Online Appendix of Imperfect Monitoring of Job Search: Structural Estimation and Policy Design

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Appendix

A Derivation of $U_k(t_{k-1})$ from the Limit of its Recursive Definition in Discrete Time

Consider time intervals of length $d\tau$. The lifetime utility of an unemployed worker at time τ can be written by the following recursive relation:

$$U_k(\tau) = \frac{\left\{y_k(\tau)d\tau + [1 - \lambda s(\tau)d\tau]U_k(\tau + d\tau) + \lambda s(\tau)d\tau\left[F[w_r(\tau)]U_k(\tau + d\tau) + \bar{F}[w_r(\tau)]\bar{W}_k(\tau + d\tau)\right]\right\}}{1 + \rho d\tau}$$
(A-1)

where $s(\tau)d\tau$ is the probability that a job offer arrives between τ and $\tau + d\tau$. Rearrangement yields

$$\rho d\tau U_k(\tau) = y_k(\tau) d\tau + \lambda s(\tau) d\tau \bar{F}[w_r(\tau)] \left[\bar{W}_k(\tau + d\tau) - U_k(\tau + d\tau) \right] + \left[U_k(\tau + d\tau) - U_k(\tau) \right].$$
(A-2)

Dividing by $d\tau$, taking the limit for $d\tau \to 0$ and using that $\lambda s(\tau)\bar{F}[w_r(\tau)] \equiv p(\tau)$ leads to the following differential equation:

$$\dot{U}_{k}(\tau) - [p(\tau) + \rho] U_{k}(\tau) = - \left[y_{k}(\tau) + p(\tau) \bar{W}_{k}(\tau) \right]$$
(A-3)

Multiplying by $P(\tau, t_{k-1})e^{-\rho(\tau-t_{k-1})}$ leads to

$$\frac{1}{d\tau} \left(U_k(\tau) P(\tau, t_{k-1}) e^{-\rho(\tau - t_{k-1})} \right) = - \left[y_k(\tau) + p(\tau) \bar{W}_k(\tau) \right] P(\tau, t_{k-1}) e^{-\rho(\tau - t_{k-1})}$$
(A-4)

Finally, integrating from t_{k-1} to t_k results in

$$U_k(t_k)P(t_k, t_{k-1})e^{-\rho(t_k - t_{k-1})} - U_k(t_{k-1}) = -\int_{t_{k-1}}^{t_k} \left[y_k(\tau) + p(\tau)\bar{W}_k(\tau) \right] P(\tau, t_{k-1})e^{-\rho(\tau - t_{k-1})}d\tau,$$
(A-5)

which after rearrangement yields equation (7), with $U_k(t_k) = \mathbb{U}_k(t_k)$ given by (8).

B Optimality Conditions with Interrupted Job Spells

This appendix explains how the first-order conditions (FOC) (9), (10) and (11) must be modified to take into account that individuals returning to unemployment after a temporary employment spell, i.e. if i = 1, face a different, presumably lower and, for simplicity, fixed, probability of negative evaluation (π_k^1), than individuals who did not yet experience such an interruption: $\pi_k^1 \neq \pi_k^0 [\bar{S}(t_k, t_{k-1})]$. Since individuals are aware of this difference, their behavior will be affected accordingly and decision variables are indexed by a superscript *i*.

In this new environment the two equivalent FOC of the reservation wage become 1

$$U_{k}^{i}(\tau) = W_{k}(w_{r}^{i}(\tau);\tau) = \frac{w_{r}^{i}(\tau) + \delta U_{k}^{1}(\tau)}{\rho + \delta} \quad \Leftrightarrow \quad w_{r}^{i}(\tau) = \rho U_{k}^{i}(\tau) - \delta \left[U_{k}^{1}(\tau) - U_{k}^{i}(\tau) \right].$$
(B-1)

$$w_{r}^{i}(\tau) + c[s^{i}(\tau)] + \delta \left[U_{k}^{1}(\tau) - U_{k}^{i}(\tau) \right] = b_{k} + \nu + p^{i}(\tau) \frac{E \left[w - w_{r}^{i}(\tau) | w > w_{r}^{i}(\tau) \right]}{\rho + \delta} + \dot{U}_{k}^{i}(\tau).$$
(B-2)

The left-hand side of (B-2), the marginal cost of continuing search (for someone who is offered a job paying the reservation wage) includes now the new term $\delta \left[U_k^1(\tau) - U_k^i(\tau)\right]$, which differs from zero if i = 0. It measures the opportunity cost of foregoing the entitlement to a lower probability of negative assessment if one accepts the job offer and loses it subsequently: In case $\pi_k^1 < \pi_k^0 \left[\bar{S}^0(t_{k-1}, t_k) \right]$ the expected lifetime utility in case of redundancy from the offered job is larger than before job acceptance, i.e. $U_k^1(\tau) > U_k^0(\tau)$. The FOC of search effort (11) is not affected if i = 0. By contrast, in case i = 1, the probability of negative evaluation is independent of $S^1(t_{k-1}, t_k)$, so that the last term on the right-hand side drops out: $\pi_k'^1 = 0$.

C Optimal Paths and Endpoint Conditions Allowing for Job Interruptions and Random Delays in the Timing of Evaluations

In this section we derive the first-order conditions and consider optimal paths for control variables, along with the corresponding endpoint conditions. The exposition focuses on the time span between two adjacent interviews (k - 1 and k) and explicitly considers the difference between the scheduled and the delay intervals. The latter ends at a random instant and the first may start with delay. We denote these sub-periods respectively $[t_{k-1}^*, t_k')$ and $[t_k', T_k^*)$, where $t_0^* = t_0$. We assume that an assessment of job search effort occurs at some random instant T_k^* within the second sub-period, i.e. the "delay interval". If t_k^* denotes the realization of T_k^* , the realized delay, $(t_k^* - t_k')$, is assumed to be the minimum of a random draw from an exponential distribution with mean 1/q and some fixed maximum delay \bar{t}_k^* , which is equal to the maximum observed delay in the data.

Let $U_{k,1}^i(\tau)$ (respectively, $U_{k,2}^i(\tau)$) denote the expected lifetime utility at time τ in the first (respectively, second) sub-period. The objective $U_{k,1}^i(\tau)$ is the same as (7) where t_{k-1}^* , t_k' , $U_{k,1}^i(t_{k-1}^*)$ and $\mathbb{U}_{k,1}^i(t_k')$ replace, respectively, t_{k-1} , t_k , $U_k(t_{k-1})$ and $\mathbb{U}_k(t_k)$. So, the problem at the start of the scheduled interval writes:

$$\max_{s^{i}(\tau),w_{r}^{i}(\tau)} U_{k,1}^{i}\left(t_{k-1}^{*}\right) = \int_{t_{k-1}^{*}}^{t_{k}^{i}} \left[y_{k}^{i}\left(\tau\right) + p^{i}\left(\tau\right)\bar{W}_{k}^{i}\left(\tau\right)\right] P^{i}\left(\tau,t_{k-1}^{*}\right)e^{-\rho(\tau-t_{k-1}^{*})}d\tau + \mathbb{U}_{k,1}^{i}\left(t_{k}^{\prime}\right)P^{i}\left(t_{k}^{\prime},t_{k-1}^{*}\right)e^{-\rho(t_{k}^{\prime}-t_{k-1}^{*})} \quad (C-1)$$

¹In Section C these FOC are derived in the more general setting allowing for random delays in the timing of interviews.

s.t. :
$$\begin{split} \mathbb{U}_{k,1}^{i}\left(t_{k}'\right) &= U_{k,2}^{i}\left(t_{k}'\right), \qquad (C-2) \\ \dot{P}^{i}(\tau, t_{k-1}^{*}) &= -p^{i}(\tau)P^{i}(\tau, t_{k-1}^{*}), \\ \dot{\bar{S}}^{i}(\tau, t_{k-1}^{*}) &= \frac{s^{i}(\tau) - \bar{S}^{i}(\tau, t_{k-1}^{*})}{\tau - t_{k-1}^{*}}, \end{split}$$

During the delay interval, the job search effort is assessed at a rate q. This assessment occurs not later than \bar{t}_k^* , as expressed on the second line of (C-3). The discount rate is augmented by the arrival rate of the meeting and, hence, is equal to $\rho + q$. Lifetime utility at t'_k can be written as

$$U_{k,2}^{i}(t_{k}') = \int_{t_{k}'}^{\bar{t}_{k}^{*}} \left[y_{k}^{i}(\tau) + p^{i}(\tau) \bar{W}_{k}^{i}(\tau) + q \mathbb{U}_{k,2}^{i}(\tau) \right] P^{i}(\tau, t_{k}') e^{-[\rho+q](\tau-t_{k}')} d\tau \\ + \mathbb{U}_{k,2}^{i}(\bar{t}_{k}^{*}) P^{i}(\bar{t}_{k}^{*}, t_{k}') e^{-[\rho+q](\bar{t}_{k}^{*}-t_{k}')}, \quad (C-3)$$

$$\mathbb{U}_{k,2}^{i}(\tau) = \pi_{k}^{i} \left[\bar{S}^{i}(\tau, t_{k-1}^{*}) \right] U_{k+1,1}^{i}(t_{k}^{*}) + \left(1 - \pi_{k}^{i} \left[\bar{S}^{i}(\tau, t_{k-1}^{*}) \right] \right) U^{+}.$$
(C-4)

This appendix assumes that the job arrival rate per unit of search effort λ is normalized to 1, an assumption made in Section 5.2.

C.1 Scheduled Interval

Consider the scheduled interval $[t_{k-1}^*, t_k')$. We first show how the optimization problem can be restated in terms of a "generalized current value Hamiltonian". Next, we solve the problem.

