Is basic income a cure for unemployment in unionized economies? A general equilibrium analysis.*

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Abstract

This paper deals with the effect of basic income schemes on the equilibrium unemployment rate. It develops a dynamic general equilibrium model of a unionized economy where the budget of the State has to be balanced in each period. Compared to a benchmark situation with an unemployment insurance, it is shown that appropriately defined basic income schemes lower the steady state unemployment rate. Moreover the dynamic adjustment induced by such reforms can be Pareto-improving.

Cet article étudie l'effet d'une allocation universelle sur le taux de chômage d'équilibre. Il présente un modèle dynamique d'équilibre général dans lequel les salaires sont formés par la négociation collective. La contrainte budgétaire de l'Etat est respectée à chaque période. En comparant à une situation de référence caractérisée par un système d'assurance chômage, on montre qu'à l'état stationnaire le taux de chômage d'équilibre est moins élevé en présence de formules appropriées d'allocation universelle. En outre, la dynamique d'ajustement suite à de telles réformes peut entraîner une amélioration au sens de Pareto.

Keywords: Basic income; unconditional income; wage bargaining; WS-PS model; unemployment.

JEL classification : H20, H55, J51, J68.

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Extended summary

This paper is concerned with the implications of a basic income in an economy characterized by an imperfect labor market. A basic or unconditional income consists of the payment of an allowance to every adult citizen. A dynamic general equilibrium model of a unionized economy is developed to analyze the effect of unconditional basic income schemes on the unemployment rate. This paper assumes a starting situation where unemployment benefits are a fixed proportion of the wage rate. Two variants of the basic income proposal are then introduced. The *full basic income scheme* replaces the existing unemployment benefits by an unconditional income at least equal to these benefits at given wages. The *partial basic income* is lower (at given wages). The unemployment benefits do not disappear but are reduced in such a way that the net income of the unemployed remains unchanged at given wages. Assuming a proportional tax on earnings and a balanced budget of the State, the following steady-state properties are proved. First, compared to a situation with an unemployment insurance system but without basic income, the equilibrium unemployment rate is always lower if a partial basic income scheme is implemented. The same holds if the unemployment insurance system is replaced by a full basic income scheme provided that the ratio between the basic income and the unemployment benefits is not too high. Second, the equilibrium unemployment rate is a decreasing function of the ratio between the partial basic income and the unemployment benefits. Third, if workers are risk averse, the equilibrium unemployment rate is an increasing function of the ratio between the full basic income and the unemployment benefits. If workers are risk neutral, the full basic income has no effect on the equilibrium unemployment rate. Fourth, the equilibrium marginal tax rate typically increases with the basic income-unemployment benefits ratio while the net wage rate generally decreases. Finally, a numerical analysis illustrates that introducing a sufficiently small partial basic income-unemployment benefits ratio can be a Pareto improvement (compared to a situation with unemployment benefits only). This property heavily depends on the level of the discount factor and on the degree of unconditionality of the basic income.

The intuition behind the impacts on unemployment goes as follows. Unions bargain in order to create a positive rent for their members. In a way or another, this amplifies the allocative inefficiency generated by unemployment benefits. Everything else equal, because the partial basic income is not withdrawn when an unemployed is hired, it reduces the reservation wage effect created by unemployment benefits. At given wages, the full basic income pushes up the instantaneous income people obtain both in unemployment and in employment. These two effects cancel out if workers are risk neutral. When workers are risk averse, the effect on utility is higher in case of unemployment. This induces a negotiated compensation for the employed workers that will eventually deteriorate the employment level.

1 Introduction

This paper is concerned with the following question : Could a basic income have favorable effects on the unemployment rate in unionized European countries? A basic or unconditional income is an income paid by the government to every member of society. This idea has taken various forms and has received various names : *social dividend* (Lange, 1936, and Meade, 1989) and *basic income* (Parker, 1989). Until recently, the economics of basic income schemes has often been reduced to an arithmetic exercise or it has looked at labor supply effects in a competitive setting. To achieve a coherent view of the implications of a basic income in Europe, one needs a richer theoretical setting. It should have the essential features of a general equilibrium analysis with an explicit budget constraint for the State. It should emphasize the working of the labor market and allow for the possibility of involuntary unemployment. Finally, it should introduce some heterogeneity between the economic agents. The model presented in this paper is an attempt to combine these requirements. It is a dynamic and general equilibrium model of a unionized economy inspired by Manning (1993) and Cahuc and Zylberberg (1996, 1999). In this non competitive labor market, ex ante identical workers become heterogeneous endogenously : Part of them are ex post unemployed while others are employed and benefit from a higher utility level.

It is assumed that an unemployment benefit system initially exists. This paper considers the case where unemployment benefits are a fixed proportion of the wage rate. Two variants of the basic income proposal are then introduced. The *full basic income scheme* replaces the existing unemployment benefits by an unconditional income that is at least equal to these benefits at given wages. The *partial basic income* is lower at given wages and does not replace the existing unemployment benefits. The unemployment benefits are reduced in such a way that the net income of the unemployed remains unchanged at given wages.

The literature about the economic effects of basic income schemes is growing rapidly. In a partial equilibrium setting, the literature on optimum income taxation has been used to cast light on the desirable levels of the Basic Income/Flat Tax proposal (see chapter 2 of Atkinson, 1995a and d'Autume, 2001a). In a general equilibrium framework, the introduction of basic income schemes has been considered in several theoretical settings: Efficiency wage models, union models and equilibrium search models. Starting with efficiency wage models, Bowles (1992) assumes that the initial total amount of income-replacing payments has to remain unchanged and is distributed equally to all citizens. This leads to a drop in the fall-back position of employees and therefore the equilibrium wage falls, too. He concludes that a small unconditional grant can be introduced without reducing the pre-grant level of profits. Atkinson (1995a) analyzes the switch from unemployment benefits financed by a payroll tax to a basic income scheme and a flat income tax. He shows that this reform reduces the unemployment and wage levels in a dual labor market. Yet, this conclusion appears to depend on the institutional features of the unemployment benefit system. Considering also a dual labor market, Groot and Peeters (1997) concludes that "a moderate basic income can be compatible with lower unemployment, higher GDP, higher real incomes for workers, lower income inequality between workers, but a lower real income for the (voluntary) unemployed" (p. 593).

Other papers evaluate basic income schemes in a unionized economy. Késenne (1993) develops a static macro model where output prices are exogenously fixed and wages are the outcome of an efficient bargain. His results emphasize the roles of labor supply responses (see also Késenne, 1991) and of fall-back positions in the bargaining process. Cahuc and Zylberberg (1996) (p. 497) introduce an unconditional allowance in a unionized economy. They explain some of the basic mechanisms developed in the present paper. Algan (2001) is based on an earlier version of this article. He assumes that the basic income is handed out to the workforce only. The basic income would replace unemployment benefits and would guarantee the same replacement ratio. His simulations illustrate a possible trade off between the level of unemployment and the intertemporal utility of the unemployed. Introducing the basic income cuts the former but can also deteriorate the latter. In such a case, the improved probability of getting a job is insufficient to compensate the cut in the *level* of unemployment benefits. The latter effect comes out because the *level* of unemployment benefits is by assumption proportional to wages and because the introduction of a basic income leads to decreasing negotiated wages. The present paper shows that this trade-off is less probable with a partial basic income and can even definitely disappear.

Chéron (2001) considers an equilibrium search framework. As in Algan (2001), the basic income would replace unemployment benefits. Dealing with endogenous search levels, the simulations of Chéron (2001) indicate that the introduction of the basic income can be Pareto-improving. Because unemployment benefits are decreasing while the probability of becoming employed increases, the discount rate used by the unemployed is a crucial parameter. Lehmann (2001) develops an equilibrium search model with two levels of skill. This author analyzes in particular how the introduction of a basic income interacts with a minimum wage legislation. Finally, there is a literature that compares alternative transfer and tax systems (comparisons between unconditional and conditional schemes can be found in the papers mentioned above and for instance also in Besley (1990), Creedy (1996), Drèze and Sneessens (1997) and Groot (1997)). Van der Linden (2000) develops a comparison between basic income schemes and reductions of pay-roll taxes.

