Optimal unemployment benefits and non-linear income taxation in a matching model with wage bargaining [*]

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Abstract

This paper characterizes the optimal level of unemployment benefits in a job-matching framework with wage bargaining, endogenous job search and risk-aversion. It is shown that taxation has to be non linear in order to decentralize the optima for each value of the bargaining power. When the State can perfectly monitor the unemployed workers' search intensity, the unemployment risk is perfectly insured and the marginal tax rate is equal to 1. When search behavior is unobservable, the optimum is characterized by imperfect unemployment insurance and by lower levels of search intensity and output. The implementation of this optimum requires lower levels of unemployment benefits, taxes and marginal tax rates.

Keywords: Unemployment, Non-linear Taxation, Unemployment Benefits, Moral Hazard, Search.

Résumé

Nous étudions le système optimal d'allocations chômage dans un modèle d'appariements comprenant des négociations salariales, un effort de recherche endogène et des agents averses vis-à-vis du risque. Nous montrons qu'une taxation non linéaire est indispensable à la décentralisation des optima et ce quelle que soit la valeur du pouvoir de négociations salariales. Lorsque l'Etat peut contrôler l'effort de recherche des chômeurs, l'assurance chômage est complète et le taux marginal est égal à 1. Dans le cas contraire, l'assurance chômage est incomplète et l'intensité de recherche est plus faible ainsi que le produit total. Dans ce dernier cas, les niveaux des allocations chômage, des taxes et le taux de taxe marginal sont plus faibles.

Mots clefs: Chômage, Taxation non linéaire, Allocations chômage, Aléa moral, Recherche d'emploi.

JEL : J64, J65, J68, H21, D82

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I Introduction

This paper addresses the issue of the optimal combination of unemployment benefits and income taxation. This question is dealt with in a general equilibrium matching model where unemployment benefits have a positive effect on wages and therefore a negative one on the arrival rate of job offers. These effects are best highlighted in an equilibrium matching framework where workers and firms bargain over the rents created by frictions on the labor market. Compared to partial equilibrium analyses, these general equilibrium effects of unemployment benefits lead to lower optimal replacement ratios. So, risk-sharing has to be less efficient in order to limit the negative impact on unemployment and more generally on allocative efficiency. Different mechanisms can however be envisaged to limit these general equilibrium effects.

Some authors have considered institutional settings where unions simultaneously bargain over wages and organize the unemployment insurance system (Holmlund and Lundborg (1989), Kiander (1993)). So doing, the union internalizes the general equilibrium effects of unemployment benefits. These papers consider however institutional arrangements that are rather specific to Nordic countries. For in most of other OECD countries, unemployment benefits are not directly determined by wage setters. Other papers have relaxed the assumption of constant unemployment benefits (Cahuc and Lehmann (2000) and Fredriksson and Holmlund (2001)). Whether one should or not introduce degressive unemployment benefits remains an open question that the present paper does not address. Sanctions (i.e. withdrawal of unemployment benefits if search effort is judged insufficient) are an alternative that allows to improve risk-sharing for those who comply with the rules (Boadway and Cuff (1999), Boone and van Ours (2000), Boone *et al* (2001)). This is expected to hold as long as search effort is observable without (too frequent) errors.

In this paper, we introduce an alternative mechanism, namely non-linear income taxation. It is now well known that progressive income taxes have a moderating effect on bargained wages (see Malcomson and Sator (1987), Lockwood and Manning (1993), Holmlund and Kolm (1995), Pissarides (1998) and Van der Linden (2002) who deals with negative income taxation). Actual policies such as the EITC in the US or the WFTC in the UK and more generally the so-called "activating policies" have been conceived in order to stimulate search effort and/or the acceptance rate of job offers but they should also influence the wage setting process (see e.g. Keen (1997)). So, both the academic literature and existing reforms point to the need of an analysis of the optimal combination of unemployment benefits and income tax progression. Given the focus of this paper, job search is treated as endogenous and the cases where search effort is observable and unobservable are contrasted. Through analytical and simulation results, this paper argues that non linear income taxation should be considered as complementary to unemployment insurance for, compared to linear taxation, it allows to enhance efficiency in the allocation of resources and in risk-sharing.