C.1.1 The Generalized Current Value Hamiltonian

Referring to (C-1) and (C-2), the Hamiltonian $H_{k,1}^i(\tau)$ of the problem during the scheduled interval is

$$\left[y_{k}^{i}(\tau) + p^{i}(\tau)\bar{W}_{k}^{i}(\tau)\right]P^{i}(\tau, t_{k-1}^{*})e^{-\rho(\tau-t_{k-1}^{*})} - \lambda_{P}^{i}(\tau)p^{i}(\tau)P^{i}(\tau, t_{k-1}^{*}) + \lambda_{S}^{i}(\tau)\frac{s^{i}(\tau) - S^{i}(\tau, t_{k-1}^{*})}{\tau - t_{k-1}^{*}},$$
(C-5)

where $\lambda_P^i(\tau)$ and $\lambda_S^i(\tau)$ are the multiplier functions associated to the state variables $P^i(\tau, t_{k-1}^*)$ and $\bar{S}^i(\tau, t_{k-1}^*)$. To get rid of $\lambda_P^i(\tau) P^i(\tau, t_{k-1}^*)$ in the last expression, consider the FOC wrt $P^i(\tau, t_{k-1}^*)$:

$$\dot{\lambda}_{P}^{i}(\tau) = -\partial H_{k,1}^{i}(\tau) / \partial P^{i}(\tau, t_{k-1}^{*}) = \lambda_{P}^{i}(\tau) p^{i}(\tau) - \left[y_{k}^{i}(\tau) + p^{i}(\tau)\bar{W}_{k}^{i}(\tau)\right] e^{-\rho(\tau - t_{k-1}^{*})}$$

Subtracting $\lambda_P^i(\tau)p^i(\tau)$ from both sides and multiplying by $P^i(\tau, t_{k-1}^*)$ yields

$$\frac{\partial}{\partial \tau} \left(\lambda_P^i(\tau) P^i(\tau, t_{k-1}^*) \right) = - \left[y_k^i(\tau) + p^i(\tau) \bar{W}_k^i(\tau) \right] P^i(\tau, t_{k-1}^*) e^{-\rho(\tau - t_{k-1}^*)}.$$

Integrating this equation from τ to t'_k gives

$$\lambda_P^i(t'_k)P^i(t'_k, t^*_{k-1}) - \lambda_P^i(\tau)P^i(\tau, t^*_{k-1}) = -\int_{\tau}^{t'_k} \left[y^i_k(x) + p^i(x)\bar{W}^i_k(x) \right] P^i(x, t^*_{k-1})e^{-\rho(x-t^*_{k-1})}dx.$$
(C-6)

The transversality condition for $P^i(t'_k, t^*_{k-1})$ is

$$\lambda_{P}^{i}(t_{k}') = \frac{\partial \left(\mathbb{U}_{k,1}^{i}\left(t_{k}'\right) P^{i}\left(t_{k}', t_{k-1}^{*}\right) e^{-\rho(t_{k}' - t_{k-1}^{*})} \right)}{\partial P^{i}(t_{k}', t_{k-1}^{*})} = \mathbb{U}_{k,1}^{i}\left(t_{k}'\right) e^{-\rho(t_{k}' - t_{k-1}^{*})}$$

Inserting this in (C-6), rearranging and using the fact that $P^i(x, t_{k-1}^*)e^{-\rho(x-t_{k-1}^*)} = P^i(x, \tau)e^{-\rho(x-\tau)}P^i(\tau, t_{k-1}^*)e^{-\rho(\tau-t_{k-1}^*)}$ delivers by (C-1)

$$\lambda_{P}^{i}(\tau)e^{\rho(\tau-t_{k-1}^{*})} = \int_{\tau}^{t_{k}^{'}} \left[y_{k}^{i}(x) + p^{i}(x)\bar{W}_{k}^{i}(x) \right] P^{i}(x,\tau)e^{-\rho(x-\tau)}dx + \mathbb{U}_{k,1}^{i}(t_{k}^{'})P^{i}(t_{k}^{'},\tau)e^{-\rho(t_{k}^{'}-\tau)} \equiv U_{k,1}^{i}(\tau).$$
(C-7)

By multiplying both sides by $e^{-\rho(\tau-t_{k-1}^*)}$ and inserting this in (C-5) one obtains

$$H_{k,1}^{i}(\tau) = \left[y_{k}^{i}(\tau) + p^{i}(\tau)\left[\bar{W}_{k}^{i}(\tau) - U_{k,1}^{i}(\tau)\right]\right]P(\tau, t_{k-1}^{*})e^{-\rho(\tau - t_{k-1}^{*})} + \lambda_{S}^{i}(\tau)\frac{s^{i}(\tau) - S^{i}(\tau, t_{k-1}^{*})}{\tau - t_{k-1}^{*}}.$$

Defining the generalized current value of any variable x during the scheduled interval as

$$\tilde{x} \equiv x \cdot \exp\left\{\int_{t_{k-1}^*}^{\tau} \left(p^i(x) + \rho\right) dx\right\} = x \cdot \exp\left\{\rho(\tau - t_{k-1}^*)\right\} / P^i(\tau, t_{k-1}^*),$$
(C-8)

and using the definition (4) of $y_k^i(\tau)$, we can define the generalized current value Hamiltonian during the scheduled interval as:

$$\tilde{H}_{k,1}^{i}(\tau) = b_{k} + \nu - c\left[s^{i}(\tau)\right] + p^{i}(\tau)\left[\bar{W}_{k}^{i}(\tau) - U_{k,1}^{i}(\tau)\right] + \tilde{\lambda}_{S}^{i}(\tau)\frac{s^{i}(\tau) - \bar{S}^{i}(\tau, t_{k-1}^{*})}{\tau - t_{k-1}^{*}}$$

Using the fact that $\bar{W}_k^i(\tau) \equiv E\left[W_k(w;\tau)|w > w_r^i(\tau)\right] = \int_{w_r^i(\tau)}^{\infty} \frac{w+\delta U_{k,1}^i}{(\rho+\delta)\bar{F}[w_r^i(\tau)]} dw$, we can write $\tilde{H}_{k,1}^i(\tau)$ in a slightly more convenient form, namely

$$\begin{split} \tilde{H}_{k,1}^{i}\left(\tau\right) &= b_{k} + v - c\left[s^{i}\left(\tau\right)\right] \\ &+ \frac{s^{i}\left(\tau\right)}{\rho + \delta} \int_{w_{r}^{i}\left(\tau\right)}^{\infty} \left\{w - \rho U_{k,1}^{i}\left(\tau\right) + \delta\left[U_{k,1}^{1}\left(\tau\right) - U_{k,1}^{i}\left(\tau\right)\right]\right\} dF\left(w\right) + \tilde{\lambda}_{S}^{i}\left(\tau\right) \frac{s^{i}\left(\tau\right) - \bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)}{\tau - t_{k-1}^{*}}. \end{split}$$

One can easily see that the FOC for the control variables are not affected if one uses $\tilde{H}_{k,1}^{i}(\tau)$ rather than $H_{k,1}^{i}(\tau)$. The FOC of the state variables need, however, a slight modification. To see this, consider $\bar{S}^{i}(\tau, t_{k-1}^{*})$, written below $\bar{S}(\tau)$ for short. The FOC for this state variable in $\tilde{H}_{k,1}^{i}(\tau)$ is

$$\frac{\partial \dot{H}_{k,1}^{i}(\tau)}{\partial \bar{S}(\tau)} = \frac{\partial H_{k,1}^{i}(\tau)}{\partial \bar{S}(\tau)} \frac{\exp\left\{\rho(\tau - t_{k-1}^{*})\right\}}{P^{i}(\tau, t_{k-1}^{*})} = -\dot{\lambda}_{\bar{S}}^{i}(\tau) \exp\left\{\rho(\tau - t_{k-1}^{*})\right\} / P^{i}(\tau, t_{k-1}^{*}) \\
= \left[p^{i}(\tau) + \rho\right] \tilde{\lambda}_{\bar{S}(\tau)}^{i}(\tau) - \dot{\bar{\lambda}}_{\bar{S}(\tau)}^{i}(\tau) \tag{C-9}$$

where the second equality follows from the FOC for $\bar{S}(\tau)$ in $H^i_{k,1}(\tau)$ and the third equality from the relationship between $\dot{\tilde{\lambda}}_{\bar{S}(\tau)}(\tau)$ and $\dot{\lambda}^i_{\bar{S}(\tau)}(\tau)$. The transversality condition is modified as follows:

$$\tilde{\lambda}_{\bar{S}}^{i}(t_{k}') \equiv \lambda_{\bar{S}}^{i}(t_{k}') \frac{\exp\left\{\rho(t_{k}' - t_{k-1}^{*})\right\}}{P^{i}(t_{k}', t_{k-1}^{*})} = \frac{\partial \mathbb{U}_{k,1}^{i}(t_{k}')}{\partial \bar{S}(t_{k}', t_{k-1}^{*})} \tag{C-10}$$

where the second equality follows from the transversality condition for $\lambda_{\bar{s}}^{i}(t'_{k})$.

C.1.2 First-Order Conditions

• Control variables

Considering search effort $s(\tau)$:

$$\frac{\partial \tilde{H}_{k,1}^{i}(\tau)}{\partial s^{i}(\tau)} = -c'\left[s^{i}(\tau)\right] + \frac{1}{\rho+\delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left\{w - \rho U_{k,1}^{i}(\tau) + \delta\left[U_{k,1}^{1}(\tau) - U_{k,1}^{i}(\tau)\right]\right\} dF(w) + \frac{\tilde{\lambda}_{S}^{i}(\tau)}{\tau - t_{k-1}^{*}} = 0$$

$$\Leftrightarrow \quad c'\left[s^{i}\left(\tau\right)\right] = \frac{1}{\rho+\delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left\{w - \rho U_{k,1}^{i}\left(\tau\right) + \delta\left[U_{k,1}^{1}\left(\tau\right) - U_{k,1}^{i}\left(\tau\right)\right]\right\} dF\left(w\right) + \frac{\tilde{\lambda}_{S}^{i}\left(\tau\right)}{\tau - t_{k-1}^{*}}.$$
 (C-11)

Considering reservation wage $w_r(\tau)$:

$$\frac{\partial H_{k,1}^{i}(\tau)}{\partial w_{r}^{i}(\tau)} = -\frac{s^{i}(\tau)}{\rho+\delta} \left\{ w_{r}^{i}(\tau) - \rho U_{k,1}^{i}(\tau) + \delta \left[U_{k,1}^{1}(\tau) - U_{k,1}^{i}(\tau) \right] \right\} f\left(w_{r}^{i}(\tau) \right) = 0$$

which leads to (B-1) (except that subscript k in the last expression between braces becomes $\{k, 1\}$ because we explicitly consider the scheduled interval here; Similar adjustments have to be implemented below each time we refer to (B-1)).

