Special Topics in Numerical Analysis and Applied Mathematics.

A 1-day conference in honour of Professor Jean Meinguet

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Université Catholique de Louvain, room de La Vallée Poussin,

Bâtiment de Hemptinne. Chemin du Cyclotron, 2. **B**-1348 Louvain-la-Neuve.

A numerical problem may not been regarded as completely solved until the degree of significance of the computed results has not been estimated. Generated errors have been extensively analyzed in the last decade, especially by resorting to ingenious backward techniques; in fact, at the present time, many computational procedures are known for which it is fairly easy to estimate the precision (for the so-called "gutartige Rechenprozesse", the rounding phenomena may even be entirely disregarded). The situation is less satisfactory when inherent errors are larger than generated errors, ...

From the abstract of "Sens et portées des arithmétiques de signification", pp.305-326 *in Programmation en mathématiques numériques, Besançon 7-14 Septembre 1966*, C.N.R.S., Paris, 1968.

The key to (50a) lies in the *extension*, stated recently by *Householder* (see [15], p.61, and [16], p.136), *of a nearly forgotten theorem by Netto* (1896), *which itself is an extension of one by Wronski* (1811). The proof is surprisingly simple: it consists in equating determinants on the two sides of a suitably chosen matrix identity expressing (with some redundance) the polynomial identity $B = AV$.

From "On the solubility of the Cauchy interpolation problem", pp. 137-163 *in Approximation Theory, Proceedings of a Symposium held at Lancaster, July 1969* (A. Talbot, editor), Academic Press, London, 1970.

For the applications we have in mind (which essentially involve finite rank Hankel matrices), the most profitable point of view is usually *sequential* rather than *functional*; that is why we prefer here, unlike the authors of [1], to speak of *singular value decomposition of matrices* (whose exceptionally fruitful idea apparently dates back to E. Beltrami and C. Jordan, 1873-1874, see [16], p.78), rather than of *Schmidt expansion of operators* (which was introduced by E. Schmidt in the quite different setting of integral equations with non-Hermitian kernels). Now, unlike the best approximation property (39) and the reminder complementing its proof, the following is conditioned by the property of H to be a Hankel matrix.

Theorem (Main results of [1] **and** [5]**).** *Let a given Hankel matrix H verify the assumption* (36)*, its positive singular values s_j being indexed according to* (38)*. Then, for each j,*

$$\begin{aligned} s_j &= \min_{B\in\mathscr{B}^{(j)}} \operatorname{lub}_2(H(Bf)) \\ &= \min_{\substack{v\in H^\infty \\ r\in\mathscr{R}^{(j)}}} \|f-v-r\|_\infty = \min_{K\in\mathscr{H}^{(j)}} \operatorname{lub}_2(H(f)-K), \end{aligned} \tag{42}$$

where $\mathscr{B}^{(j)}, \mathscr{R}^{(j)}$ and $\mathscr{H}^{(j)}$ are defined in section 4.2.

From "A simplified presentation of the Adamjan-Arov-Krein approximation theory", pp.217-248 *in Computational Aspects of Complex Analysis, Proceedings of the NATO Advanced Study Institute held at Braunlage, Harz, Germany, July 26-August 6, 1982.* (H. Werner, L. Wuytack, E. Ng, H.J. Bünger, editors), D.Reidel, Dordrecht, 1983.

$$\text{Condition } (*) \quad \int_{X:=e^{-i\theta_0}}^{Y:=e^{+i\theta_0}} \frac{w^2dw}{\sqrt{(w^2-X^2)(w^2-Y^2)}} = 0$$

$$\text{Poser: } w^2 = X^2\cos^2\varphi + Y^2\sin^2\varphi \qquad (X \leftrightarrow \varphi = 0, Y \leftrightarrow \varphi = \pi/2).$$

$$(*) \Leftrightarrow \int_0^{\pi/2} \frac{(X^2\cos^2\varphi + Y^2\sin^2\varphi)^{1/2}d\varphi}{i} = 0$$

$$E(\underbrace{\sqrt{1-e^{4i\theta_0}}}_{=k\Leftrightarrow k'=e^{2i\theta_0}}) := \int_0^{\pi/2} \sqrt{1-(1-e^{4i\theta_0})\sin^2\varphi}\, d\varphi = 0$$

$$\Rightarrow E(\underbrace{-i\operatorname{tg}\theta_0}_{=\frac{1-k'}{1+k'}}) = \frac{E(k)+k'K(k)}{1+k'} = \frac{k'}{1+k'}K(k) \searrow$$

alors que cf. Erdélyi II, p.319

$$K(-i\operatorname{tg}\theta_0) = \frac{1+k'}{2}K(k) \nearrow$$

... $\Longleftrightarrow$

$$\boxed{K(\sin\theta_0) = 2E(\sin\theta_0)}$$

From a private communication, December 30-31, 1985.