In a general equilibrium framework, the optimization of unemployment benefits cannot be treated without dealing with the sharing of net income between firm-owners and workers. This can be done by imposing a minimal level of profits irrespective of the values of parameters such as workers' risk aversion or the marginal disutility of job search. The distribution of net income between firms-owners and workers is then fixed without an explicit normative criterion. We prefer to deal with this distributional problem in an explicit way. Assuming risk-neutral firm owners, we maximize a social welfare function that adds the expected utility of a worker facing the risk of unemployment and profits weighted by an explicit parameter. So doing, we adjust profits in an optimal way conditional on an explicit social weight attributed to them.

In this paper, search intensity and tightness determine efficiency. High search intensity and tightness improve the matching process and increase employment but they imply costs. A high level of search intensity (respectively tightness) implies a high disutility for unemployed workers (respectively high costs of hiring for the representative firm-owner). This corresponds to the standard allocative efficiency problem highlighted by Hosios (1990) and Pissarides (2000) in a matching framework. Equity consists in sharing the net output between the workers (taken as a whole) and the representative firm-owner. The insurance problem consists then in sharing the unemployment risk among workers facing the risk of unemployment.

This paper proposes a synthesis between the Hosios-Pissarides literature and the literature about optimal unemployment insurance. The first one is concerned with efficiency and generally neglects insurance and redistributive issues. The second one is concerned with the latter issues but typically ignores firms behavior and the wage setting process.

In general, there is a trade-off between allocative efficiency and the equity and insurance dimensions. However, under perfect information about search intensity, it will turn out that the first-best levels of search intensity and tightness maximize the allocative efficiency criterion, namely total output net of costs. In addition, workers are perfectly insured against the unemployment risk (the marginal utility in employment and unemployment being both equal to the social weight of profits). This solution can be decentralized by setting the marginal tax rate at 1. This prevents insiders from capturing rents when they negotiate their wage.

The assumption of perfect information about search intensity seems implausible. In this paper, the second-best setting is characterized by the polar assumption that unemployed workers' search intensity cannot be observed by the State. Then, the trade-off between allocative efficiency and the equity and insurance dimensions cannot be avoided any more. Full insurance against the unemployment risk is no more possible since this would induce unemployed people not to search anymore. We show that, at the second-best optimum, search intensity is lower and tightness (measured in efficiency units) is higher than at the first-best optimum. Conditional on any value of the workers' bargaining power, the decentralization of the second-best optimum then implies lower marginal tax rate, lower levels of tax and typically lower unemployment benefits than at the first-best optimum. Simulation results suggest that the unemployment rate and the expected utility of workers are higher at the second best. Simulations also emphasize the substantial losses in efficiency and/or insurance if taxation has to be proportional to income.

The paper is organized as follows. Section II describes the structure of the economy. Section III is devoted to the equilibrium, section IV to the first-best optimum and its decentralization, section V to the second best optimum. Some numerical simulations are presented in section VI and section VII concludes.

II Assumptions and Notations

The economy is made of a continuum of homogenous risk-averse workers, a representative risk-neutral firm and the State. There are no financial markets. Workers can either be employed or unemployed. Jobs can either be filled or vacant. Agents are infinitely lived.

The model is based on the assumption that the matching between unemployed workers and vacant jobs is a time-consuming and costly process due to various frictions on the labor market. Assume a continuous-time setting. The flow of hires M is a function $M(S,v)$ of the number of job-seekers measured in efficiency units S and of the number of vacancies v. It is standard to assume that this function is increasing and concave in both arguments (with $M(0,v) = M(S,0) = 0$) and that returns to scale are constant (see e.g. Pissarides (2000)). Denoting by e the average search intensity and by u the mass of unemployed workers, one has $S = e \cdot u$. Let $\theta \equiv v/S$ be tightness on the labor market (measured in efficiency units). The rate at which a vacant job is filled is $m(\theta)$ with $m(\theta) = \frac{M(S,v)}{v} = M(1/\theta, 1)$, and $m'(.) < 0$. An unemployed with search intensity $e_i \geq 0$ flows out of unemployment at a rate $e_i \cdot \alpha(\theta) = \frac{e_i}{e} \cdot \frac{M(e \cdot u, v)}{u}$, with $\alpha(\theta) \equiv M(1,\theta) = \theta \cdot m(\theta)$ and $\alpha'(\theta) > 0$, $\alpha''(.) < 0$. Job matches end at the exogenous rate q.

We normalize the size of the labor force to 1. In steady state, equality between entries and exits implies the "Beveridge curve" equation:

$$e \cdot \alpha(\theta) = q(1-u) \qquad \Leftrightarrow \qquad u = \frac{q}{q + e \cdot \alpha(\theta)} \tag{1}$$

that negatively links the unemployment rate to tightness θ.