• State variables

For average search effort $\bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)$:

$$\begin{aligned} \frac{\partial \ddot{H}_{k,1}^{i}\left(\tau\right)}{\partial \bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)} &= -\frac{\tilde{\lambda}_{S}^{i}\left(\tau\right)}{\tau - t_{k-1}^{*}} = \left[p^{i}\left(\tau\right) + \rho\right] \tilde{\lambda}_{S}^{i}\left(\tau\right) - \dot{\tilde{\lambda}}_{S}^{i}\left(\tau\right) \\ \Leftrightarrow \quad \frac{\dot{\tilde{\lambda}}_{S}^{i}\left(\tau\right)}{\tilde{\lambda}_{S}^{i}\left(\tau\right)} &= p^{i}\left(\tau\right) + \rho + \frac{1}{\tau - t_{k-1}^{*}}. \end{aligned}$$

Acknowledging that $\dot{\tilde{\lambda}}_{S}^{i}(\tau)/\tilde{\lambda}_{S}^{i}(\tau) = \frac{\partial}{\partial \tau} \ln \tilde{\lambda}_{S}^{i}(\tau), \quad p^{i}(\tau) = -\frac{\dot{P}^{i}(\tau,t_{k-1}^{*})}{P^{i}(\tau,t_{k-1}^{*})} = -\frac{\partial}{\partial \tau} \ln P^{i}(\tau,t_{k-1}^{*})$ and $\frac{1}{\tau-t_{k-1}^{*}} = \frac{\partial}{\partial \tau} \ln (\tau - t_{k-1}^{*}),$ the last result can be written as

$$\frac{\partial}{\partial \tau} \ln \tilde{\lambda}_{S}^{i}(\tau) = -\frac{\partial}{\partial \tau} \ln P^{i}(\tau, t_{k-1}^{*}) + \rho + \frac{\partial}{\partial \tau} \ln(\tau - t_{k-1}^{*}).$$

Integrating from τ to t_k' and rearranging

$$\ln \frac{\tilde{\lambda}_{S}^{i}(t_{k}')}{\tilde{\lambda}_{S}^{i}(\tau)} = -\ln P^{i}(t_{k}',\tau) + \rho(t_{k}'-\tau) + \ln \frac{t_{k}'-t_{k-1}^{*}}{\tau-t_{k-1}^{*}}$$

$$\Leftrightarrow \quad \frac{\tilde{\lambda}_{S}^{i}(t_{k}')}{\tilde{\lambda}_{S}^{i}(\tau)} = \frac{t_{k}'-t_{k-1}^{*}}{\tau-t_{k-1}^{*}} \frac{e^{\rho(t_{k}'-\tau)}}{P^{i}(t_{k}',\tau)}.$$

Applying the transversality condition

$$\tilde{\lambda}_{S}^{i}\left(t_{k}^{\prime}\right) = \frac{\partial \mathbb{U}_{k,1}^{i}\left(t_{k}^{\prime}\right)}{\partial \bar{S}^{i}\left(t_{k}^{\prime}, t_{k-1}^{*}\right)} = \frac{\partial U_{k,2}^{i}\left(t_{k}^{\prime}\right)}{\partial \bar{S}^{i}\left(t_{k}^{\prime}, t_{k-1}^{*}\right)}$$

we finally see that

$$\tilde{\lambda}_{S}^{i}(\tau) = \frac{\tau - t_{k-1}^{*}}{t_{k}^{\prime} - t_{k-1}^{*}} \frac{\partial U_{k,2}^{i}(t_{k}^{\prime})}{\partial \bar{S}^{i}(t_{k}^{\prime}, t_{k-1}^{*})} P^{i}(t_{k}^{\prime}, \tau) e^{-\rho(t_{k}^{\prime} - \tau)}.$$
(C-12)

Inserting this result, together with (B-1), into (C-11) we obtain first order condition for search effort:

$$c'\left[s^{i}\left(\tau\right)\right] = \frac{1}{\rho+\delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left[w - w_{r}^{i}\left(\tau\right)\right] dF\left(w\right) + \frac{1}{t'_{k} - t^{*}_{k-1}} \frac{\partial U^{i}_{k,2}\left(t'_{k}\right)}{\partial \bar{S}^{i}\left(t'_{k}, t^{*}_{k-1}\right)} P^{i}\left(t'_{k}, \tau\right) e^{-\rho(t'_{k} - \tau)}$$
(C-13)

Using definition (1) in the paper (adding superscript *i*), we can write $\bar{S}^i(\tau, t_{k-1}^*) = \bar{S}^i(t'_k, t_{k-1}^*) \frac{t'_k - t_{k-1}^*}{\tau - t_{k-1}^*} + \bar{S}^i(\tau, t'_k) \frac{\tau - t'_k}{\tau - t_{k-1}^*}$. Substituting this expression and (C-4) into (C-3) and partially differentiating the latter equation with respect to $\bar{S}^i(t'_k, t_{k-1}^*)$ yields

$$\frac{1}{t'_{k} - t^{*}_{k-1}} \frac{\partial U^{i}_{k,2}\left(t'_{k}\right)}{\partial \bar{S}^{i}\left(t'_{k}, t^{*}_{k-1}\right)} = \int_{t'_{k}}^{\bar{t}^{*}_{k}} q \frac{\partial \pi^{i}_{k}\left[\bar{S}^{i}\left(x, t^{*}_{k-1}\right)\right]}{\partial \bar{S}^{i}\left(x, t^{*}_{k-1}\right)} \frac{\left[U^{i}_{k+1,1}\left(t^{*}_{k}\right) - U^{+}\right]}{x - t^{*}_{k-1}} P^{i}\left(x, t'_{k}\right) e^{-\left[\rho + q\right]\left(x - t'_{k}\right)} dx + \frac{\partial \pi^{i}_{k}\left[\bar{S}^{i}\left(\bar{t}^{*}_{k}, t^{*}_{k-1}\right)\right]}{\partial \bar{S}^{i}\left(\bar{t}^{*}_{k}, t^{*}_{k-1}\right)} \frac{\left[U^{i}_{k+1,1}\left(t^{*}_{k}\right) - U^{+}\right]}{\bar{t}^{*}_{k} - t^{*}_{k-1}} P^{i}\left(\bar{t}^{*}_{k}, t'_{k}\right) e^{-\left[\rho + q\right]\left(\bar{t}^{*}_{k} - t'_{k}\right)}. \quad (C-14)$$

This expression can then be substituted in (C-13) to obtain the FOC of job search effort. In the simpler problem considered in Section 4.3, we ignored that (i) individuals who return to unemployment after an interruption are treated differently when their job search effort is evaluated and also that (ii) the evaluation could be randomly delayed. This means that in that section we could in (C-13) (i) ignore the superscript *i* and (ii) replace t_{k-1}^* and t'_k by t_{k-1} and t_k . Moreover, instead of substituting $\frac{1}{t'_k - t^*_{k-1}} \frac{\partial U^i_{k,2}(t'_k)}{\partial \bar{S}^i(t'_k, t^*_{k-1})}$ by (C-14), we now replace $\frac{\partial U_k(t_k)}{\partial \bar{S}(t_k, t_{k-1})}$ by $\pi'_k \left[\bar{S}(t_k, t_{k-1})\right] [U_{k+1}(t_k) - U^+]$, where the last expression follows from (8). If we implement these changes and normalize $\lambda = 1$, then (C-13) corresponds to the FOC of search effort (11) in the main text. Because of the existence of delays, Eq. (C-14) shows that the effect of search effort on lifetime utility is no longer evaluated at the end of the considered sub-period t_k , but rather at some random instant on the delay interval. The integral on the right-hand side of (C-14) takes the expectation of this impact over the random timing of the assessment over the delay interval $[t'_k, \bar{t}^*_k]$, while the last term is the expected impact at the maximum delay. This clearly demonstrates that the delay reduces the incentives of the monitoring scheme.

C.1.3 The evolution of the optimal controls and the lifetime utility in the scheduled interval.

We now show that differentiating the FOC and the value of unemployment expressed in (C-1) with respect to time yields a system of differential equations that describes the evolution of the optimal controls and the lifetime utility in the scheduled interval. This system can be solved backwards from the endpoint conditions for all the paths at t'_k . Endpoint conditions for reservation wage and search effort are found by solving the system of FOC (B-1), in which (again) subscript k has to be replaced by $\{k, 1\}$ since we look at consider here the scheduled interval, and (C-13). By differentiating these two equations with respect to time we obtain the optimal paths of the control variables within the scheduled interval. The optimal path for the reservation wage is

$$\dot{w}_{r}^{i}(\tau) = \rho \dot{U}_{k,1}^{i}(\tau) - \delta [\dot{U}_{k,1}^{1}(\tau) - \dot{U}_{k,1}^{i}(\tau)].$$
(C-15)

and for the search effort is likewise

$$\begin{split} \dot{s}^{i}\left(\tau\right) &= \frac{1}{\left(\rho + \delta\right)c''\left[s^{i}\left(\tau\right)\right]} \int_{w_{r}^{i}(\tau)}^{\infty} \left[-\dot{w}_{r}^{i}\left(\tau\right)\right] dF\left(w\right) \\ &+ \frac{p^{i}\left(\tau\right) + \rho}{c''\left[s^{i}\left(\tau\right)\right]} \frac{1}{t'_{k} - t^{*}_{k-1}} \frac{\partial U^{i}_{k,2}\left(t'_{k}\right)}{\partial \bar{S}^{i}\left(t'_{k}, t^{*}_{k-1}\right)} P^{i}(t'_{k}, \tau) e^{-\rho(t'_{k} - \tau)} \end{split}$$

Once i = 1, the last term drops out, since accumulated search effort $\bar{S}^1(t'_k, t^*_{k-1})$ does not affect the probability of a negative evaluation during the delay interval. Otherwise, when i = 0, the term containing the derivative multiplied by the expected discounted capital loss due to negative evaluation can be substituted out using eq.(C-13). Thus, optimal path for search effort reads for $i \in \{0, 1\}$:

$$\dot{s}^{i}(\tau) = -\frac{\dot{w}_{r}^{i}(\tau)\bar{F}\left(w_{r}^{i}(\tau)\right)}{(\rho+\delta)c''[s^{i}(\tau)]} + (1-i)\frac{p^{i}(\tau)+\rho}{c''[s^{i}(\tau)]}\left[c'\left[s^{i}(\tau)\right] - \frac{1}{\rho+\delta}\int_{w_{r}^{i}(\tau)}^{\infty}\left[w-w_{r}^{i}(\tau)\right]dF\left(w\right)\right].$$
 (C-16)