This paper is organized as follows. Section 2 presents the model. Section 3 deals with the dynamic properties of the model. Section 4 develops the main theoretical results in a steady state. Section 5 looks at the dynamic adjustment path when a partial basic income scheme is introduced. Section 6 assesses the findings of the previous sections. It includes in particular is a brief extension to the case where the size of the labor force is endogenous. Section 7 concludes the paper.

2 The model

This model draws upon Manning (1991, 1993) and Cahuc and Zylberberg (1999). Let us consider a small economy facing an exogenous interest rate r[1]. Assume a deterministic setting with, in each period t, n identical firms, N homogeneous workers and M inactive individuals (n, N and M are exogenous). Section 6 considers the case where N and M are

[1] There is implicitly an international financial market with perfect mobility.

endogenous. Each of the n firm owners bargains over wages with a firm-specific union.[2] The former decides unilaterally on employment and on the level of investment. Firms and workers are infinitely lived agents with perfect foresight. They share a common discount factor $\beta = \frac{1}{1+r}$. In a given period t, the sequence of decisions is as follows :

1. Each firm decides upon its current investment level which will increase its capital stock in $t+1$. So, the capital stock is predetermined during the current period.

2. A decentralized bargaining over the current wage level takes place in each firm (wages are only set for one period). If an agreement is reached, the employees receive a *net* real wage w_t at the end of the period.[3] Otherwise, workers immediately leave the firm and start searching a job. In firms where there is a collective agreement, the firm determines labor demand for the current period. Given w_t, the employment level is fixed by labor demand and production occurs. In the absence of a collective agreement, nothing is produced during the current period. Yet, the firm will have the opportunity to bargain and to hire workers (without hiring costs) in $t+1$.

3. A proportional tax on earnings, τ_t, is adjusted so that the current public budget constraint balances (no other taxes are introduced).

4. At the end of the period, an exogenous fraction q of the employees leaves the firm and enters unemployment.

To present the model, let us move backwards.

Workers

Each of the N homogeneous workers supplies one unit of labor. His instantaneous utility function is $v(R_t)$, where R_t denotes net real income in period t ($v' > 0, v'' \leq 0$). Let us assume a constant relative risk aversion utility function :

$$v(R_t) \equiv \frac{R_t^\lambda}{\lambda}, \text{ where } \lambda \leq 1, \lambda \neq 0. \tag{1}$$

At the end of period t, each employee leaves the firm with an exogenous probability q, $0 < q < 1$. He is then unemployed at the beginning of period $t+1$ and will be hired by a firm with probability a_{t+1}. This probability is endogenous (see below). Hence, in period t, the intertemporal discounted utility of a job in a given firm, V_e^t, is given by the following expression :

$$V_e^t = \frac{(w_t + B_t)^\lambda}{\lambda} \ + \ \beta\{q[a_{t+1}\overline{V_e^{t+1}} + (1 - a_{t+1})V_u^{t+1}] \ + \ (1-q)V_e^{t+1}\}, \tag{2}$$

where B_t denotes the level of the basic income at time t, $\overline{V_e^{t+1}}$ is the intertemporal discounted utility of a job on average in the economy in period $t+1$ and V_u^{t+1} is the intertemporal discounted utility of being unemployed in $t+1$. Both actually are (perfectly) anticipated utilities. $\overline{V_e^{t+1}}$ is of the same form as (2) with only one difference : The average net real wage in the economy, $\overline{w_t}$, replaces w_t.

[2]If the wage bargain takes place at the sectoral level, all the results obviously remain unchanged if the model of the firm developed below is reinterpreted as the one of the sector.

[3]To save on notations, no subscript is added to designate the firm.

Let Z_t be the exogenous level of unemployment benefits. The instantaneous utility of an unemployed, v_u^t, is equal to $\frac{Z_t^\lambda}{\lambda}$ with a partial basic income (i.e. when $B_t < Z_t$) and $\frac{B_t^\lambda}{\lambda}$ with a full basic income (i.e. $B_t \geq Z_t$). The intertemporal discounted utility of being unemployed at time t, V_u^t, is given by

$$V_u^t = v_u^t + \beta\{a_{t+1}\overline{V_e^{t+1}} + (1 - a_{t+1})V_u^{t+1}\}. \tag{3}$$

Firms

The n identical firms produce an homogeneous good and sell it on a competitive market at a price normalized to 1. Let L_t and K_t denote the level of labor and capital in a given firm. Assume a Cobb-Douglas technology with constant returns to scale : $(AL_t)^\alpha K_t^{1-\alpha}, A > 0, 1 > \alpha > 0$.[4] According to the sequence of decisions explained above, the capital stock is predetermined when bargaining takes place. Conditional on K_t, labor demand can be written as :

$$L_t = K_t A^{\frac{\alpha}{1-\alpha}} \left(\frac{w_t(1+\tau_t)}{\alpha}\right)^{\frac{1}{\alpha-1}}. \tag{4}$$

Let $\pi_t(K_t)$ be current optimal profits net of investment :

$$\pi_t(K_t) \equiv \max_{L_t}[(AL_t)^\alpha K_t^{1-\alpha} - w_t(1+\tau_t)L_t] = (1-\alpha)K_t \left(\frac{w_t(1+\tau_t)}{\alpha A}\right)^{\frac{\alpha}{\alpha-1}}. \tag{5}$$

Wage-setting

Following Manning (1991, 1993), assume that the union's objective is the product $L_t^\psi(V_e^t - V_g^t)$, where ψ is a nonnegative parameter representing union's preferences for employment relative to an intertemporal rent for currently occupied workers. Redundant workers are assumed to be immediately rehired in another firm with probability a_t. Hence, the outside option is

$$V_g^t = a_t\overline{V_e^t} + (1 - a_t)V_u^t. \tag{6}$$

Assume that the current real wage w_t is set to maximize a Nash product. Remember that w_t is only set for the current period during which the capital stock is given. Without an agreement, the workers leave immediately the firm. Their utility is then equal to V_g^t. Cahuc and Zylberberg (1999) show that the firm's component in the Nash product, i.e. the difference between intertemporal discounted profits in case of an agreement and in the absence of an agreement, is simply π_t. It is plausible and therefore assumed that the firm-specific union and the firm owner take the tax rate τ_t, the average wage $\overline{w}_t$, the unemployment outflow rate a_t and the level of benefits, Z_t and B_t, as given when they bargain over wages. Remembering (5) and ignoring constant and predetermined terms, the Nash program writes

$$\max_{w_t}(w_t)^{\frac{\alpha(1-\gamma)}{\alpha-1}} L_t^{\psi\gamma}(V_e^t - V_g^t)^\gamma, \tag{7}$$

[4] Allowing A to vary exogenously with t would not change the conclusions of this paper.

where γ is the so-called bargaining power of the union, $0 \leq \gamma \leq 1$, and L_t is given by (4). The first-order condition of this problem can be written as

$$V_e^t - V_g^t = \mu w_t (w_t + B_t)^{\lambda - 1}, \text{ with } \mu \equiv \frac{\gamma(1-\alpha)}{\alpha(1-\gamma) + \psi\gamma} \geq 0. \tag{8}$$

Notice that the intertemporal rent of an employee, $V_e^t - V_g^t$, is positive if $\gamma > 0$. This property holds true in equilibrium. The second-order condition is satisfied if $\mu < 1$, namely if $\gamma < \frac{\alpha}{1-\psi}$. This inequality is assumed to hold.

