Fonds National de la Recherche Scientifique

Groupe de Contact en Analyse Numérique

Numerical methods in ordinary and partial differential equations

Vendredi 5 décembre 1997, Friday December 5, 1997

Faculté Polytechnique de Mons

Laboratoire d'Automatique, boulevard Dolez 31, Mons

9.30-10.00 Accueil Welcome.

10.00-11.00 W. Schiesser (Lehigh University, Bethlehem, USA)
Space and time adaptivity in the numerical integration of partial differential equations.

11.00-11.30 Café coffee

11.30-12.00 Tanja Van Hecke (Universiteit Gent, Vakgroep Toegepaste Wiskunde en Informatica)
Mono-implicit Runge-Kutta-Nyström Methods : efficient IRKN methods.

12.00-12.30 Jo Simoens (K.U.Leuven Department of Computer Science)
Fast Direct Solvers as Preconditioner in Waveform Relaxation.

12.30-14.30 Repas de midi (lunch).

14.30-15.00 Bart Sijnave (Vakgroep Toegepaste Wiskunde en Informatica, Universiteit Gent)
Continuation of codimension 2 equilibrium bifurcations in CONTENT.

15.00-15.15 break

15.15-16.15 P. Zegeling (Math. Dept., Utrecht University, The Netherlands)
An adaptive grid method based on smoothed equidistribution and its application to PDEs with higher-order derivatives.

16.15-16.30 Café coffee

16.30-17.00 Gérard Degrez (von Karman Institute)
Implicit upwind residual distribution schemes for Euler/Navier-Stokes equations.

17.00-17.30 Alain Vande Wouwer (Faculté Polytechnique de Mons)
Numerical Experiments with the Moving Finite Element Method

Space and Time Adaptivity in the Numerical Integration of Partial Differential Equations

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An essential aspect of the numerical integration of partial differential equations (PDEs) is the monitoring and control of numerical errors. In this paper, we review three basic approaches to adaptive error control (for discretization errors in space and time):

1. h refinement: Variation of the space and time integration intervals.

2. p refinement: Variation in the order of the space and time approximations.

3. r refinement: Adaptive movement of the grid points to regions of large variations in the solution.

These three strategies can be implemented manually (by observing the solution and making adjustments), or automatically (by having the computer code make the adjustments).

We illustrate these strategies within the method of lines (MOL), which is a flexible approach to the numerical integration of systems of ordinary differential equations (ODEs), differential-algebraic equations (DAEs) and PDEs.

Some representative numerical methods will be reviewed and illustrated through computer codes for: (a) a system of two initial-value ODEs of arbitrary stiffness and (b) a special case of the Einstein field equations of general relativity.

The talk will conclude with a brief discussion of the software available for MOL analysis, including a set of Matlab "m" files and the Fortran code for the Einstein field equations that will be provided to the attendees.

Mono-implicit Runge-Kutta-Nyström Methods : efficient IRKN methods

T. Van Hecke, Research Assistent of the University of Gent,
G. Vanden Berghe, M. Van Daele
and H. De Meyer, Research Director at the National Fund for Scientific Research (N.F.W.O. Belgium)

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Implicit Runge-Kutta-Nyström methods (IRKN) are determined by the Butcher tableau

$$\begin{array}{c|c} c & \mathrm{A} \\ \hline & \bar{b}^T \\ & b^T \end{array}$$

with $c, \bar{b}, b \in \mathbb{R}^{s\times 1}$ and $A \in \mathbb{R}^{s\times s}$. IRKN methods are expensive and explicit RKN methods have bad stability properties. We consider the subclass of mono-implicit methods (MIRKN) for which a strictly lower triangular matrix $X \in \mathbb{R}^{s\times s}$ and a vector $v \in \mathbb{R}^{s\times 1}$ exist such that the Butcher matrix A can be written as

$$A = X + v\bar{b}^T + wb^T \, .$$

Moreover, since we aim at the construction of P-stable MIRKN methods, we require that the 2×2 matrix stability function

$$M(z) = \begin{bmatrix} 1 + z\bar{b}^T(I - zA)^{-1}e & 1 + z\bar{b}^T(I - zA)^{-1}c \\ zb^T(I - zA)^{-1}e & 1 + zb^T(I - zA)^{-1}c \end{bmatrix} ,$$

whereby $e \in \mathbb{R}^{s\times 1}$ denotes the vector with unit entries, satisfies for all real $z < 0$ both the conditions $\det M(z) = 1$ and $(\mathrm{Tr} M(z))^2 < 4$.

An analysis is made of P-stable FSAL methods with $v = c$. It will also be shown how the required LU-decomposition can be simplified. Symplectic methods are discussed within the class of MIRKN methods.

Fast Direct Solvers as Preconditioner in Waveform Relaxation

Jo Simoens

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Waveform relaxation is an iterative technique for solving large systems of ODEs. It is the continuous-in-time analogue of iterative methods for stationary problems based on matrix splitting, such as the Jacobi and Gauss–Seidel iterations. In this talk we consider its application for solving semi-discretised parabolic initial-boundary value problems.