Both paths depend on the evolution of the value of unemployment: $\dot{w}_r^i(\tau)$ explicitly and $\dot{s}^i(\tau)$ implicitly via $\dot{w}_r^i(\tau)$. Evolution of the value of unemployment can be obtained simply by differentiating (C-1) at any $\tau \in [t_{k-1}^*, t_k')$. We get

$$\begin{split} \dot{U}_{k,1}^{i}\left(\tau\right) &= \int_{\tau}^{t'_{k}} \left[y_{k}^{i}\left(x\right) + p^{i}\left(x\right)\bar{W}_{k}^{i}\left(x\right)\right] \frac{\partial}{\partial\tau} \left(P^{i}\left(x,\tau\right)e^{-\rho\left(x-\tau\right)}\right) dx - \left[y_{k}^{i}\left(\tau\right) + p^{i}\left(\tau\right)\bar{W}_{k}^{i}\left(\tau\right)\right] \\ &+ \mathbb{U}_{k,1}^{i}\left(t'_{k}\right) \frac{\partial}{\partial\tau} \left(P^{i}\left(t'_{k},\tau\right)e^{-\rho\left(t'_{k}-\tau\right)}\right) \\ &= \rho U_{k,1}^{i}\left(\tau\right) - y_{k}^{i}\left(\tau\right) - p^{i}\left(\tau\right) \left[\bar{W}_{k}^{i}\left(\tau\right) - U_{k,1}^{i}\left(\tau\right)\right]. \end{split}$$

Inserting into the last expression eq. (B-1), which tells us here that $U_{k,1}^{i}(\tau) = \frac{1}{\rho+\delta} [w_{r}^{i}(\tau) + \delta U_{k,1}^{1}(\tau)]$, and the expression for $\bar{W}_{k}^{i}(\tau)$ we finally get

$$\dot{U}_{k,1}^{i}(\tau) = \frac{\rho}{\rho+\delta} w_{r}^{i}(\tau) - y_{k}^{i}(\tau) - \frac{s^{i}(\tau)}{\rho+\delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left[w - w_{r}^{i}(\tau) \right] dF(w) + \frac{\rho\delta}{\rho+\delta} U_{k,1}^{1}(\tau) \,. \tag{C-17}$$

Differential equations (C-15), (C-16) and (C-17) form a system that describes the evolution of optimal controls and the lifetime utility in the scheduled interval. This system can be solved backwards from the endpoint conditions for all the paths at t'_k . Endpoint condition for the utility function at t'_k is given by $U^i_{k,1}(t'_k) = U^i_{k,2}(t'_k)$. Endpoint conditions for reservation wage and search effort are found by solving the system of FOC (B-1) and (C-13) evaluated at t'_k .

$$\begin{cases} w_{r}^{i}\left(t_{k}'\right) = \rho U_{k,2}^{i}\left(t_{k}'\right) - \delta \left[U_{k,2}^{1}\left(t_{k}'\right) - U_{k,2}^{i}\left(t_{k}'\right)\right] \\ c'\left[s^{i}\left(t_{k}'\right)\right] = \frac{1}{\rho + \delta} \int_{w_{r}^{i}\left(t_{k}'\right)}^{\infty} \left[w - w_{r}^{i}\left(t_{k}'\right)\right] dF\left(w\right) + \frac{1}{t_{k}' - t_{k-1}^{*}} \frac{\partial U_{k,2}^{i}\left(t_{k}'\right)}{\partial \bar{S}^{i}\left(t_{k}', t_{k-1}^{*}\right)} \end{cases}$$

for $\left\{w_{r}^{i}\left(t_{k}'\right),s^{i}\left(t_{k}'\right)\right\}$, where we have already invoked that $U_{k,1}^{i}\left(t_{k}'\right)=U_{k,2}^{i}\left(t_{k}'\right)$.

C.2 Delay Interval

C.2.1 The Generalized Current Value Hamiltonian

Referring to (C-3) and (C-4), the definition of the generalized current value of any variable x is now:

$$\tilde{x} \equiv x \cdot \exp\left\{\int_{t'_k}^{\tau} \left(p^i(x) + \rho + q\right) dx\right\} = x \cdot \exp\left\{\left(\rho + q\right)\left(\tau - t'_k\right)\right\} / P^i(\tau, t'_k),\tag{C-18}$$

Following the same steps as in the scheduled interval, the generalized current value Hamiltonian in the delay period reads

$$\tilde{H}_{k,2}^{i}(\tau) = b_{k} + \nu - c\left[s^{i}(\tau)\right] + p^{i}(\tau)\left[\bar{W}_{k}^{i}(\tau) - U_{k,2}^{i}(\tau)\right] + q\mathbb{U}_{k,2}^{i}(\tau) + \tilde{\lambda}_{S}^{i}(\tau)\frac{s^{i}(\tau) - \bar{S}^{i}(\tau, t_{k-1}^{*})}{\tau - t_{k-1}^{*}}.$$

Using the definition of $\bar{W}_k^i(\tau)$, we can write $\tilde{H}_{k,2}^i(\tau)$ in a slightly more convenient form, namely

$$\begin{split} \tilde{H}_{k,2}^{i}\left(\tau\right) &= b_{k} + \nu - c\left[s^{i}\left(\tau\right)\right] + q\mathbb{U}_{k,2}^{i}\left(\tau\right) \\ &+ \frac{s^{i}\left(\tau\right)}{\rho + \delta} \int_{w_{r}^{i}\left(\tau\right)}^{\infty} \left\{w - \rho U_{k,2}^{i}\left(\tau\right) + \delta\left[U_{k,2}^{1}\left(\tau\right) - U_{k,2}^{i}\left(\tau\right)\right]\right\} dF\left(w\right) + \tilde{\lambda}_{S}^{i}\left(\tau\right) \frac{s^{i}\left(\tau\right) - \bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)}{\tau - t_{k-1}^{*}}. \end{split}$$

C.2.2 First-order conditions

• Control Variables

FOC for control variables repeat the derivations in C.1.1 leading us to (B-1), in which from now on subscript $\{k, 2\}$ replaces k since we look at the delay interval, and to

$$c'\left[s^{i}\left(\tau\right)\right] = \frac{1}{\rho+\delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left\{w - \rho U_{k,2}^{i}\left(\tau\right) + \delta\left[U_{k,2}^{1}\left(\tau\right) - U_{k,2}^{i}\left(\tau\right)\right]\right\} dF\left(w\right) + \frac{\tilde{\lambda}_{S}^{i}\left(\tau\right)}{\tau - t_{k-1}^{*}}.$$
 (C-19)

• State variables

For average search effort $\bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)$:

$$\begin{aligned} \frac{\partial \tilde{H}_{k,2}^{i}(\tau)}{\partial \bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)} &= q \frac{\partial \pi_{k}^{i}\left[\bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)\right]}{\partial \bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)} \left[U_{k+1,1}^{i}\left(t_{k}^{*}\right) - U^{+}\right] - \frac{\tilde{\lambda}_{S}^{i}\left(\tau\right)}{\tau - t_{k-1}^{*}} &= \left[p^{i}\left(\tau\right) + \rho + q\right]\tilde{\lambda}_{S}^{i}\left(\tau\right) - \dot{\tilde{\lambda}}_{S}^{i}\left(\tau\right) \\ \Leftrightarrow \quad \dot{\tilde{\lambda}}_{S}^{i}\left(\tau\right) - \left(p^{i}\left(\tau\right) + \rho + q + \frac{1}{\tau - t_{k-1}^{*}}\right)\tilde{\lambda}_{S}^{i}\left(\tau\right) &= -q \frac{\partial \pi_{k}^{i}\left[\bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)\right]}{\partial \bar{S}^{i}\left(\tau, t_{k-1}^{*}\right)} \left[U_{k+1,1}^{i}\left(t_{k}^{*}\right) - U^{+}\right] \end{aligned}$$

Multiplying both sides with $P^i(\tau, t'_k) \exp\{-\int_{t'_k}^{\tau} [\rho + q + 1/(x - t^*_{k-1})]dx\}$ we recognize that

$$\begin{split} \dot{\tilde{\lambda}}_{S}^{i}\left(\tau\right)P^{i}\left(\tau,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\tau}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx} \\ &-\left(p^{i}\left(\tau\right)+\rho+q+\frac{1}{\tau-t_{k-1}^{*}}\right)\tilde{\lambda}_{S}^{i}\left(\tau\right)P^{i}\left(\tau,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\tau}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx} \\ &=-q\frac{\partial\pi_{k}^{i}\left[\bar{S}^{i}\left(\tau,t_{k-1}^{*}\right)\right]}{\partial\bar{S}^{i}\left(\tau,t_{k-1}^{*}\right)}\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right)-U^{+}\right]P^{i}\left(\tau,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\tau}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx} \\ \Leftrightarrow \quad \frac{\partial}{\partial\tau}\left(\tilde{\lambda}_{S}^{i}\left(\tau\right)P^{i}\left(\tau,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\tau}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx}\right) \\ &=-q\frac{\partial\pi_{k}^{i}\left[\bar{S}^{i}\left(\tau,t_{k-1}^{*}\right)\right]}{\partial\bar{S}^{i}\left(\tau,t_{k-1}^{*}\right)}\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right)-U^{+}\right]P^{i}\left(\tau,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\tau}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx}. \end{split}$$

Integrating the last result from τ to \bar{t}^*_k we see that

$$\begin{split} \tilde{\lambda}_{S}^{i}\left(\bar{t}_{k}^{*}\right)P^{i}\left(\bar{t}_{k}^{*},t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\bar{t}_{k}^{*}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx}} - \tilde{\lambda}_{S}^{i}\left(\tau\right)P^{i}\left(\tau,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{\tau}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx} \\ &= -\int_{\tau}^{\bar{t}_{k}^{*}}q\frac{\partial\pi_{k}^{i}\left[\bar{S}^{i}\left(x,t_{k-1}^{*}\right)\right]}{\partial\bar{S}^{i}\left(x,t_{k-1}^{*}\right)}\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right)-U^{+}\right]P^{i}\left(x,t_{k}^{\prime}\right)e^{-\int_{t_{k}^{\prime}}^{x}\left[\rho+q+1/(x-t_{k-1}^{*})\right]dx}dx. \end{split}$$