Investment

At the beginning of any period t, the level of investment, I_t, is chosen in order to maximize $\Pi_t(K_t) = \pi_t(K_t) - I_t + \beta\Pi_{t+1}(K_{t+1})$, subject to $K_{t+1} = I_t + (1-\delta)K_t$, where δ is the depreciation rate (common to all firms).[5] This problem is solved in Cahuc and Zylberberg (1999). Since the technology is homogeneous of degree one, the first-order conditions for profit maximization only determine the capital-labor ratio. From these conditions, another important relationship can be derived. The anticipated wage w_{t+1} should be given by:

$$(1 + \tau_{t+1})w_{t+1} = C, \text{ where } C = \alpha A \left(\frac{\delta + r}{1-\alpha}\right)^{\frac{\alpha-1}{\alpha}} > 0. \tag{9}$$

This equation implies that the anticipated real wage cost is fixed by the structural parameters characterizing the firm and the economy r, δ, α, A. Since firms are identical, this anticipated wage is the same in each of them. Expression (9) is the familiar 'real input prices frontier' found in models where returns to scale are constant and competition on the market for goods is perfect. In these models, the firm breaks even if the marginal cost is equal to the price of output. Equation (9) expresses this condition.

The equilibrium

Since all firms and unions' characteristics are identical, in equilibrium, $w_t = \overline{w_t}$ and $V_e^t = \overline{V_e^t}$. Then (6) implies that

$$V_e^t - V_g^t = (1 - a_t)(V_e^t - V_u^t). \tag{10}$$

This establishes a relationship between the exit rate from unemployment, a_t, the rent employed workers get compared to redundant ones and the difference in intertemporal utilities between an employed and an unemployed. a_t can be rewritten as a function of the current and previous unemployment rates (hence, a_t is endogenous). For the current unemployment level is made of those who where unemployed at the beginning of this period and who are not currently hired. After division by the size of the labor force, N, this definition writes

$$u_t = (1 - a_t)\left(u_{t-1} + q(1 - u_{t-1})\right), \tag{11}$$

[5] In the initial period, $t = 0$, the capital stock is exogenously fixed. Hence, the following holds in all periods except this one. Furthermore, as investment in the current period is only available for production in the next period, the so-called 'hold-up' problem (Grout, 1984) does not appear here. On this issue, see Cahuc and Zylberberg (1999).

where u_t is the unemployment rate in period t.

Let us assume that the replacement ratio is constant in each period t ($\frac{Z_t}{w_t} = z, 0 < z < 1$)[6] and that the basic income is proportional to the level of the unemployment benefits ($B_t = \xi Z_t$, $\xi \geq 0$). Let

$$\mathcal{I}(\xi) \equiv \begin{cases} 1 & \text{if } \xi < 1 \text{ (the partial basic income case)} \\ \xi & \text{if } \xi \geq 1 \text{ (the full basic income case).} \end{cases} \tag{12}$$

Combining (2), (3), (8), (10) and (11) leads to the following wage-setting ('WS') equation :

$$\frac{1+\xi z}{\mu\lambda}\left(1 - \left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) + \beta(1-q)\left(\frac{w_{t+1}}{w_t}\right)^{\lambda} = \frac{q+(1-q)u_{t-1}}{u_t}. \tag{13}$$

The current net real wage rate is a function of the current and past unemployment rate and of the anticipated wage rate in $t+1$. The parameters influencing the shape of this relationship are r, μ, λ, q, z and ξ.

The budget of the State is assumed to be balanced in each period. The extent to which the M inactive individuals and the n firm owners are eligible for the basic income obviously influences this budget constraint. Let ν be the ratio of the eligible inactive population to the workforce ($\nu \leq \frac{M+n}{N}$). The balanced budget can be written as :

$$\tau_t(1-u_t) = \begin{cases} z(u_t + \xi(1-u_t+\nu)) & \text{if a partial basic income applies} \\ \xi z(1+\nu) & \text{if a full basic income applies.} \end{cases} \tag{14}$$

From (14), the marginal tax rate τ_t increases with the unemployment rate.

3 Dynamics and steady state

For $t \geq 1$, the assumption of perfect foresight implies that the net wage instantaneously reaches its long-term value defined by equation (9).[7] Combining equations (9) and (14) yields the following 'price-setting curve' ('PS') :

$$w_t = \begin{cases} \frac{C(1-u_t)}{1+\xi z(1+\nu)-(1+(\xi-1)z)u_t} & \text{if a partial basic income applies} \\ \frac{C(1-u_t)}{1+\xi z(1+\nu)-u_t} & \text{if a full basic income applies.} \end{cases} \tag{15}$$

This relationship between the net wage rate and the unemployment rate is downward-sloping and concave. The ratio $\frac{w_{t+1}}{w_t}$ can now be derived from (15) and substituted in the 'WS' equation (13). The latter is then a second-order scalar nonlinear difference equation where the current unemployment rate u_t is a function of the lagged unemployment level u_{t-1} and the future one u_{t+1}. With one predetermined variable, the saddle point property is required in order to have a unique nonexploding solution. Appendix 1 shows that this dynamic system is locally stable around the steady state and that the equilibrium is a saddle point.[8] The dynamic adjustment of the model will be further analyzed in Section 5.

[6]This assumption is supported by Figure 2.2 in OECD (1996). Other assumptions such as $Z_t = zw_{t-1}$ would lead to similar conclusions in steady state.

[7]Consequently, the net real wage accommodates any change in the tax rate.

[8]This holds for plausible values of the parameters. However, Appendix 1 explains that this property is lost if z and the equilibrium unemployment rate u^* are extremely high when $\lambda < 0$.

In steady state, the unemployment rate, the marginal tax rate and the net real wage rate are constant. Therefore, the wage-setting equation (13) defines the equilibrium unemployment rate u^* :

$$u^* = \frac{q}{\frac{1+\xi z}{\mu\lambda}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right)-(1-\beta)(1-q)}. \tag{16}$$

$0 \leq u^* \leq 1$ if

$$\mu \leq \frac{1+\xi z}{\lambda\left(1-\beta(1-q)\right)}\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right). \tag{17}$$

Whatever the sign of λ, the right-hand side of (17) is positive. It could be lower than 1, imposing an upper-bound upon μ. Henceforth, condition (17) is assumed to hold.[9]

Knowing u^*, the steady state marginal tax rate τ^* and net real wage rate w^* are easily computed from, respectively, (14) and (15). Figure 1 illustrates this solution. The equilibrium (w^*, u^*) is at the intersection of the vertical wage-setting curve 'WS' (16) and the downward-sloping price-setting curve 'PS' (15).

Union-firm bargaining generates a positive rent for the employed workers (if $\gamma > 0$). From (8), (2), (3) and (1), the latter is equal to :

$$V_e^* - V_g^* = \mu\,(w^*)^{\lambda}(1+\xi z)^{\lambda-1} > 0 \text{ if } \gamma > 0. \tag{18}$$

$$V_e^* - V_u^* = (w^*)^{\lambda}\,\frac{(1+\xi z)^{\lambda}-(\mathcal{I}(\xi)z)^{\lambda}}{\lambda[1-\beta(1-q)(1-a^*)]} > 0. \tag{19}$$

Substituting (18) and (19) into (10) indicates how the steady-state exit rate from unemployment, a^*, is influenced by the bargaining process :

$$\mu = \frac{1-a^*}{\lambda[1-\beta(1-q)(1-a^*)]}\,[1+\xi z]\left(1-\left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) \tag{20}$$

INSERT FIGURE 1 APPROXIMATELY HERE.

4 The effect of basic income schemes in steady state

This section is concerned with the impact of the basic income schemes on the equilibrium unemployment rate, the net wage and the marginal tax rate. To avoid clutter, no superscript is added to indicate that the endogenous variables are at their steady-state level. This section starts with a result summarizing standard properties of the 'WS-PS' model. Next it turns to the main results of the paper.

[9] According to unreported numerical simulations, this inequality is satisfied for plausible values of the parameters.