Let r be the discount rate common to workers and firms. An employed worker has an instantaneous utility function $v(\omega)$, where ω denotes her after-tax income. An unemployed worker has an instantaneous utility $v(z-d(e))$ where z denotes her untaxed unemployment benefits. We assume $v'(.) > 0$, $d(.) \geq 0, d'(.) > 0$ and $d''(.) \geq 0$ (with $\lim_{e\to 0} d'(e) = 0$ and $\lim_{e\to\infty} d'(e) = +\infty$). The risk aversion assumption implies $v''(.) < 0$. For the unemployed, a possible interpretation of our specification is that $d(e)$ denotes the monetary cost of job-search activities. Then, $z - d(e)$ would stand for the net level of consumption of the unemployed. However, on top of expenses related to job-search activities, $d(e)$ can also capture the disutility of search effort.

The representative firm is made of L filled jobs and v vacant jobs. Each filled job produces a flow of y units of output, whereas each vacant job costs c per unit of time. With a normalized labor force, one *ex-post* has $L = 1 - u$. χ is a lump-sum transfer or tax paid to or by the representative firm, with $\chi \lesseqgtr 0$ [1]. Hence, the representative firm-owner's income flow is:

$$\Pi = (1-u)(y-w) - c \cdot v + \chi \tag{2}$$

where w is the gross wage.

A tax T is levied on each filled job by the government [2], with $T = w - \omega$. These resources are used to finance unemployed benefits and the transfer to the firm-owner. The public budget has to balance at any point in time, so:

$$u \cdot z + \chi = T(1-u) \tag{3}$$

This budget constraint together with (2) gives the following aggregate resource constraint:

$$(1-u)\omega + z \cdot u + \Pi + c \cdot v = (1-u)y \tag{4}$$

Rearranging this expression one gets

$$(1-u)\omega + u(z - d(e)) + \Pi = Y \tag{5}$$

where $Y \equiv (1-u)y - u \cdot d(e) - c \cdot v$ stands for total output net of search and vacancy costs. "Efficiency" will be achieved when Y is maximized. The redistribution problem consists in sharing this net output between employed workers (ω), unemployed workers ($z - d(e)$) and the firm-owner (Π).

[1] We allow for such lump sum transfer so as to guarantee to the State a complete set of fiscal tools. We therefore consider that second best considerations can arise only for a single reason: the inobservability of search intensity.

[2] Therefore T captures the income tax and social security contributions, if any.

III The Market equilibrium

III.1 The representative firm

Intertemporal profits as of time t are:

$$\mathbb{P}_t = \int_t^{+\infty} e^{-r(\tau - t)} \cdot \Pi_\tau \cdot d\tau \tag{6}$$

At time $t = 0$, the representative firm-owner maximizes:

$$\max_v \quad \mathbb{P}_0 \qquad s.t. \quad \dot{L} = m(\theta) \cdot v - q \cdot L$$

taking tightness θ as given. Since marginal productivity y is constant, the same conditions would be reached in the standard model where each firm holds only one job. In this case, let J denotes the intertemporal expected value of a filled vacancy and J^v the expected value of an open vacancy. J and J^v verify the following equations:

$$r \cdot J - \dot{J} = y - w + q(J^v - J) \tag{7}$$

$$r \cdot J^v - \dot{J}^v = -c + m(\theta)(J - J^v) \tag{8}$$

Assuming free entry of vacancies implies $J^v = \dot{J}^v = 0$. Hence, in a steady-state equilibrium:

$$J = \frac{c}{m(\theta)} = \frac{y-w}{r+q} \tag{9}$$

This leads to:

$$w = \phi(\theta) \equiv y - \frac{c(r+q)}{m(\theta)} \tag{10}$$

This relationship between the gross wage w and tightness θ is downward-sloping. The higher w, the lower the value of a filled job J, so the less vacancies in the economy and the lower tightness θ. Since θ is measured in efficiency units, one should note that this relation does not depend on search intensity e. The equivalence between the free entry condition (9) and the inverse labor demand equation (10) is straightforward.