Applying the transversality condition

$$\tilde{\lambda}_{S}^{i}(\bar{t}_{k}^{*}) = \frac{\partial \mathbb{U}_{k,2}^{i}(\bar{t}_{k}^{*})}{\partial \bar{S}^{i}(\bar{t}_{k}^{*}, t_{k-1}^{*})} = \frac{\partial \pi_{k}^{i}\left[\bar{S}^{i}(\bar{t}_{k}^{*}, t_{k-1}^{*})\right]}{\partial \bar{S}^{i}(\bar{t}_{k}^{*}, t_{k-1}^{*})} \left[U_{k+1,1}^{i}(t_{k}^{*}) - U^{+}\right]$$

and rearranging further shows that

$$\begin{split} \tilde{\lambda}_{S}^{i}(\tau) &= \int_{\tau}^{\bar{t}_{k}^{*}} q \frac{\partial \pi_{k}^{i} \left[\bar{S}^{i} \left(x, t_{k-1}^{*} \right) \right]}{\partial \bar{S}^{i} \left(x, t_{k-1}^{*} \right)} \left[U_{k+1,1}^{i} \left(t_{k}^{*} \right) - U^{+} \right] P^{i} \left(x, \tau \right) e^{-\int_{\tau}^{x} \left[\rho + q + 1/(z - t_{k-1}^{*}) \right] dz} dx \\ &+ \frac{\partial \pi_{k}^{i} \left[\bar{S}^{i} \left(\bar{t}_{k}^{*}, t_{k-1}^{*} \right) \right]}{\partial \bar{S}^{i} \left(\bar{t}_{k}^{*}, t_{k-1}^{*} \right)} \left[U_{k+1,1}^{i} \left(t_{k}^{*} \right) - U^{+} \right] P^{i} \left(\bar{t}_{k}^{*}, \tau \right) e^{-\int_{\tau}^{\bar{t}_{k}^{*}} \left[\rho + q + 1/(x - t_{k-1}^{*}) \right] dx} \\ \Leftrightarrow \quad \frac{\tilde{\lambda}_{S}^{i} \left(\tau \right)}{\tau - t_{k-1}^{*}} = \int_{\tau}^{\bar{t}_{k}^{*}} q \frac{\partial \pi_{k}^{i} \left[\bar{S}^{i} \left(x, t_{k-1}^{*} \right) \right]}{\partial \bar{S}^{i} \left(x, t_{k-1}^{*} \right)} \frac{\left[U_{k+1,1}^{i} \left(t_{k}^{*} \right) - U^{+} \right]}{x - t_{k-1}^{*}} P^{i} \left(x, \tau \right) e^{-\left[\rho + q \right] \left(x - \tau \right)} dx \\ &+ \frac{\partial \pi_{k}^{i} \left[\bar{S}^{i} \left(\bar{t}_{k}^{*}, t_{k-1}^{*} \right) \right]}{\partial \bar{S}^{i} \left(\bar{t}_{k}^{*}, t_{k-1}^{*} \right)} \frac{\left[U_{k+1,1}^{i} \left(t_{k}^{*} \right) - U^{+} \right]}{\bar{t}_{k}^{*} - t_{k-1}^{*}} P^{i} \left(\bar{t}_{k}^{*}, \tau \right) e^{-\left[\rho + q \right] \left(\bar{t}_{k}^{*} - \tau \right)}. \end{split}$$

Inserting this result, together with (B-1), into (C-19) leads to:

$$\begin{aligned} c'\left[s^{i}\left(\tau\right)\right] &= \frac{1}{\rho+\delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left\{w - w_{r}^{i}(\tau)\right\} dF\left(w\right) \\ &+ \int_{\tau}^{\bar{t}_{k}^{*}} q \frac{\partial \pi_{k}^{i}\left[\bar{S}^{i}\left(x, t_{k-1}^{*}\right)\right]}{\partial \bar{S}^{i}\left(x, t_{k-1}^{*}\right)} \frac{\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right) - U^{+}\right]}{x - t_{k-1}^{*}} P^{i}\left(x, \tau\right) e^{-\left[\rho+q\right]\left(x-\tau\right)} dx \\ &+ \frac{\partial \pi_{k}^{i}\left[\bar{S}^{i}\left(\bar{t}_{k}^{*}, t_{k-1}^{*}\right)\right]}{\partial \bar{S}^{i}\left(\bar{t}_{k}^{*}, t_{k-1}^{*}\right)} \frac{\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right) - U^{+}\right]}{\bar{t}_{k}^{*} - t_{k-1}^{*}} P^{i}\left(\bar{t}_{k}^{*}, \tau\right) e^{-\left[\rho+q\right]\left(\bar{t}_{k}^{*}-\tau\right)}. \end{aligned}$$
(C-20)

The interpretation is similar as during the delay interval.

C.2.3 The evolution of the optimal controls and the lifetime utility in the delay interval.

Differentiating equations (B-1) and (C-20) with respect to time we get the optimal paths of the control variables during the interview delay. The optimal path for the reservation wage is

$$\dot{w}_{r}^{i}(\tau) = \rho \dot{U}_{k,2}^{i}(\tau) - \delta \left[\dot{U}_{k,2}^{1}(\tau) - \dot{U}_{k,2}^{i}(\tau) \right].$$
(C-21)

The optimal path for search effort is likewise the time-derivative of (C-20)

$$\begin{split} c''\left[s^{i}\left(\tau\right)\right]\dot{s}^{i}\left(\tau\right) &= -\frac{\dot{w}_{r}^{i}\left(\tau\right)\bar{F}\left(w_{r}^{i}\left(\tau\right)\right)}{\rho+\delta} - q\frac{\left(\pi_{k}^{i}\left[\bar{S}^{i}\left(\tau,t_{k-1}^{*}\right)\right]\right)'}{\tau-t_{k-1}^{*}}\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right)-U^{+}\right]\right.\\ &+ \left(p^{i}\left(\tau\right)+\rho+q\right)\left[\int_{\tau}^{\bar{t}_{k}^{*}}q\frac{\left(\pi_{k}^{i}\left[\bar{S}^{i}\left(x,t_{k-1}^{*}\right)\right]\right)'}{x-t_{k-1}^{*}}\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right)-U^{+}\right]P^{i}\left(x,\tau\right)e^{-\left[\rho+q\right]\left(x-\tau\right)}dx\right.\\ &+ \frac{\left(\pi_{k}^{i}\left[\bar{S}^{i}\left(\bar{t}_{k}^{*},t_{k-1}^{*}\right)\right]\right)'}{\bar{t}_{k}^{*}-t_{k-1}^{*}}\left[U_{k+1,1}^{i}\left(t_{k}^{*}\right)-U^{+}\right]P^{i}\left(\bar{t}_{k}^{*},\tau\right)e^{-\left[\rho+q\right]\left(\bar{t}_{k}^{*}-\tau\right)}\right].\end{split}$$

Once i = 1, all terms but the first drop out, since $\forall \tau : (\pi_k^1 [\bar{S}^i(\tau, t_{k-1}^*)])' = 0$ on the delay interval. Otherwise, when i = 0, the last term in the square brackets can be substituted out using eq. (C-20). With this substitution, optimal path for search effort reads

$$\dot{s}^{i}(\tau) = -\frac{\dot{w}_{r}^{i}(\tau)\bar{F}\left(w_{r}^{i}(\tau)\right)}{(\rho+\delta)c''\left[s^{i}\left(\tau\right)\right]} - \frac{q}{c''\left[s^{i}\left(\tau\right)\right]} \frac{\left(\pi_{k}^{i}\left[\bar{S}^{i}\left(\tau,t_{k-1}^{*}\right)\right]\right)'}{\tau-t_{k-1}^{*}} \left[U_{k+1,1}^{i}\left(t_{k}^{*}\right) - U^{+}\right] + (1-i)\frac{p^{i}\left(\tau\right) + \rho + q}{c''\left[s^{i}\left(\tau\right)\right]} \left[c'\left[s^{i}\left(\tau\right)\right] - \frac{1}{\rho+\delta}\int_{w_{r}^{i}(\tau)}^{\infty} \left\{w - w_{r}^{i}\left(\tau\right)\right\}dF\left(w\right)\right]. \quad (C-22)$$

Finally note that with $q \to 0$, i.e. when the end of the delay interval is deterministic meaning that delay interval becomes just another scheduled interval, the path in (C-22) reduces to the path in (C-16).

Like before, optimal paths for both control variables depend on the evolution of the value of unemployment. The evolution of the value of unemployment can be obtained by differentiating (C-3) at any $\tau \in [t'_k, \bar{t}^*_k)$. We get

$$\begin{split} \dot{U}_{k,2}^{i}\left(\tau\right) &= \int_{\tau}^{t_{k}^{*}} \left[y_{k}^{i}\left(x\right) + p^{i}\left(x\right)\bar{W}_{k}^{i}\left(x\right) + q\mathbb{U}_{k,2}^{i}\left(x\right)\right]\frac{\partial}{\partial\tau}\left(P^{i}\left(x,\tau\right)e^{-\left[\rho+q\right]\left(x-\tau\right)}\right)dx \\ &= \left(\rho+q\right)U_{k,2}^{i}\left(\tau\right) - y_{k}^{i}\left(\tau\right) - p^{i}\left(\tau\right)\left[\bar{W}_{k}^{i}\left(\tau\right) - U_{k,2}^{i}\left(\tau\right)\right] - q\mathbb{U}_{k,2}^{i}\left(\tau\right). \end{split}$$

Inserting into the last expression eq. (B-1), which tells us that $U_{k,2}^{i}(\tau) = \frac{1}{\rho+\delta} \left[w_{r}^{i}(\tau) + \delta U_{k,2}^{1}(\tau) \right]$, and the expression for $\bar{W}_k^i(\tau)$ we finally get

$$\dot{U}_{k,2}^{i}(\tau) = \frac{\rho + q}{\rho + \delta} w_{r}^{i}(\tau) - y_{k}^{i}(\tau) - \frac{s^{i}(\tau)}{\rho + \delta} \int_{w_{r}^{i}(\tau)}^{\infty} \left[w - w_{r}^{i}(\tau) \right] dF(w) + \delta \frac{\rho + q}{\rho + \delta} U_{k,2}^{1}(\tau) - q \mathbb{U}_{k,2}^{i}(\tau) . \quad (C-23)$$

Differential equations (C-21), (C-22) and (C-23) form a system that describes evolution of the optimal controls and the lifetime utility in the delay interval. This system can be solved backwards from the endpoint conditions for all the paths at \bar{t}_k^* . Endpoint conditions are found by solving the system of FOC (B-1) and (C-20) together with eq.(C-3), all evaluated at \bar{t}_k^* :

$$\begin{cases} w_r^i\left(\bar{t}_k^*\right) = \rho U_{k,2}^i\left(\bar{t}_k^*\right) - \delta \left[U_{k,2}^1\left(\bar{t}_k^*\right) - U_{k,2}^i\left(\bar{t}_k^*\right)\right] \\ c'\left[s^i\left(\bar{t}_k^*\right)\right] = \frac{1}{\rho + \delta} \int_{w_r^i\left(\bar{t}_k^*\right)}^{\infty} \left[w - w_r^i\left(\bar{t}_k^*\right)\right] dF\left(w\right) + \frac{\left(\pi_k^i\left[\bar{S}^i\left(\bar{t}_k^*, t_{k-1}^*\right)\right]\right)'}{t_k^* - t_{k-1}^*} \left[U_{k+1,1}^i\left(t_k^*\right) - U^+\right] \\ U_{k,2}^i\left(\bar{t}_k^*\right) = \pi_k^i \left[\bar{S}^i\left(\bar{t}_k^*, t_{k-1}^*\right)\right] U_{k+1,1}^i\left(t_k^*\right) + \left(1 - \pi_k^i \left[\bar{S}^i\left(\bar{t}_k^*, t_{k-1}^*\right)\right]\right) U^+ \end{cases}$$

for $\left\{w_r^i(\bar{t}_k^*), s^i(\bar{t}_k^*), U_{k,2}^i(\bar{t}_k^*)\right\}$, where for after the last interview $(k=3) U_{4,1}^i(t_k^*) \equiv U^-$.

