+(-.781307141439894121243153903398e-4 +(.977683729452561788511104251535e-4
-.650948211813855652340284354509e-4 *y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y)*y

where y = x - 1.5. It requires 18 multiplications and 19 additions (including one for the computation of y).

Robert Israel isr...@xxxxxxxxxxx Department of Mathematics http://www.math.ubc.ca/~israel University of British Columbia Vancouver, BC, Canada

> Why not leave that series expansion in Tchebycheff polynomials, (as it is most certainly computed internally), evaluating the Tchebycheff polynomials using the recurrence relation, rather than converting the result to a polynomial to be evaluated using Horner's rule, isn't that more stable? All those minus signs give me reason to worry about the practical accuracy of that formula despite its theoretical error bound.

Also, I'd choose the interval [1/2,1) rather than [1,2] which seems more in keeping with how numbers are stored internally.

$>$ Just some ideas. $>$ Jen

the true minimax polynomial is a bit better better than the chebyshev expansion. but your second idea is no longer valid: in IEEE the mantissa is normalized to be in [1,2] rather then [.5,1] (as earlier) this is due to the implied "thought" leading 1 (not stored) if $x \neq 0$. we did not ask the original questioner whether this makes sense at all. if he works on a cpu without division he also might have to work on a cpu without IEEE or floating point at all. this makes it a bit harder but the ideas presented here would nethertheless be applicable.

For the problem at hand, what does it matter whether an implied 1 is stored or not? Whether you choose to normalize your x to [1,2] or [.5,1] will just change the exponent, not the mantissa. Does a shift of 1 in the exponents cause a significant change in the speed of floating-point arithmetic?

Robert Israel

TRANSCENDENTAL FUNCTIONS

COMMON LOGARITHMS

x	$\log_{10} x$	x	$\log_{10} x$
200	30102 99957	250	39794 00087
201	30319 60574	251	39967 37215
202	30535 13694	252	40140 05408
203	30749 60379	253	40312 05212
204	30963 01674	254	40483 37166
205	31175 38611	255	40654 01804
206	31386 72204	256	40823 99653
207	31597 03455	257	40993 31233
208	31806 33350	258	41161 97060
209	32014 62861	259	41329 97641

Record: 1 Title: Table of reciprocals of the integers from 100,000 through 200,009 / Prepared by the Mathematical Tables Project, Work Projects Administration of the Federal Works Agency. Conducted under the sponsorship of the National Bureau of Standards. Lyman J. Briggs, director, National Bureau of Standards, official sponsor. Arnold N. Lowan, technical director, Mathematical Tables Projectlink to cataloged record Name as creator: United States. National Bureau of Standards. Computation Laboratory United States. Work Projects Administration. Mathematical Tables Project Physical Description: viii, 201, [2] p. ; 28 cm Type: Tables Date: 1943 Topic: Reciprocals Mathematics Data Source: Smithsonian Institution Libraries

Record: 2 Title: Tables of the moment of inertia and section modulus of ordinary angles, ..., 1941

Record: 3 Title: Tables of circular and hyperbolic sines and cosines for radian arguments. ... Date: 1939

Record: 4 Title: Tables of the exponential function e [superscript x] ... 1939 ... Works Progress Administration, Mathematical Tables Project. Tables of the exponential function. Washington: National Bureau of Standards, 1942.

11. Works Progress Administration, Mathematical Tables Project. Tables of natural logarithms. Volumes I and II, Washington: National Bureau of Standards, 1941.

12. Works Progress Administration, Mathematical Tables Project. Tables of probability functions. Volume II, Washington: National Bureau of Standards, 1942.

Citation: David Alan Grier, "Gertrude Blanch of the Mathematical Tables Project," *IEEE Annals of the History of Computing*, vol. **19**, no. 4, pp. 18-27, Oct.-Dec. 1997.

Abramowitz and Stegun: Handbook of Mathematical Functions An electronic copy of the tenth printing of this famous reference. `http://www.math.ucla.edu/~cbm/aands/`

`http://www.convertit.com/Go/Bioresearchonline/Reference/AMS55.ASP`

`http://www-lsp.ujf-grenoble.fr/recherche/a3t2/a3t2a2/bahram/biblio/abramowitz`

`http://www.math.sfu.ca/~cbm/aands/intro.htm`

Grier, David Alan: Irene Stegun, the Handbook of mathematical functions, and the lingering influence of the New Deal. *Amer. Math. Monthly* **113** (2006), no. 7, 585–597.


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EEX
CLX
CLR
7
8
9

Tables of Mathematical Functions by Milton Abramowitz and Irene A. Stegun in PDF and OCRized

`http://www.lacim.uqam.ca/~plouffe/articles/Abramowitz&Stegun.pdf`

Et maintenant?

Grands problèmes: discrétisation,

OU méthodes spectrales: Boyd, Trefethen,. . .