Result 1 *The equilibrium unemployment rate increases with the interest rate r, the separation rate q, the mark-up μ and the replacement ratio z. On the contrary, the equilibrium unemployment rate is lower the more relative risk averse workers are.*

The proof is left to Appendix 2. Before the main propositions of this paper are formally derived, let us intuitively explain the mechanisms through which the basic income influences the steady-state unemployment rate (16). Unions bargain in order to create a positive rent for their members. In a way or another, this amplifies the allocative inefficiency generated by unemployment benefits. Everything else equal, because the *partial* basic income is not withdrawn when an unemployed is hired, it reduces the reservation wage effect created by unemployment benefits. Therefore, the unemployment rate goes down. At given wages, the *full* basic income pushes up the instantaneous income people obtain both in unemployment and in employment. These two effects cancel out if workers are risk neutral. When workers are risk averse, the effect on utility is higher in case of unemployment. This induces a negotiated compensation for the employed workers (see (19)) that will eventually deteriorate the employment level (a^* shrinks; see (20)). This effect was already announced by Cahuc and Zylberberg (1996) (p. 497).

Behind this general intuitive explanation, two underlying mechanisms are actually at work. The first one is related to the literature about wage-setting in the presence of a non linear tax system (see e.g. Lockwood and Manning (1993)). According to this literature, an increase in the progressivity of taxation, loosely speaking, acts as an incentive for wage moderation. Similarly, Cahuc and Zylberberg (1996) have shown that an increase in progressivity decreases the equilibrium unemployment rate (see pp. 502-506). This result applies here, too. The progressivity of taxation is measured by the so-called coefficient of residual income progression η_w^R, i.e. the elasticity of the net income of an employed worker ($R_t = B_t + w_t$) with respect to w_t. With a basic income, $\eta_w^R = (1 + \xi z)^{-1} < 1$. Therefore, the first term in the denominator of (16) is $\left(\eta_w^R\right)^{-1}$. It is easily seen that the progressivity increases with the level of the basic income. So, the basic income has a first favorable effect on the equilibrium unemployment rate because it introduces some progressivity in taxation.

The second mechanism is captured by the ratio $\frac{z\mathcal{I}(\xi)}{1+\xi z}$ in (16). The latter is simply the 'effective replacement ratio' $\frac{max(Z_t,B_t)}{w_t+B_t}$. The *partial* basic income favors in-work net income (everything else equal, an increase in ξ raises net earnings without affecting the replacement ratio z). Hence, $V_e - V_u$ becomes higher (see (19) with $\mathcal{I}(\xi) = 1$). To keep the equilibrium condition (10), the exit rate from unemployment has to increase (see (20)). On the contrary, the *full* basic income influences both in-work net income and the income of the jobless workers. Everything else equal, as the full basic income rises, the latter increases relatively more. So, $V_e - V_u$ becomes now lower and this eventually pushes up the unemployment rate. Therefore, with a full basic income, the first and the second mechanism have opposite effects on unemployment in steady-state .

Result 2 *Compared to the case with an unemployment insurance system but without basic income, the equilibrium unemployment rate is always lower if a partial basic income scheme is implemented. The same holds when the unemployment insurance system is replaced by a full basic income scheme if the ratio between the basic income and the unemployment benefit, ξ ($\xi > 1$), is lower than $1 + \frac{z}{1-z}$, where z is the replacement ratio.*

Proof Let u_z denote the equilibrium unemployment rate when there is an unemployment insurance system (with a replacement ratio z) and no basic income scheme. u_z is immediately obtained by putting $\xi = 0$ in (16). The equilibrium unemployment rate u_z is higher than u defined in (16) if

$$\frac{1 - z^\lambda}{\lambda} < \frac{1}{\lambda}\left(1 - \left[\frac{z\mathcal{I}(\xi)}{1 + \xi z}\right]^\lambda\right).$$

The latter condition is verified if $1 + \xi z > \mathcal{I}(\xi)$. The last inequality is always satisfied in the case of a partial basic income $(0 < \xi < 1, \mathcal{I}(\xi) = 1)$. With a full basic income, the same conclusion holds if $\xi < 1 + \frac{z}{1-z}$. ■

Result 3 *The equilibrium unemployment rate decreases with the ratio between the partial basic income and the unemployment benefits, ξ. In the full basic income case, the equilibrium unemployment rate increases with (resp., is independent of) ξ if workers are risk averse (resp., risk neutral). The equilibrium real net wage decreases with ξ.*

Proof Consider first the partial basic income case $(0 < \xi < 1)$. Carrying out the first-order partial derivative of (16) with respect to ξ yields

$$\frac{\partial u}{\partial \xi} = -\frac{qz}{D^2\mu\lambda}\left[1 - (1-\lambda)\left(\frac{z}{1+\xi z}\right)^\lambda\right] \ < \ 0, \tag{21}$$

where D is the denominator of (16). The expression between brackets is strictly positive if $\lambda = 1$. The same is true if $0 < \lambda < 1$ since $0 < \left(\frac{z}{1+\xi z}\right)^\lambda < 1$. When λ is negative, the expression between brackets is negative. Therefore, the whole expression is negative, too. In the full basic income case $(\xi \geq 1)$, it can easily be checked that (16) is independent of ξ and z if workers are risk-neutral $(\lambda = 1)$. If $\lambda < 1$,[10] let $\kappa = \frac{\xi z}{1+\xi z}$, $0 < \kappa < 1$. κ increases with ξ. The denominator of (16) can now be rewritten as $\frac{1-\kappa^\lambda}{\mu\lambda(1-\kappa)} - (1-\beta)(1-q)$. In a two-dimensional space, consider the two points $E \equiv (\kappa, v(\kappa))$ and $F \equiv (1, v(1))$, where v is the utility function (1). $\frac{1-\kappa^\lambda}{\lambda(1-\kappa)}$ is the slope of the chord EF. From the properties of the function v, this slope decreases with κ. Therefore, the equilibrium unemployment rate increases with κ and, hence, with ξ.

Combining this result and (15), it is easily seen that the real net wage decreases with ξ in the full basic income case. The same conclusion cannot be derived for the partial basic income. For there are cases where the decrease in unemployment (21) outweighs the partial effect $\frac{\partial w_t}{\partial \xi} < 0$ derived from (15). However, unreported numerical simulations show that this only happens for small values of ξ in cases where the equilibrium unemployment rate is very high (say, above 35%) in the absence of a basic income. ■

As a corollary, the equilibrium marginal tax rate typically increases with ξ (see (9)). The bold face curves in Figure 1 illustrates Result 3 in the case of a partial basic income (with a full basic income the curve 'WS' shifts to the right).

[10] I thank Etienne Lehmann for suggesting the following proof.

5 Dynamic effects of basic income schemes

The results of the previous section can be rephrased in a clear-cut qualitative message. If, for whatever reason, the unions' power and preferences and the replacement ratio z are given, the introduction of a *partial* basic income lowers the equilibrium unemployment rate. Moreover, the lowest unemployment level is reached when $\xi = 1$. This value of the basic income-unemployment benefit ratio is recommended if the reduction of the unemployment rate is the unique government's goal. If this is the case, it is nevertheless interesting to look at the speed of adjustment of the unemployment rate. However, a sound normative analysis should not only consider a criterion such as the unemployment rate. In a welfarist perspective, the utility levels of the agents should be taken into account. In particular, one should be concerned with the effect of a basic income on the well-being of the least well-off (namely the unemployed). This effect cannot be signed unambiguously. Let us see why. Introducing a basic income typically increases the tax rate and hence reduces the net wage. Under the assumption of a constant replacement ratio z, the *level* of the unemployment benefit decreases in the same proportion. So, introducing a *partial* basic income has an unfavorable effect on the consumption level of the unemployed and a favorable one on their intertemporal utility through the improvement of the hiring rate. The net effect on this utility clearly depends on the discount rate and on the dynamics of the net wage and the unemployment rate. This section is devoted to the analysis of these issues. It will be restricted to partial basic income proposals. A full discussion of the dynamics when $\xi \geq 1$ would require substantially more space. Let us briefly sketch the main effects of these reforms. Consider first the case where $\xi = 1$. Since, the tax rate increases with ξ in the neighborhood of 1, the net wage and hence the consumption level of those currently unemployed are lower than in the *partial* basic income case. From simulations made by Algan (2001) and by the author,[11] this effect outweighs the improvement in the unemployment rate. Therefore, the intertemporal utility of those currently unemployed shrinks. Increasing ξ above one can be an answer. The net wage will further decrease and the unemployment rate will start increasing. However, the fact that the consumption level of the unemployed is now proportional to ξ can be sufficient to compensate these unfavorable effects on the utility of the unemployed. There are cases where moving from $\xi = 0$ to $\xi > 1$ is Pareto-improving. However, there can be little doubt that the level of taxes needed to finance a substantial basic income-net wage ratio raises problems of political feasibility. Moreover, more general models featuring an underground economy or the investment made by individuals (say, in education or training) would highlight the drawbacks of such high taxation levels (see Section 6).