Identification D

Consider the FOC (10) and (11) of the reservation wage and job search effort. Let us first focus on individuals who are notified, did not interrupt their unemployment spell, are not yet evaluated, and are in the scheduled interval. These individuals are entitled to an UB that is equal to b_h . To stress that we are considering a group of individuals that is homogeneous in b_h , x and u we drop in the notation the explicit dependence on these: $c_0 \equiv e^{\mathbf{x}'\zeta_{\varepsilon}+u}$, $\nu \equiv \nu(x)$, $\delta \equiv \delta(x)$, and $\bar{S}(t_1, t_0) \equiv \bar{S}^0(t_1, t_0; \mathbf{x}; u)$. If we insert the specifications chosen in Section 5.1 into these FOC and use $s(\tau) = \tilde{s}(\tau)/\lambda$, we obtain:

$$w_{r}(\tau) + c_{0} \left(e^{\frac{\varepsilon}{\lambda}\tilde{s}(\tau)} - 1 \right) = b_{h} + \nu + p(\tau) \frac{E\left[w - w_{r}(\tau) | w > w_{r}(\tau) \right]}{\rho + \delta} + \dot{U}_{k}(\tau)$$

$$c_{0} \frac{\varepsilon}{\lambda} e^{\frac{\varepsilon}{\lambda}\tilde{s}(\tau)} = \frac{\bar{F}\left[w_{r}(\tau) \right] E\left[w - w_{r}(\tau) | w > w_{r}(\tau) \right]}{\rho + \delta} - \frac{\beta_{1}}{\lambda} \frac{e^{-\alpha_{1} - \frac{\beta_{1}}{\lambda}}\tilde{S}(t_{1}, t_{0})}{t_{1} - t_{0}} \left[U_{2}(t_{1}) - U^{+} \right] P(t_{1}, \tau) e^{-\rho(t_{1} - \tau)}.$$

in which $\tilde{\tilde{S}}(t_1, t_0) = \int_{t_0}^{t_1} \tilde{s}(\tau) d\tau / (t_1 - t_0)$. The following in these FOC is either known or can be identified from the observed data: (i) $w_r(\tau)$, $\bar{F}[w_r(\tau)]$ and $E[w-w_r(\tau)|w>w_r(\tau)]$ can, with the functional forms assumed in Section 5.1, be identified from the observed distribution of accepted wages; (ii) $p(\tau)$, $P(t_1, \tau)$, $\tilde{s}(\tau)$ and $\bar{S}(t_1, t_0)$ can be identified from the observed transitions to employment (given that $\bar{F}[w_r(\tau)]$ is known); (iii) $[U_2(t_1) - U^+]$ and $\dot{U}_k(\tau)$ are identified if the parameters in all future monitoring stages are identified,

which is the case if one proceeds by backward induction; (iv) α_1 and β_1/λ can be identified from the observed fractions of negative evaluations at the first interview and from $\tilde{S}(t_1, t_0)$; (v) δ can be identified from the observed transitions from employment back to unemployment; (vi) b_h , t_0 and t_1 are observed in the data.

This leaves us with the following unknown parameters: c_0 , ε , λ , ν , β_1 and ρ . Clearly, with two FOC we can identify at most two unknowns. However, if we consider another monitoring regime, e.g. individuals who have been evaluated negatively at the first assessment, this provides us with an additional set of independent FOC, because (i) behavior changes (the distribution of accepted wages and the transition rate to employment is affected), (ii) expectations about the future change (which affects the terms $[U_2(t_1) - U^+]$ and $\dot{U}_k(\tau)$), (iii) the probability of negative evaluation no longer matters in case of a positive evaluation or prior to the notification, and (iv), as from the second interview, the benefit level may change in case of a sanction. Therefore, as we are simultaneously considering many monitoring regimes, we have a sufficient number of independent FOC to identify the unknown parameters. However, increasing the number of FOC in this way is not helpful in identifying ε and λ on the one hand, and β_1 and λ on the other hand, because these unknown parameters appear in these FOC as ratio's: $\frac{\varepsilon}{\lambda}$ and $\frac{\beta_1}{\lambda}$. This therefore reveals an identification problem. It can be checked that this problem does not disappear if another, e.g. isoelastic, specification is chosen for the cost of effort. We therefore normalize $\lambda = 1$.

E Derivation of the Sample Average Probability of Negative Evaluation

To obtain the sample averages of the probabilities of negative evaluation $\bar{\pi}_k$ (for $k \in \{1, 2, 3\}$) we account for the facts that not every individual ι in the sample has the same probability to be assessed for a second or third time, and that the sanction probability depends on the average realized search effort $\bar{S}^0(t_k^*, t_{k-1}^*; v_j)$, on whether one was temporarily employed prior to the meeting (i = 0 or i = 1), and on unobservables. We therefore appropriately aggregate individual sanction probabilities across e, u and $\bar{S}^0_{\iota}(\tau, t_{k-1}^*; v_j)$ into the expected probability of negative evaluation $E\pi_{k\iota}$ for each notified individual ι and then weigh this expected probability by the relative probability $PE_{k\iota}$ that the k^{th} evaluation takes place.

More precisely, subscript ι refers to a notified individual characterized by a specific UB level, gender and schooling level. $E\pi_{k\iota}$ denotes the expected probability of negative evaluation for a notified individual ι conditional on being evaluated for the k^{th} time ($k \in \{1, 2, 3\}$) and irrespective of having experienced an employment spell since the last evaluation (i = 0 or i = 1). $PE_{k\iota}$ denote the probability that the k^{th} evaluation takes place for individual ι . Then, if N denotes the number of notified individuals, one can write the sample average probability as a weighted average of expected individual probabilities:

$$\bar{\pi}_k = \sum_{\iota=1}^N \frac{P E_{k\iota}}{\sum_{j=1}^N P E_{kj}} E \pi_{k\iota}$$
(E-1)

where

$$PE_{k\iota} = \prod_{l=0}^{k-1} E\pi_{l\iota}$$
(E-2)

and where $E\pi_{0\iota} \equiv 1$. In the model one cannot escape a first evaluation ($PE_{1\iota} = 1$), since the duration counter that determines whether an evaluation will take place is temporarily halted rather than reset to zero if an individual leaves unemployment for employment. Since employment spells are exponentially distributed and since \bar{t}_1^* , the maximum duration at which the evaluation takes

place, is finite, individuals will always eventually return to unemployment and be evaluated with probability one.² A second evaluation can only take place if one is negatively evaluated at the first: $PE_{2\iota} = E\pi_{1\iota}$. Finally, a third evaluation takes place only if the evaluation at the previous two was negative: $PE_{3\iota} = E\pi_{1\iota}E\pi_{2\iota}$.

We now derive $E\pi_{k\iota}$. For compactness, we abstract from the vector of observed characteristics **x**. The probability of negative evaluation depends on whether the unemployment spell was interrupted by employment (i = 1) or not (i = 0), and if i = 0 on the timing τ of the interview within the delay interval $(\tau \in [t'_k, \bar{t}^*_k])$ and on the average search effort $\bar{S}^0_{\iota}(\tau, t^*_{k-1}; v_j)$ of individual ι with unobserved mass point v_j :

$$\pi_{k\iota}^{0} \left[\bar{S}_{\iota}^{0}(\tau, t_{k-1}^{*}; v_{j}) \right] = \exp \left\{ -\kappa_{k} (\alpha_{1, v_{j}} + \beta_{1} \bar{S}_{\iota}^{0}(\tau, t_{k-1}^{*}; v_{j}) \right\}$$
(E-3)

$$\pi_k^1 = \exp(-\kappa_k \gamma_1) \tag{E-4}$$

where $\kappa_1 \equiv 1$. The expected probability of negative evaluation $E\pi_{k\iota}$ is a weighted average of these probabilities, where the weights depend on the probability of their realization. We first derive these probabilities conditional on the unobserved mass points and denote these by $E\pi_{k\iota}(v_j)$. Subsequently, we write the unconditional probability as a function of the conditional probabilities.

$$E\pi_{k\iota}(v_j) = \left\{ \left[1 - e^{-\int_{t_{k-1}^*}^{t_k'} p_{\iota}^0(z;v_j)dz} \right] + e^{-\int_{t_{k-1}^*}^{t_k'} p_{\iota}^0(z;v_j)dz} \int_{t_k'}^{\bar{t}_k^*} p_{\iota}^0(\tau;v_j)e^{-\int_{t_k'}^{\tau} \left[p_{\iota}^0(z;v_j) + q \right]dz} d\tau \right\} \pi_k^1 \right. \\ \left. + e^{-\int_{t_{k-1}^*}^{t_k'} p_{\iota}^0(z;v_j)dz} \left\{ \int_{t_k'}^{\bar{t}_k^*} q e^{-\int_{t_k'}^{\tau} \left[p_{\iota}^0(z;v_j) + q \right]dz} \pi_{k\iota}^0 \left[\bar{S}_{\iota}^0(\tau, t_{k-1}; v_j) \right] d\tau \right. \\ \left. + e^{-\int_{t_k'}^{\bar{t}_k^*} \left[p_{\iota}^0(z;v_j) + q \right]dz} \pi_{k\iota}^0 \left[\bar{S}_{\iota}^0(\bar{t}_k^*, t_{k-1}; v_j) \right] \right\}$$

$$(E-5)$$

for $k \in \{1, 2, 3\}$. The expression contains four terms. The first two terms weigh the probability of negative evaluation for i = 1 (π_k^1) by the probability of having found employment before the k^{th} interview. This occurs if employment is found during the scheduled interval $[t_{k-1}^*, t_k')$ (first term) or if employment is found during the delay interval $[t_k', \bar{t}_k^*)$ before an interview takes place (second term). The third term weighs for each $\tau \in [t_k', \bar{t}_k^*)$ the probability of negative evaluation for i = 0 $(\pi_{k\iota}^0 [S_{\iota}^0(\tau, t_{k-1}^*; v_j)])$ by the probability that an evaluation occurs before employment is found and integrates ("sums") this over the delay interval. The last term is the probability of negative evaluation for i = 0 if it takes place at the end of the delay interval $([S_{\iota}^0(\bar{t}_k^*, t_{k-1}^*; v_j)])$ weighted by the probability of neither having the k^{th} interview nor a transition to employment before \bar{t}_k^* .