"Chebyshev and Fourier Spectral Methods" by John P. Boyd , First edition (out of print), Springer-Verlag (1989), 792 pp. ... (Adobe .pdf; File size about 3.2 MB); Chebyshev and Fourier Spectral ... `www-personal.umich.edu/~jpboyd/BOOK_Spectral2000.html`

Trefethen, SPECTRAL METHODS IN MATLAB Forty short m-files which do everything from demonstrating spectral accuracy on functions of varying smoothness to solving the Poisson, biharmonic, ... `www.comlab.ox.ac.uk/nick.trefethen/spectral.html`

Trefethen numerical ODE/PDE textbook Perhaps a reasonable way to cite this book would be: Lloyd N. Trefethen, Finite Difference and Spectral Methods for Ordinary and Partial Differential ... `www.comlab.ox.ac.uk/nick.trefethen/pdetext.html`

Ultra-haute précision

1. Calculation of π to 100,000 Decimals Daniel Shanks, John W. Wrench, Jr. *Mathematics of Computation*, Vol. **16**, No. 77 (Jan., 1962), pp. 76-99

The Computation of π to 29,360,000 Decimal Digits Using Borweins' Quartically Convergent Algorithm, David H. Bailey *Mathematics of Computation*, Vol. **50**, No. 181 (Jan., 1988), pp. 283-296

`http://crd.lbl.gov/~dhbailey/`

$$\pi = \sum_{k=0}^{\infty} \frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right)$$

http://numbers.computation.free.fr/Constants/constants.html

Mathematical Constants By Steven R. Finch Published by Cambridge University Press, 2003 ISBN 0521818052, 9780521818056 602 pages

http://www.apfloat.org/apfloat_java/

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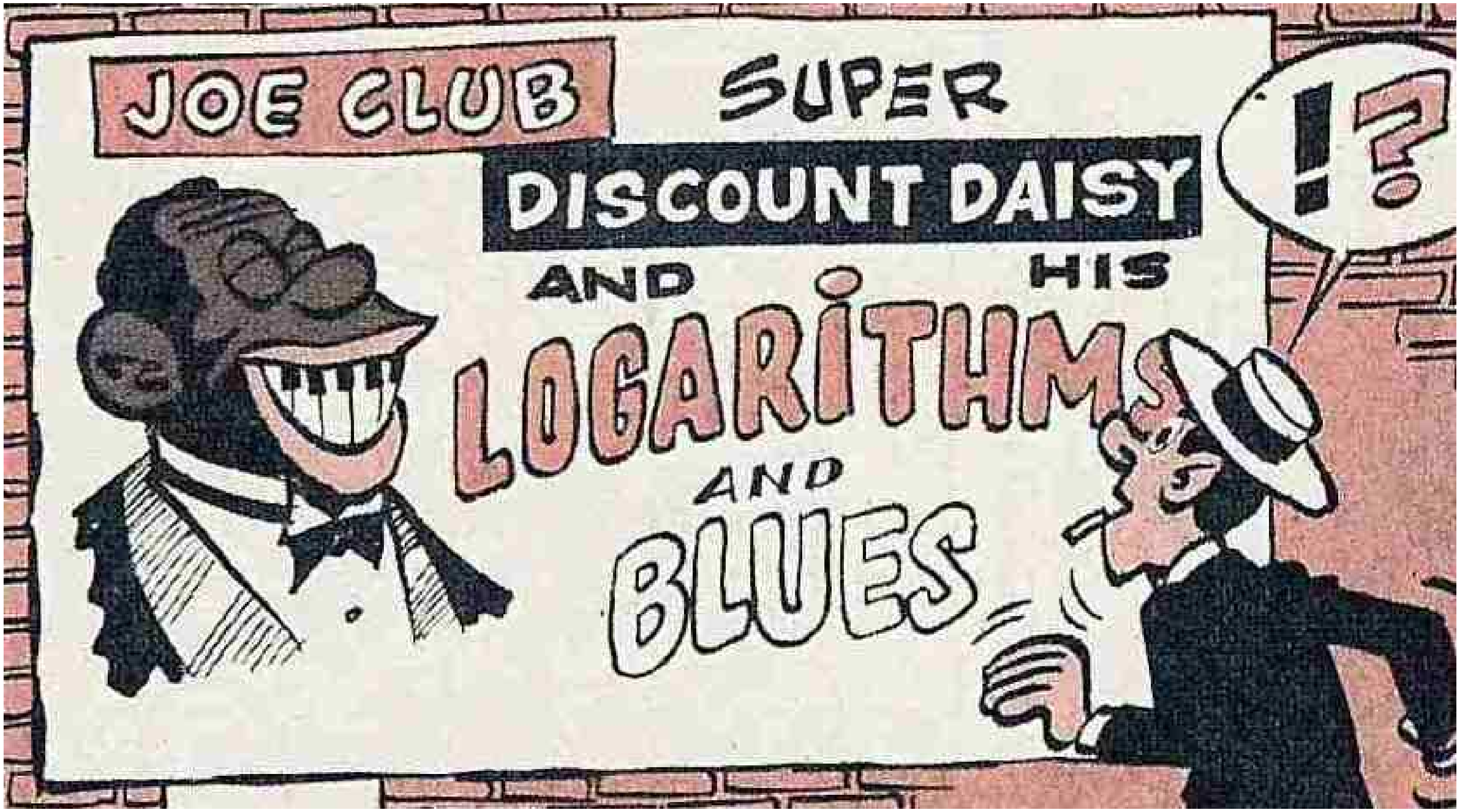
Bug reports, questions and comments should be sent to the author. Last updated: May 25th, 2008

Annie A. M. Cuyt, Brigitte Verdonk, Haakon Waadeland: Efficient and Reliable Multiprecision Implementation of Elementary and Special Functions. *SIAM J. Scientific Computing* **28**(4): 1437-1462 (2006)
Brigitte Verdonk, Annie A. M. Cuyt, Dennis Verschaeren: A precision- and range-independent tool for testing floating-point arithmetic I: basic operations, square root, and remainder. *ACM Trans. Math. Softw.* **27**(1): 92-118 (2001) II: conversions. *ibid.* **27**(1): 119-140 (2001)

Maurice Tillieux 1971



XVIIe siècle. Emprunté du latin scientifique logarithmus, composé à partir du grec logos, relation, rapport , et arithmos, nombre .

http://www.esnips.com/doc/953d20c1-cff2-4948-9333-1a3b2dac1d40/Logarithm---Dabru-Fasano.mp3