Consider the following numerical example based on plausible values of the parameters. Since wages are set determined for one period, let us assume that such a period last a year. A is normalized to 1. Let $\alpha = 0.7, \gamma = 0.6, \psi = 0$, hence $\mu = 0.64, \lambda = -1$ (relative risk aversion = 2), $\beta = 0.95$ ($r \approx 5\%$), $\nu = 0.1, q = 0.2, \delta = 0.1$ and $z = 0.4$. In other words, the bargaining is modeled as the maximization of an asymmetric Nash product and the unions do not value the level of employment[12]. The assumption $\nu = 0.1$

[11] They are not reported but available upon request.

[12] The assumption $\psi = 0$ is in accordance with the so-called seniority model. Moreover, sufficiently close to the steady state, each union member is certain to keep his job since new hirings should compensate the

means that, on average in the EU, about one-quarter of the inactive population aged 18-64 would be eligible for a basic income. Considering only the population aged 18-64 stems from the focus of this paper on the unemployment insurance mechanism.[13] Moreover, it is here assumed that some *participation* criteria restrict eligibility.[14] In this context, the assumption of an eligibility rate of 25% is simply an example. Remember that ν only influences the marginal tax rate. So, the level of ν affects net earnings. But linear taxes have no effect on the steady-state unemployment rate in the 'WS-PS' model. The value of the separation rate q is in accordance with the results of Burda and Wyplosz (1994). $z = 0.4$ is an hypothesis that could be supported by the data available in OECD (1996).

Figures 2 to 7 show the dynamic effects of the introduction of a small partial basic income ($\xi = 0.2$, i.e. a basic income-net wage ratio, ξz, equal to 8%). Most of the adjustment typically occurs within a year. The decline of the unemployment rate is monotonous (from 8.7% when $\xi = 0$ to 7.1% when $\xi = 0.2$). Such an improvement is still insufficient to avoid a sharp increase in the tax rate τ (from about 4% to 12%; see Figure 3). Therefore, the net wage and the level of unemployment benefits have to decrease (see Figure 4). The relative change amounts to -7.3%. Compared to a situation without basic income, the *instantaneous* income of an employed worker, $w_t + B_t$, remains nearly unchanged (see Figure 5). Let us now look at the results in an *intertemporal perspective*. Considering intertemporal utility levels, the introduction of a partial basic income is in this example a Pareto improvement. For, compared to a situation without basic income, the levels of intertemporal utility of people currently unemployed (Figure 6) and employed (Figure 7) and of the eligible inactive population are raised while the well-being of the ineligible inactive people remains unaffected. Although unreported simulation results indicate that this property is not atypical, there is no claim that it applies for a very wide range of values for the parameters. Yet, the fact that the introduction of a partial basic income can be a Pareto improvement is as such a valuable result.

This property heavily depends on the level of the discount factor[15] and on the degree of unconditionality of the basic income.[16] This degree is a parameter controlled by public authorities. It could be set optimally according to a normative criterion. On the contrary, the discount rate used by the unemployed is typically a given parameter. Its value could be much higher than the one assumed above. In that case, the decrease in consumption for the unemployed could outweigh the improvement in employment prospects. As long as ν remains quite low, this drawback can be avoided by the following alternative. Let the current income of the unemployed be the maximum of B_t and Z_t, with $B_t = bw_t, 0 \leq b < 1$ and $Z_t = z(B_t + w_t) = z(1 + b)w_t, 0 \leq z < 1$. This means that the replacement ratio $\frac{Z_t}{w_t}$ is now positively affected by the basic income-net wage ratio b. Let $z = 0.4, b = 0.2$.

number of quits that occurred at the end of the previous period. Hence, the assumption $\psi = 0$ is plausible in the neighborhood of the steady state.

[13]Other components of the Welfare State such as retirement pensions and family benefits have been left aside.

[14]About the concept of *participation income*, see Atkinson (1995b).

[15]As β shrinks, the unemployed more heavily discount the future gains coming from an improved probability of hiring.

[16]As ν increases, the trade-off between the well-being of the workforce and the one of the inactive population' often becomes unavoidable. Notice however that the contribution of "inactive" people to the well-being of the whole society is entirely neglected. This points to necessary extensions.

Compared to the previous simulations, the basic income-net wage ratio is now substantially higher (20% compared to 8%). So does the replacement ratio : $\frac{Z_t}{w_t} = 0.48$. Let us keep all other parameters unchanged, except r. For the purpose of illustration, consider a huge discount rate, $r = 50\%$ ($\beta = 0.667$), for all agents. This modification influences the initial values. More importantly, the current consumption levels of the employed and the unemployed decrease when the partial basic income is introduced but the effect is small (-1%, see Figures 11 and 10). The unemployment rate still decreases but to a much lower extent (see Figure 8). Even with such a large discount rate, the intertemporal utility of both types of workers increases (see Figures 12 and 13). The tax rate increases more than before (compare Figures 3 and 9).

INSERT FIGURES 2 to 13 APPROXIMATELY HERE.

6 Assessing the results of this paper

The (partial) basic income scheme appears to be a very powerful instrument to fight unemployment. However, is this a robust result? To answer that question, d'Autume (2001b) confirms what was suggested above, namely that the effectiveness of basic income proposals should be checked in models featuring more heterogeneity among individuals. This heterogeneity could come from differences in skill. However, such differences are not sufficient. With two skill groups and negotiated wages, simulations made by Lehmann (2001) lead to a relative decrease in unemployment that is similar to the one found in the previous section. To generate new conclusions, one should probably combine differences in skill *and* investment made by workers and/or by firms (in training or in education). Then, the higher (marginal) tax rates needed to finance basic income schemes would probably affect this investment in a negative way. This would affect economic performances. This discussion is also present in the literature about tax progressivity in imperfect labor markets where the amount of training is endogenous (see Fuest and Huber, 1998).

Endogeneizing the size of the labor force is another channel through which the robustness of the previous results can be assessed. Van der Linden (1999a) introduces "informal activities" (home production or jobs in the underground economy). These activities are by assumption not observed by tax authorities. Nor do they entitle individuals to a minimum income guarantee. In a 'WS-PS' model, the participation rate has no long-run effect on the unemployment rate but it influences the balanced-budget tax rate and the net wages. Then, in steady state, handing out an unconditional income to every adult (that is putting ν equal to 1) has a negative effect on the participation rate and on the intertemporal utility levels of the unemployed and the employed workers. On the contrary, introducing an 'active citizen's income' ($\nu = 0$) turns out to have positive effects on participation and on the welfare of each group. However, in a different institutional setting, even an unconditional income ($\nu = 1$) can stimulate participation. Assume for instance that every inactive individual benefits from a means-tested minimum income guarantee (MIG, for short). A partial unconditional income (below the MIG) can be introduced without directly affecting the net income of jobless people. It is well known that the MIG creates a (financial) unemployment trap. Since the importance of this trap is reduced by the partial unconditional income, participation to the (formal) labor market will go up.