The expected probability of negative evaluation, unconditional on v_j can then be expressed as follows:

$$E\pi_{k\iota} = \frac{\sum_{j=1}^{2} Q_j e^{-\int_0^{t_0^*} p_{\iota}^0(z;v_j) dz} \prod_{l=0}^k E\pi_{l\iota}(v_j)}{\sum_{j=1}^{2} Q_j e^{-\int_0^{t_0^*} p_{\iota}^0(z;v_j) dz} \prod_{l=0}^{k-1} E\pi_{l\iota}(v_j)}$$
(E-6)

for $k \in \{1, 2, 3\}$ and where $E\pi_{0\iota}(v_j) \equiv 1$. Each conditional expected probability of negative evaluation $E\pi_{k\iota}(v_j)$ is weighted by the conditional probability that the mass point is equal to v_j , conditional on individual ι of type v_j being evaluated for the k^{th} time, and, hence, conditional on k-1 negative evaluation, i.e. it is weighted by $\left[Q_j e^{-\int_0^{t_0^*} p_{\iota}^0(z;v_j)dz} \prod_{l=0}^{k-1} E\pi_{l\iota}(v_j)\right] / \left[\sum_{j=1}^2 Q_j e^{-\int_0^{t_0^*} p_{\iota}^0(z;v_j)dz} \prod_{l=0}^{k-1} E\pi_{l\iota}(v_j)\right]$, where $e^{-\int_0^{t_0^*} p_{\iota}^0(z;v_j)dz}$ is the probability that individual ι of type v_j survives in unemployment until notification.

 $^{^{2}}$ This is an approximation, since in reality the duration counter determining the moment of evaluation is reset to zero after an uninterrupted full time employment spell of 12 months.

F Likelihood Contributions

To write down the likelihood contribution of an unemployed individual, consider first the probability of surviving in unemployment until some given moment t. For that, given the normalization $\lambda = 1$, let $p^0(\tau; \mathbf{x}, u)$ be given by (5) with superscript i = 0 denoting that the worker with type (\mathbf{x}, u) did not interrupt unemployment since t_0 , and let $p^+(\mathbf{x}, u) \equiv s^+(\mathbf{x}, u)\bar{F}(w_r^+(\mathbf{x}, u))$ designate the exit rate in a stationary environment with a flat benefit b_h . Furthermore define $\underline{t}'_k \equiv \min\{t, t'_k\}, \ \underline{t}^*_k \equiv \min\{t, t^*_k\}$ and $t'_4 \equiv t^*_4 \equiv \infty$. With these definitions, the probability of surviving in unemployment until t, being notified at t^*_0 and being evaluated at $\{t^*_k\}_{k=1}^3$, conditional on the type (\mathbf{x}, u) and on the outcome of the notification $(O_0 \equiv 1)$ and of the evaluations $(\{O_k\}_{k=1}^3)$ is

$$\mathcal{P}\left\{t, \{t_{k}^{*}\}_{k=0}^{3} | \mathbf{x}, u, \{O_{k}\}_{k=0}^{3}\right\} = \exp\left\{-p^{+}(\mathbf{x}, u)\underline{t}_{0}^{*}\right\}$$

$$\times \exp\left\{-1\left[t \ge t_{0}^{*}\right]\sum_{k=1}^{4}O_{k-1}\left[\int_{t_{k-1}^{*}}^{t_{k}^{*}}p^{0}\left(\tau; \mathbf{x}, u\right)d\tau + 1\left[t \ge t_{k}^{*}\right]\int_{t_{k}^{'}}^{t_{k}^{*}}\left[p^{0}\left(\tau; \mathbf{x}, u\right) + q\right]\right]d\tau\right\}$$

$$\times \prod_{k=1}^{3}q^{1\left[t \ge t_{k}^{*}\right]}\exp\left\{-\sum_{k=1}^{3}\left(1 - O_{k}\right)p^{+}(\mathbf{x}, u)\left(t - t_{k}^{*}\right)\right\},$$
(F-1)

Note that if $O_3 = 1, \forall \tau > t_3^*, i \in \{0, 1\} : p^i(\tau; \mathbf{x}, u) = p^-(\mathbf{x}, u) \equiv s^-(\mathbf{x}, u) \overline{F}(w_r^-(\mathbf{x}, u))$ where the superscript '-' designates the levels achieved when the entitlement to unemployment insurance has been lost, i.e. when the individual has been sanctioned. The first term on the right-hand side in (F-1) is the survivor rate in unemployment between entry into unemployment and t or the notification t_0^* , depending on which of the two comes first. The term following on the next line gives for each k the survivor rates in the scheduled interval $[t_{k-1}^*, t_k']$ and in the delay interval $[t_k', t_k^*)$. In the latter interval re-employment and the occurrence of an evaluation are competing risks, which explains the presence of q in the expression. However, if an evaluation takes place, the worker still remains unemployed. Consequently, the probability of surviving in unemployment after t_k^* is the density of being evaluated at t_k^* times the probability of surviving in unemployment beyond t_k^* . Since this density at t_k^* is the product of the arrival rate of evaluation q and the corresponding survivor function, this explains the presence of q in the last term on the third line on the right-hand side of (F-1). The last term also contains the survivor rate in unemployment after a positive evaluation at any interview k.

The density of the duration t spent in unemployment before exiting to a job, conditional on (\mathbf{x}, u) reads

$$g^{0}(t|\mathbf{x}, u) \equiv p^{0}(t; \mathbf{x}, u) \mathcal{P}\left\{t, \{t_{k}^{*}\}_{k=0}^{3} | \mathbf{x}, u, \{O_{k}\}_{k=0}^{3}\right\}$$
(F-2)

where we neglect for notational convenience the dependence on $\{t_k^*\}_{k=0}^3$ and $\{O_k\}_{k=0}^3$. The duration data are grouped into monthly intervals. We account for this grouping by integrating over the corresponding time intervals and by assuming that at most one transition occurs within an interval. Conditional on **x** and the elapsed unemployment duration at selection t_s , but marginal on the unobserved factor u affecting the cost of search, an individual contribution of an unemployment spell lasting d_u months followed by an employment spell of length d_e paying the observed wage w^o writes for uncensored unemployment durations:

$$\ell(d_u, d_e, w^o; t_s) = \frac{\sum_{j=1}^2 Q_j \int_{d_u-1}^{d_u} \left\{ g^0\left(t|\mathbf{x}; v_j\right) \left[f_o(w^o; t, \mathbf{x_1}, v_j) \right]^{c_w} dt \right\}}{\sum_{j=1}^2 Q_j \exp\left\{ -p^+(\mathbf{x}; v_j) t_s \right\}} \left[e^{-\delta(\mathbf{x_1})d_e} - c_e e^{-\delta(\mathbf{x_1})(d_e+1)} \right]$$
(F-3)

where the density f_o has been defined in (15), $c_e = 0$ if the employment spell that follows the transition from unemployment is right censored ($c_e = 1$ otherwise), and $c_w = 0$ if the wage upon this transition is unobserved ($c_w = 1$ otherwise). For unemployment durations censored between durations d_{u-1} and d_u , the contribution to the likelihood becomes:

$$\ell(d_u, d_e, w^o; t_s) = \frac{\sum_{j=1}^2 Q_j \left(1 - \int_0^{d_u - 1} g^0 \left(t | \mathbf{x}, v_j \right) dt \right)}{\sum_{j=1}^2 Q_j \exp\left\{ -p^+(\mathbf{x}; v_j) t_s \right\}}$$
(F-4)

Finally, the likelihood function for the realized evaluation outcomes at the first interview conditional on it taking place at time t_1^* is

$$\ell(O_{1},i;t_{1}^{*}) = \frac{\sum_{j=1}^{2} \left(Q_{j} \mathcal{P}\left\{ t_{1}^{*}, \{t_{k}^{*}\}_{k=0}^{3} | \mathbf{x}, v_{j}, \{O_{k}\}_{k=0}^{3} \right\} \left[\exp\left\{ -\left(\alpha_{1,v_{j}} + \beta_{1} \bar{S}^{0}\left(t_{1}^{*}, t_{0}^{*}; \mathbf{x}, v_{j}\right)\right) \right\}^{O_{1}}}{\sum_{j=1}^{2} Q_{j} \mathcal{P}\left\{ t_{1}^{*}, \{t_{k}^{*}\}_{k=0}^{3} | \mathbf{x}, v_{j}, \{O_{k}\}_{k=0}^{3} \right\}} \times \left(1 - \exp\left\{ -\left(\alpha_{1,v_{j}} + \beta_{1} \bar{S}^{0}\left(t_{1}^{*}, t_{0}^{*}; \mathbf{x}, v_{j}\right)\right) \right\} \right)^{1-O_{1}} \right]^{1-O_{1}} \right]^{1-O_{1}} \left[\exp\left\{ -\gamma_{1} \right\}^{O_{1}} \left(1 - \exp\left\{ -\gamma_{1} \right\} \right)^{1-O_{1}} \right]^{i}$$
(F-5)

for $i \in \{0, 1\}$.