The waveform relaxation method parallellises very well if the preconditioner is highly parallellisable. We discuss the use as preconditioner of direct parabolic PDE solvers with known optimal parallel complexity. These preconditioners are derived from direct solvers for elliptic PDEs, and use the FFT in the spatial dimensions and cyclic reduction in the time dimension. Fine and coarse-grain variants of the parallel algorithm are presented.

The convergence rate of the resulting waveform relaxation iteration is shown to be independent of the mesh size used in the spatial discretisation. We conclude by comparing the method with some other waveform relaxation and multigrid methods for parabolic problems.

Continuation of codimension 2 equilibrium bifurcations in CONTENT

B. Sijnave, W. Govaerts (Fund for Scientific Research (F.W.O. Belgium)
and Yu. A. Kuznetsov (CWI, Amsterdam (The Netherlands))

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In this talk, we discuss recent extensions to CONTENT, an interactive software package developed at CWI (Amsterdam) by Yu. A. Kuznetsov and V. V. Levitin, to compute and pathfollow numerically all codimension 2 bifurcations of the solutions of the equilibrium equations $F(u, \alpha) = 0$ associated with the dynamical system

$$\dot{u} = F(u, \alpha) \qquad u, F(u) \in \mathbb{R}^n, \alpha \in \mathbb{R}^m \ .$$

We will deal with Bogdanov-Takens points (`BT`), Zero-Hopf points (`ZH`), Double Hopf points (`DH`) and Generalized Hopf points (`GH`) which can be found while following a curve of single Hopf points and also with cusp points (`CP`) that can be found on limit point curves. Also, detection of several codimension 3 bifurcations will be discussed.

We illustrate the use of this software with several model computations for all types of the above mentioned bifurcations. One of the presented models is a realistic model of a neuron, developed at Cornell University by J. Guckenheimer and coworkers.

An adaptive grid method based on smoothed equidistribution and its application to PDEs with higher order derivatives.

Dr. P. A. Zegeling

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Abstract: Traditional numerical techniques to solve time-dependent PDEs integrate on a uniform spatial grid that is kept fixed on the entire time interval. If the solutions have regions of high spatial activity, a standard fixed-grid technique is computationally inefficient, since to afford an accurate numerical approximation, it should contain, in general, a very large number of grid points.

The grid on which the PDE is discretized then needs to be locally refined. Moreover, if the regions of high spatial activity are moving in time, like for steep moving fronts in reaction-diffusion or hyperbolic equations, then techniques are needed that also adapt (move) the grid in time: continuously deforming grid methods, which are also denoted by the term r-refinement.

In this talk I will describe an adaptive moving-grid method that is based on an equidistribution principle supplied with smoothing both in the time and space direction. In one space dimension this is rather straightforward and it can theoretically be shown that the ratios of adjacent spatial grid cells are forced to be bounded from below and above, due to the smoothing. In two dimensions, an adaptive moving grid can be defined by applying the 1D-principle along the two coordinate directions. Numerical results will be shown in one and two space dimensions for, among others, reaction-diffusion models and higher-order PDEs in time and space, such as wave equations, KWdV-equation and the Extended Fisher-Kolmogorov PDE model.

Implicit upwind residual distribution schemes for Euler/Navier-Stokes equations.

Gérard Degrez

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A review is made of implicit residual distribution schemes for the solution of compressible flows on unstructured grids which have been extensively studied at the von Karman Institute over the past few years.

Space discretization is considered first. The main motivation for the development of upwind residual distribution discretizations is to genuinely account for the multidimensional flow physics, contrary to state-of-the-art finite volume solvers which are based on 1D Riemann solvers. The basic principles of residual distribution schemes are explained in detail for the linear scalar advection equation. It is shown in particular that more classical schemes such as the SUPG finite element and Lax-Wendroff schemes can be viewed as residual distribution schemes. Then, their generalization to non-commuting hyperbolic systems is presented. The application of upwind residual schemes to the non-linear Euler equations of gas dynamics requires two additional ingredients, i.e. suitable decompositions of the system and a conservative linearization, which are discussed next. Finally, extension to viscous flow problems is achieved by a Galerkin finite element discretization of the viscous terms.

Having selected the space discretization scheme, there remains to specify the iterative solution strategy for the resulting set of non-linear algebraic equations. We consider here an implicit time-stepping/damped Newton strategy. It is shown that, thanks to the compactness of the space discretization, it is possible to compute the Jacobian matrix needed by the damped Newton algorithm economically both in terms of CPU time and storage. Linear systems are solved iteratively using preconditioned Krylov subspace algorithms (GMRES, BiCG-STAB). Finally, the presentation concludes with illustrative applications to a wide range of inviscid, laminar and turbulent flows.

Numerical Experiments with the Moving Finite Element Method

Alain Vande Wouwer

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Automatic Control Lab

In this talk, the moving finite element and the gradient weighted moving finite element methods for solving partial differential equations in one space dimension are briefly described. Several test-examples from science and engineering are used to illustrate the method features, including the selection of an initial node distribution, the parameter tuning and the use of matrix preconditioning.