Since the trap is first of all a disincentive to take part-time jobs, the development of such jobs would be a corollary. This is clearly stated in Van Parijs, Jacquet and Salinas (2001).

7 Conclusion

To contribute to the debate about the consequences of a basic income, this paper has developed a dynamic and general equilibrium model in which collective bargaining causes unemployment. Ex ante identical workers are heterogeneous ex post : Some of them are unemployed while others are employed and benefit from a higher utility level. The analysis has assumed that an unemployment insurance system initially exists with unemployment benefits proportional to the net wage. Two reforms have been considered : The so-called partial and full basic income schemes. Both have been taken proportional to the level of unemployment benefits. The coefficient of proportionality is by assumption lower than one when the basic income is partial and higher or equal to one in the case of the full scheme. Whatever the reform under consideration, the public budget has to be balanced.

The paper has shown that two mechanisms are at work. The partial basic income increases the progressivity of taxation and it favors in-work net income. Both mechanisms lower the equilibrium unemployment rate. In the full basic income case, the first mechanism still has a favorable effect on unemployment. However, as the full basic income rises, out-of-work income increases relatively more than net earnings. This raises the equilibrium unemployment rate.

The analytical results of the paper have been derived in steady state. First, compared to a situation with an unemployment insurance system but without basic income, the equilibrium unemployment rate is always lower if a partial basic income scheme is implemented. The same holds if the unemployment insurance system is replaced by a full basic income scheme provided that the ratio between the basic income and the unemployment benefits is not too high. Second, the equilibrium unemployment rate is a decreasing function of the ratio between the partial basic income and the unemployment benefits. Third, if workers are risk averse, the equilibrium unemployment rate is an increasing function of the ratio between the full basic income and the unemployment benefits. If workers are risk neutral, the full basic income has no effect on the equilibrium unemployment rate. Fourth, the equilibrium marginal tax rate typically increases with the basic income-unemployment benefits ratio while the net wage rate generally decreases.

From the dynamic analysis, it can be concluded that convergence to the steady state is very rapid. In an intertemporal perspective, a sufficiently small partial basic income-unemployment benefits ratio can be a Pareto improvement. To reach such a conclusion, two conditions should be fulfilled. First, unemployed people should not discount future incomes too much. With high discount rates, this paper has shown that alternative schemes can however be designed in order to improve the intertemporal utility levels of both the unemployed and the employed. Second, the basic income should not be handed out to a too large fraction of the inactive population. Otherwise, the tax rate increases too sharply. However, a thorough discussion of the last feature requires a model that deals with the contribution of inactive people to the well-being of the whole society.

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Appendix 1

This appendix deals with the dynamic properties of the equilibrium. For $t \geq 1$, the ratio $\frac{w_{t+1}}{w_t}$ in equation (13) can be derived from equation (15). It can be then checked that for $t \geq 1$ the unemployment rate fluctuates according to the following second-order equations :

$$\theta_1 \left(\frac{\theta_2 - \theta_3 u_t}{\theta_2 - \theta_3 u_{t+1}} \frac{1 - u_{t+1}}{1 - u_t} \right)^{\lambda} - \frac{q + (1-q)u_{t-1}}{u_t} + \theta_4 = 0, \tag{22}$$

with a partial basic income scheme and

$$\theta_1 \left(\frac{\theta_2 - u_t}{\theta_2 - u_{t+1}} \frac{1 - u_{t+1}}{1 - u_t} \right)^{\lambda} - \frac{q + (1-q)u_{t-1}}{u_t} + \theta_4 = 0, \tag{23}$$

with the full basic income scheme. In these expressions,

$$\begin{aligned}
\theta_1 &= \beta(1-q),\ 0 < \theta_1 < 1, \\
\theta_2 &= 1 + \xi z(1+\nu),\ \theta_2 > 1, \\
\theta_3 &= 1 + (\xi - 1)z,\ 0 < \theta_3 < 1 \text{ since } \xi < 1, \\
\theta_4 &= \frac{1 + \xi z}{\mu \lambda} \left(1 - \left[\frac{z\mathcal{I}(\xi)}{1 + \xi z} \right]^{\lambda} \right),\ \theta_4 > 0.
\end{aligned}$$

Let $j = 1$ refer to the partial basic income and $j = 2$ to the full one. Let us look at the local behavior of the second-order scalar nonlinear difference equations (22) and (23) around their corresponding steady state u^{j*} (defined in (16)). The linearized difference equation can be converted to an equivalent first-order planar map :

$$\begin{pmatrix} u_{t+1} - u^{j*} \\ u_t - u^{j*} \end{pmatrix} = \mathcal{A}^j \begin{pmatrix} u_t - u^{j*} \\ u_{t-1} - u^{j*} \end{pmatrix} \tag{24}$$

In this expression, $\mathcal{A}^j$ is the 2×2 matrix

$$\begin{pmatrix} 1 + \frac{q + (1-q)u^{j*}}{\zeta^j (u^{j*})^2} & -\frac{1-q}{\zeta^j u^{j*}} \\ 1 & 0 \end{pmatrix} \tag{25}$$

where

$$\begin{aligned}
\zeta^1 &= \lambda \theta_1 \frac{\theta_2 - \theta_3}{(\theta_2 - \theta_3 u^{1*})(1 - u^{1*})}, \\
\zeta^2 &= \lambda \theta_1 \frac{\theta_2 - 1}{(\theta_2 - u^{2*})(1 - u^{2*})}.
\end{aligned}$$

For $j = 1, 2$, since $\theta_2 > 1$ and $\theta_3 \in]0, 1[$, it can be checked that ζ^j has the same sign as λ.

The characteristic polynomial is $P^j(\omega) = \omega^2 - (tr\mathcal{A}^j)\omega + (det\mathcal{A}^j)$ where $tr\mathcal{A}^j = 1 + \frac{q+(1-q)u^{j*}}{\zeta^j(u^{j*})^2}$ and $det\mathcal{A}^j = \frac{1-q}{\zeta^j u^{j*}}$ (with $sgn(det\mathcal{A}^j) = sgn(\lambda)$).

With one predetermined variable, the saddle point property is required in order to have a unique nonexploding solution. This property is guaranteed if

$$(tr\mathcal{A}^j)^2 - 4(det\mathcal{A}^j) = \left(1 + \frac{q}{\zeta^j(u^{j*})^2}\right)^2 + \frac{1-q}{\zeta^j u^{j*}}\left(\frac{2q}{\zeta^j(u^{j*})^2} + \frac{1-q}{\zeta^j u^{j*}} - 2\right) > 0. \quad (26)$$

and if $[P^j(1) < 0$ and $P^j(-1) > 0]$ or $[P^j(1) > 0$ and $P^j(-1) < 0]$, where

$$\begin{aligned} P^j(1) &= -\frac{q}{\zeta^j(u^{j*})^2}, \text{ with } sgn[P^j(1)] = -sgn[\lambda], \\ P^j(-1) &= 1 + tr\mathcal{A}^j + det\mathcal{A}^j = 2 + \frac{q + 2(1-q)u^{j*}}{\zeta^j(u^{j*})^2}. \end{aligned}$$

Let us check these conditions.

The case where $\lambda > 0$

There is no proof that condition (26) is always satisfied. Yet, numerical simulations show that it is verified for $0.01 < q < 0.4$ and $0 \leq u^{j*} \leq 1$. The former interval covers the range of plausible values for q. The steady-state unemployment rate u^{j*} is a function of all the parameters. However, remember that condition (17) is assumed to hold. This condition implies that $0 \leq u^{j*} \leq 1$. Therefore, condition (26) should be considered as fulfilled. Furthermore, without having to refer to the parameters of the model, it is easily seen that $P^j(1) < 0$ and $P^j(-1) > 0$. Hence, the linearized dynamic system (24) has the saddle point property.