G Solving the Optimal Control Problem and Estimation

Estimation requires that the optimal control problem described in Section 4.4 has to be solved at each iteration of the numerical optimization. Given a vector of all parameters of the model, for each sampled individual the problem is solved, both for e = 0 and e = 1, by backward induction in the following steps:

- Step 0: The stationary problems are solved in case of a positive evaluation and in case of a sanction after a third interview; U^+ and U^- are calculated.
- Step 1.1: Given U^+ and U^- , the FOC for control variables are solved at \bar{t}_3^* to determine the endpoint conditions for the paths of control variables at \bar{t}_3^* . First we solve for endpoint conditions under effort-independent evaluation (e = 1), since for e = 1 FOC depend only on the knowledge of U^+ and U^- . Then we solve for endpoint conditions under effort-dependent evaluation (e = 0), as for e = 0 FOC require knowledge of $U_{3,2}^1(\bar{t}_3^*)$, available now from the former solution. Moreover, these FOC also require knowledge of $\pi^0 \left[\bar{S}^0(\bar{t}_3^*, t_2^*) \right]$, which itself contains an integral of the yet unknown path of the search effort. An initial guess for this probability is taken.
- Step 1.2: Given the endpoint conditions of Step 1.1, the system of differential equations that describe the evolution of the optimal paths of control variables is solved in the interval $[t'_3, \bar{t}^*_3)$. This system is obtained by the differentiation of the FOC for control variables with respect to time. First we solve for optimal paths under effort-independent evaluation (e = 1). Then we solve for optimal paths under effort-dependent evaluation (e = 0), since the solution of the system of differential equations in this case requires knowledge of the path of $U^1_{3,2}(\tau), \tau \in [t'_3, \bar{t}^*_3)$, available now from the former solution. Moreover, this system also requires knowledge of $\pi^0 \left[\bar{S}^0(\bar{t}^*_3, \tau) \right], \tau \in [t'_3, \bar{t}^*_3)$, for which the initial guess is maintained for the moment. Using both solutions, $U^1_{3,2}(t'_3)$ and $U^0_{3,2}(t'_3)$ at the scheduled date of the third interview t'_3 are computed.
- Step 1.3: Given $U_{3,2}^1(t'_3)$ and $U_{3,2}^0(t'_3)$ from Step 1.2, the FOC for control variables are solved at t'_3 to determine the endpoint conditions for the paths of control variables at t'_3 . The endpoint conditions are solved first for the effort-independent evaluation, followed by the effort-dependent evaluation (for the same reason as in Step 1.1).
- Step 1.4: Given the endpoint conditions of Step 1.3 the system of differential equations that describe the evolution of the optimal paths of control variables is solved in the interval $[t_2^*, t_3')$. First we solve for optimal paths under effort-independent evaluation, followed by effort-dependent

evaluation (for the same reason as in Step 1.2). Likewise, the system of differential equations under effort-dependent evaluation requires knowledge of $\pi^0 \left[\bar{S}^0(t'_3, \tau) \right], \tau \in [t_2^*, t'_3)$, for which the initial guess is currently maintained.

- Step 1.5: The solution of Steps 1.1-1.4 provides us with the optimal path of search effort $s^0(\tau)$ on $[t_2^*, \bar{t}_3^*)$. This is used to update the initial guess about $\pi^0 [\bar{S}^0(\bar{t}_3^*, \tau)], \tau \in [t_2^*, \bar{t}_3^*)$, and Steps 1.1-1.4 are repeated again until convergence in $s^0(\tau)$. Upon convergence the value of the lifetime utility $U_{3,1}^0(t_2^*)$ at the actual date of the second interview is evaluated.
- Step 2: We go back to Step 1.1, replace U^- by $U^0_{3,1}(t_2^*)$, as calculated in Step 1.5, and iterate until convergence. The result is the lifetime utility $U^0_{2,1}(t_1^*)$ at the actual date of the first interview.
- Step 3: We continue in this way until arriving at t_0^* , the moment of notification.³

The above described solution algorithm takes the vector of all parameters of the model as given. Parameters of the model are described by the following likelihood functions: (F-3)-(F-4) determine all parameters but $\{\alpha_{1,v_j}\beta_1\}_{j=1,2}$, and (F-5) determines $\{\alpha_{1,v_j}\beta_1\}_{j=1,2}$. Consequently the estimation is performed in two stages:

- STAGE 1: For the initial values of $\{\alpha_{1,v_j}\beta_1\}_{j=1,2}$ and the rest of the parameters, (F-3)-(F-4) are maximized conditional on $\{\alpha_{1,v_j}\beta_1\}_{j=1,2}$. The resulting estimates are used to compute, based on Steps 0 to 3, the average search effort at the first interview $\bar{S}^0(t_1^*, t_0^*)$ for all individuals who are observed to have the first interview.
- STAGE 2: Given $\bar{S}^0(t_1^*, t_0^*)$ from Stage 1, (F-5) is maximized with respect to $\{\alpha_{1,v_j}\beta_1\}_{j=1,2}$. $\{\alpha_{k,v_j}\beta_k\}_{j=1,2}$ with k = 2, 3 are updated as described in Section 5.2 and in Appendix C. Based on these new parameter estimates Steps 0 to 3 are implemented as input for Stage 1.

Stages 1 and 2 are iteratively repeated until convergence in all parameters of the model.

H Intertemporal Indicators

The intertemporal indicators are the lifetime utility in unemployment denoted U, the lifetime earnings of an unemployed denoted X and government's discounted expenditures denoted Y. Below, we only write down the expressions for the first subperiod, $[t_0, t_1^*]$, where t_0 is the month of notification. The expressions for the next subperiods are analogous and are therefore not reported. For computational considerations we take intertemporal indicators conditional on the expected duration of interview delays, rather then explicitly model their expected values with respect to the distribution of random delay times.

1. "Lifetime Utility U" defined as the expected discounted stream of benefits and leisure net of search costs plus future labor earnings of an unemployed. Limited to the first subperiod, this measure is nothing but the lifetime value of unemployment for i = 0 (with expectations being taken at the moment of notification t_0).

$$U_{1}^{0}(t_{0}) = \int_{t_{0}}^{t_{1}^{\prime}} \left[\left(b_{h} + \nu - c \left[s^{0}(x) \right] \right) + p \left(w_{r}^{0}(x), s^{0}(x) \right) \bar{W}_{1}^{0}(x) \right] P^{0}(x, t_{0}) e^{-\rho(x-t_{0})} dx + P^{0}\left(t_{1}^{\prime}, t_{0} \right) e^{-\rho(t_{1}^{\prime}-t_{0})} \left\{ \int_{t_{1}^{\prime}}^{t_{1}^{\ast}} \left[\left(b_{h} + \nu - c \left[s^{0}(x) \right] \right) + p \left(w_{r}^{0}(x), s^{0}(x) \right) \bar{W}_{1}^{0}(x) + q \mathbb{U}_{1}^{0}(x) \right] \times P^{0}\left(x, t_{1}^{\prime} \right) e^{-[\rho+q](x-t_{1}^{\prime})} dx \right\} + P^{0}\left(t_{1}^{\ast}, t_{0} \right) e^{-\rho(t_{1}^{\ast}-t_{0}^{\prime})-q(t_{1}^{\ast}-t_{1}^{\prime})} \mathbb{U}_{1}^{0}\left(t_{1}^{\ast} \right)$$

 $^{^{3}}$ Detailed expressions of the systems of endpoint conditions and optimal paths at each step are provided in Section C of this online appendix.

where $\mathbb{U}_{1}^{0}(x) = \pi_{1}^{0} \left[\bar{S}^{0}(x, t_{0}) \right] U_{2}^{0}(t_{1}^{*}) + \left(1 - \pi_{1}^{0} \left[\bar{S}^{0}(x, t_{0}) \right] \right) U^{+}.$

2. "Lifetime Earnings X" defined as the expected discounted stream of labor earnings. This measure excludes instantaneous utility of unemployment (benefits and leisure net of search costs) from the lifetime utility U. In the first subperiod,

$$\begin{split} X_{1}^{0}\left(t_{0}\right) &= \int_{t_{0}}^{t_{1}'} \left[\left(0+0-0\right)+p\left(w_{r}^{0}\left(x\right),s^{0}\left(x\right)\right)\bar{W}_{X_{1}}^{0}\left(x\right)\right]P^{0}\left(x,t_{0}\right)e^{-\rho\left(x-t_{0}\right)}dx \\ &+P^{0}\left(t_{1}',t_{0}\right)e^{-\rho\left(t_{1}'-t_{0}\right)}\left\{\int_{t_{1}'}^{t_{1}'} \left[\left(0+0-0\right)+p\left(w_{r}^{0}\left(x\right),s^{0}\left(x\right)\right)\bar{W}_{X_{1}}^{0}\left(x\right)+q\mathbb{X}_{1}^{0}\left(x\right)\right] \\ &\times P^{0}\left(x,t_{1}'\right)e^{-\left[\rho+q\right]\left(x-t_{1}'\right)}dx\right\}+P^{0}\left(t_{1}^{*},t_{0}\right)e^{-\rho\left(t_{1}^{*}-t_{0}\right)-q\left(t_{1}^{*}-t_{1}'\right)}\mathbb{X}_{1}^{0}\left(t_{1}^{*}\right) \end{split}$$

where

$$\bar{W}_{X_{1}}^{0}(x) = \int_{w_{r}(x)}^{\infty} \frac{1}{\rho + \delta} \left[w + \delta X_{1}^{1}(x) \right] \frac{f(w)}{\bar{F}(w_{r}(x))} dw.$$

$$\mathbb{X}_{1}^{0}(x) = \pi_{1}^{0} \left[\bar{S}^{0}(x, t_{0}) \right] X_{2}^{0}(t_{1}^{*}) + \left(1 - \pi_{1}^{0} \left[\bar{S}^{0}(x, t_{0}) \right] \right) X^{+},$$

 X_1^1 designating the same expression as X_1^0 when the unemployment spell has been interrupted by temporary employment (i = 1) and X^+ having the same interpretation as U^+ above.

3. "Government's Discounted expenditures Y" includes the discounted stream of benefit payments (unemployment insurance and means-tested assistance benefits) augmented with the fixed cost \tilde{c} per capita and per meeting with a caseworker. Y takes into account the risk that someone who exits to a job eventually returns in unemployment after being laid-off. Considering again the first period after notification only, we have:

$$Y_{1}^{0}(t_{0}) = \int_{t_{0}}^{t_{1}'} \left[(b_{h} + 0 - 0) + p(w_{r}^{0}(x), s^{0}(x)) \times \frac{\delta Y_{1}^{1}(x)}{\rho + \delta} \right] P^{0}(x, t_{0}) e^{-\rho(x - t_{0})} dx \\ + P^{0}(t_{1}', t_{0}) e^{-\rho(t_{1}' - t_{0})} \left\{ \int_{t_{1}'}^{t_{1}^{*}} \left[(b_{h} + 0 - 0) + p(w_{r}^{0}(x), s^{0}(x)) \times \frac{\delta Y_{1}^{1}(x)}{\rho + \delta} + q[\mathbb{Y}_{1}^{0}(x) + \tilde{c}] \right] \\ \times P^{0}(x, t_{1}') e^{-[\rho + q](x - t_{1}')} dx \right\} + P^{0}(t_{1}^{*}, t_{0}) e^{-\rho(t_{1}^{*} - t_{0}') - q(t_{1}^{*} - t_{1}')} [\mathbb{Y}_{1}^{0}(t_{1}^{*}) + \tilde{c}]$$

where

$$\mathbb{Y}_{1}^{0}(x) = \pi_{1}^{0} \left[\bar{S}^{0}(x, t_{0}) \right] Y_{2}^{0}(t_{1}^{*}) + \left(1 - \pi_{1}^{0} \left[\bar{S}^{0}(x, t_{0}) \right] \right) Y^{+},$$

 Y_1^1 designating the same expression as Y_1^0 when the unemployment spell has been interrupted by temporary employment $(i = 1) Y^+$ having the same interpretation as U^+ above.