The case where $\lambda < 0$

Here, $(tr\mathcal{A}^j)^2 - 4(det\mathcal{A}^j)$ and $P^j(1)$ are always positive but $P^j(-1)$ is not necessarily negative. A numerical analysis shows that $P^j(-1)$ can become positive for very high values of z and low values of ξ (creating mass unemployment). This is illustrated in Figure 14. For the values of the parameters assumed in Section 5 ($\beta = 0.95$), this figure shows that $P^j(-1)$ becomes positive when $z \geq 0.8$ and $\xi = 0$ ($u = 57\%$). For more commonly observed value of z, unreported simulations show that $P^j(-1)$ remains negative even for extreme values of the other parameters (like q or γ). To sum up, the saddle point property holds for plausible values of the parameters.

INSERT FIGURE 14 APPROXIMATELY HERE.

Appendix 2

This appendix proves Result 1. Let D be the denominator of (16). Carrying out the appropriate first-order derivatives yields :

$$\begin{aligned}
\frac{\partial u}{\partial r} &= \frac{\partial u}{\partial \beta}\frac{\partial \beta}{\partial r} = \frac{q(1-q)}{D^2(1+r)^2} > 0, \\
\frac{\partial u}{\partial q} &= \frac{1}{D}\left(1 - \frac{q(1-\beta)}{D}\right) > 0 \text{ since } u < 1, \\
\frac{\partial u}{\partial \mu} &= \frac{q}{D^2\mu^2}\frac{1+\xi z}{\lambda}\left(1 - \left[\frac{z\mathcal{I}(\xi)}{1+\xi z}\right]^{\lambda}\right) > 0, \\
\frac{\partial u}{\partial z} &= -\frac{q\xi}{\mu\lambda D^2}\left[1 - \left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)^{\lambda}\left(1 + \frac{\lambda}{\xi z}\right)\right] > 0.
\end{aligned}$$

The last property cannot be shown analytically if $\xi < 1$. Yet, an unreported numerical analysis shows that it is verified for plausible values of the parameters. For $\xi \geq 1$, let $\kappa = \frac{\xi z}{1+\xi z}$, $0 < \kappa < 1$. The denominator of (16) can now be rewritten as $\frac{1-\kappa^{\lambda}}{\mu\lambda(1-\kappa)} - (1-\beta)(1-q)$. As it is explained in the proof of Result 3, $\frac{1-\kappa^{\lambda}}{\mu\lambda(1-\kappa)}$ decreases with κ. Since κ increases with z, it is immediately seen that u increases with z.

Finally, relative risk aversion equals $1-\lambda$ and

$$\frac{\partial u}{\partial \lambda} = \frac{q(1+\xi z)}{\mu\lambda D^2}\left(\frac{1}{\lambda}\left(1 - \left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)^{\lambda}\right) + \ln\left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)\left(\frac{z\mathcal{I}(\xi)}{1+\xi z}\right)^{\lambda}\right) > 0,$$

again on the basis of a numerical simulation. ■

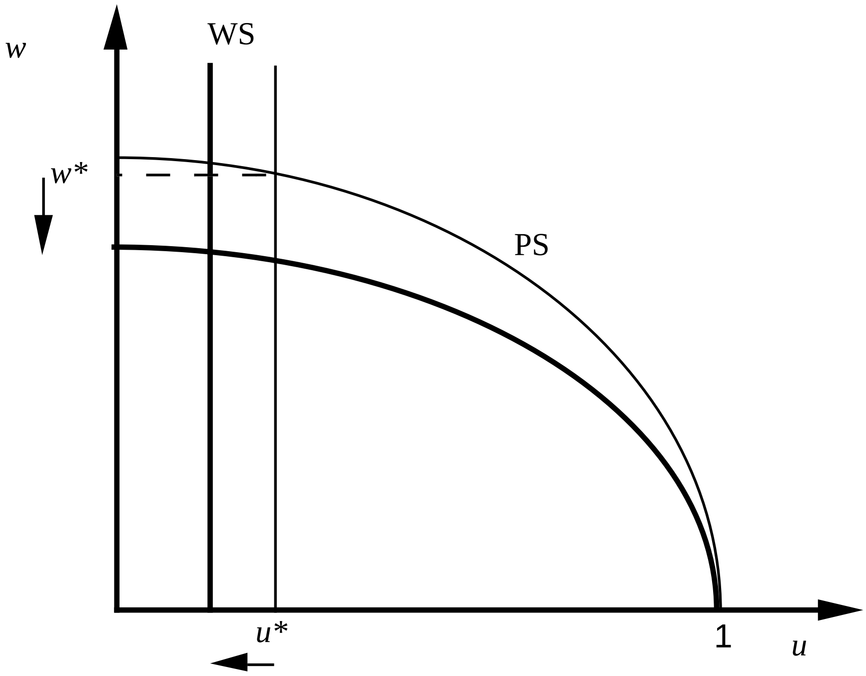


Figure 1: The steady-state equilibrium

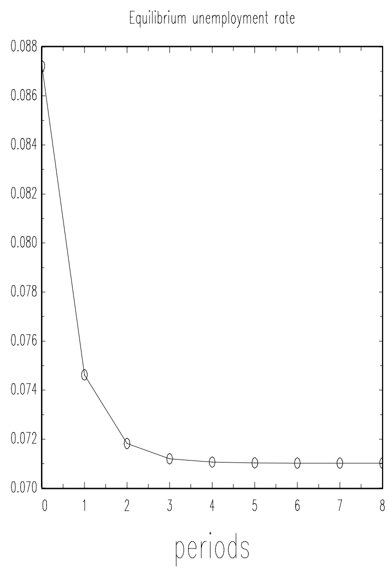


Figure 2: Dynamics of the unemployment rate u_t

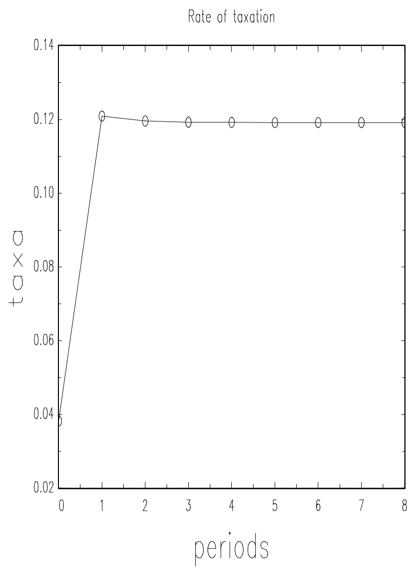
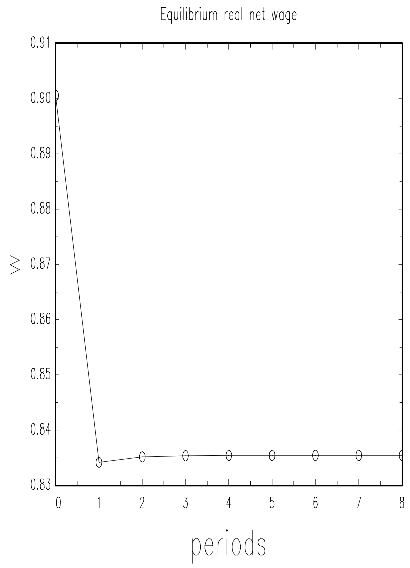


Figure 3: Dynamics of the tax rate τ_t



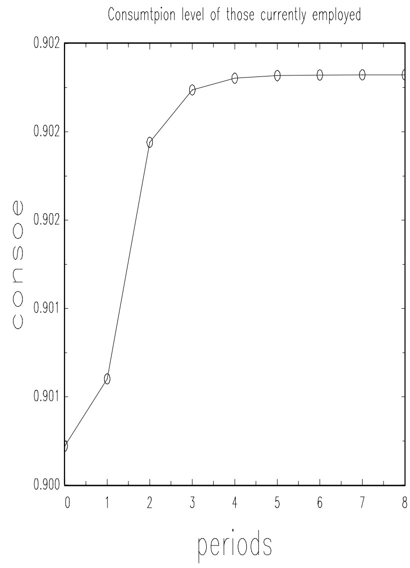
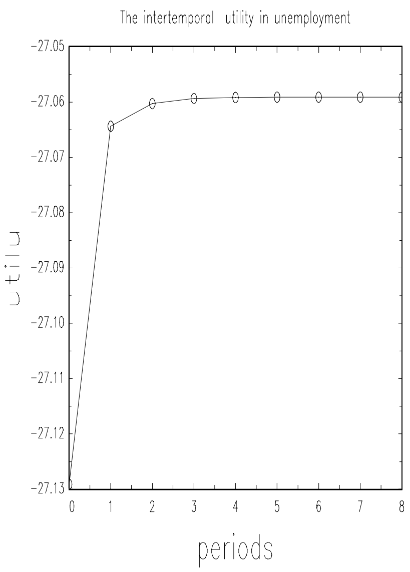


Figure 4: Dynamics of the net wage w_t

Figure 5: Dynamics of consumption for employed people $(1 + \xi z)w_t$



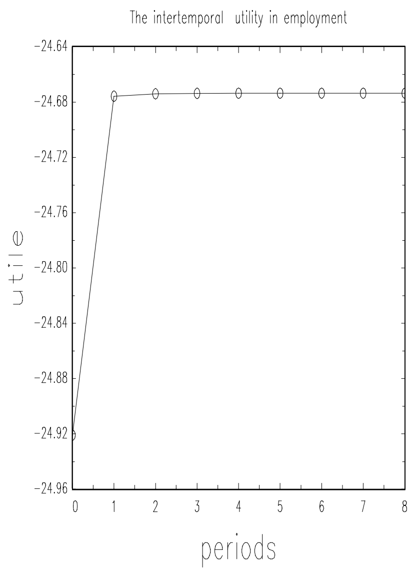
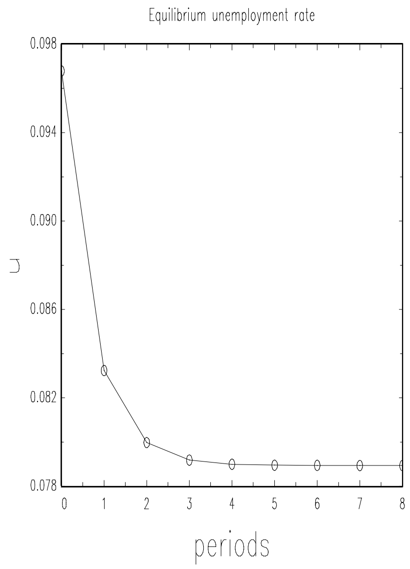


Figure 6: Dynamics of the intertemporal utility for the unemployed V_u^t

Figure 7: Dynamics of the intertemporal utility for the employed V_e^t



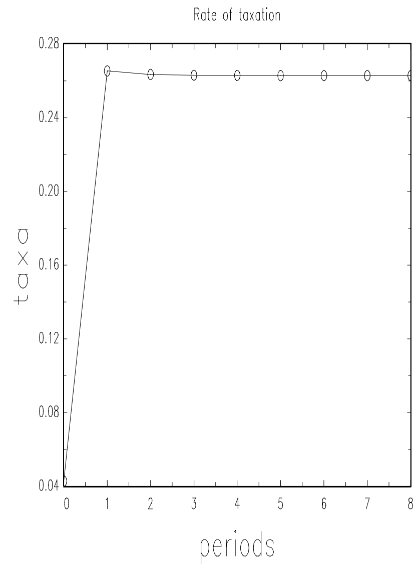
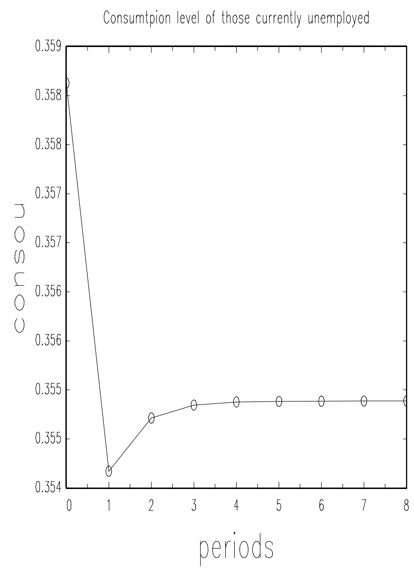


Figure 8: Dynamics of the unemployment rate u_t

Figure 9: Dynamics of the tax rate τ_t



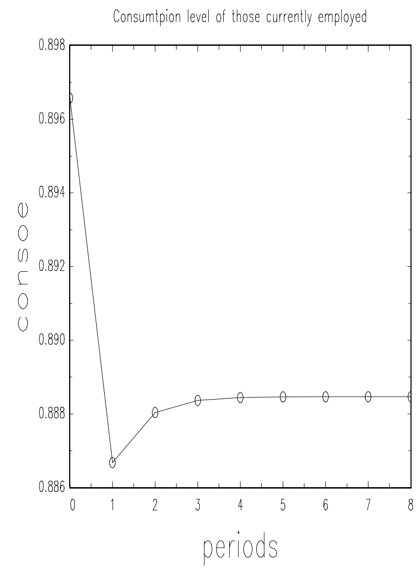


Figure 10: Dynamics of consumption for unemployed people $z(1+b)w_t$

Figure 11: Dynamics of consumption for employed people $(1+b)w_t$

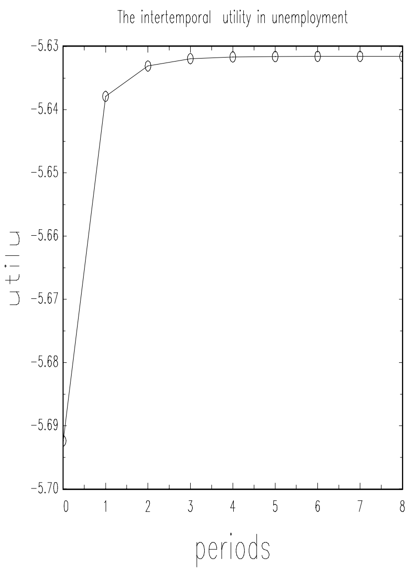


Figure 12: Dynamics of the intertemporal utility for the unemployed V_u^t

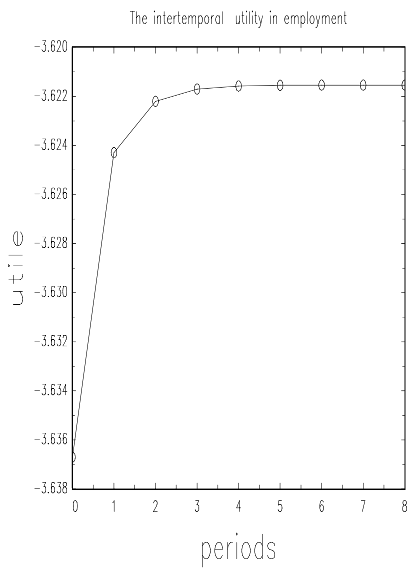


Figure 13: Dynamics of the intertemporal utility for the employed V_e^t

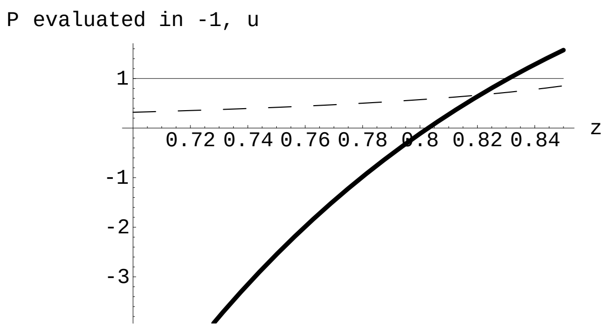


Figure 14: The characteristic polynomial evaluated at -1 (thick line) and the steady-state unemployment rate (interrupted line) as a function of z when there is no basic income (the case where $\lambda < 0$)