III.2 Search Behavior

Let V and V^u denote the expected lifetime utility of respectively an employed and an unemployed worker. V solves:

$$r \cdot V - \dot{V} = v(w - T) + q(V^u - V) \tag{11}$$

Two cases will be considered. The one where search intensity is observable will be introduced later. When search cannot be observed, an unemployed worker has to choose her search intensity at any point in time. With a search intensity e_i, her instantaneous utility is $v(z - d(e_i))$ and her expected "capital gain" is $e_i \cdot \alpha(\theta)(V - V^u)$. Hence, she solves:

$$r \cdot V^u - \dot{V}^u = \max_{e_i} \{v(z - d(e_i)) + e_i \cdot \alpha(\theta)(V - V^u)\} \tag{12}$$

taking V, V^u, $\dot{V}^u$ and θ as given. The first-order condition of this problem is [3]:

$$0 = \alpha(\theta)(V - V^u) - d'(e) \cdot v'(z - d(e)) \tag{13}$$

At a steady state where $\dot{V} = \dot{V}^u = 0$, equation (13) together with equations (11) and (12) implicitly defines the optimal search level e according to $0 = S(\theta, w, e)$ with:

$$S(\theta, w, e) \equiv \alpha(\theta)(v(w - T) - v(z - d(e))) - d'(e) \cdot v'(z - d(e))(r + q + e \cdot \alpha(\theta)) \tag{14}$$

It can be verified (see Appendix 1) that the following partial derivatives have unambiguous signs [4]: $S'_e < 0$, $S'_w > 0$, $S'_\theta > 0$. Therefore, the optimal search intensity increases with w and θ. It can be checked that an increase in T lowers search intensity (since $S'_T = -S'_w < 0$) while a rise in the level of unemployment benefits has an ambiguous effect on e. With the chosen instantaneous utility function, an increase in z reduces the marginal disutility of search effort [5]. It also decreases the marginal gain of search. Hence the ambiguous net effect on e.

Microeconometric estimations generally lead to the conclusion that the individual exit rate out of unemployment is negatively affected by the level of unemployment benefits. From this evidence, the case where:

$$S'_z < 0 \tag{15}$$

is the most reasonable.

III.3 The Wage Bargain

A match generates a surplus that is shared between the worker and the firm-owner. Let γ be the exogenous bargaining power of the worker, with $0 < \gamma < 1$. The gross wage rate maximizes the following Nash product:

$$\max_w \quad (V - V^u)^\gamma (J - J^v)^{1-\gamma}$$

[3] The second-order condition is satisfied since $d''(.) \geq 0$ and $v''(.) < 0$. It should be noticed that similar conclusions would be obtained if the utility of unemployed worker i was denoted by $v(z - c_i)$ and if her exit rate from unemployment was written as $p(c_i) \cdot \alpha(\theta)$, provided that $c \equiv d(e)$ and $p(.) \equiv d^{-1}(.)$.

[4] For any function $f(., ..., .)$, f'_x denotes the partial derivatives of f with respect to x.

[5] This effect would not be present if the instantaneous utility function was separable.

The level of taxes T is a function of the gross wage w. The wage setters integrate that a marginal rise of the gross wage of an amount Δw changes the level of taxes by $T_m \cdot \Delta w$, where T_m denotes the marginal tax rate. Taking this relationship and θ as given, the first-order condition of the previous maximization can be written as :

$$0 = \gamma \frac{v'(w - T) \cdot (1 - T_m)}{V - V^u} - \frac{1 - \gamma}{J - J^v} \tag{16}$$

Combining this equality with (11) and (12) and the free entry condition (9) yields at a steady state $WS(\theta, w, e) = 0$ with :

$$WS(\theta, w, e) \equiv v(w - T) - v(z - d(e)) - \frac{c \cdot \gamma \cdot (1 - T_m)}{1 - \gamma} \frac{r + q + e \cdot \alpha(\theta)}{m(\theta)} \cdot v'(w - T) \tag{17}$$

This equation defines the wage-setting curve. From Appendix 1 one has: $WS'_\theta < 0$, $WS'_w > 0$, $WS'_e = -\frac{S(\theta,w,e)}{r+q+e\cdot\alpha(\theta)}$, $WS'_T < 0$, $WS'_{T_m} > 0$ and $WS'_z < 0$. Conditional on e, the wage-setting curve is therefore upward-sloping in a (θ, w) space. If the marginal tax rate is fixed and θ and e are given, increasing the level of taxes T has a (gross) wage push effect. On the contrary, for given levels of taxes T, tightness θ and search intensity e, a more progressive tax schedule will put a downward pressure on the negotiated wage. This is the standard result highlighted among others by Malcomson and Sator (1987), Lockwood and Manning (1993) Kolm and Holmlund (1995), Pissarides (1998). More generous unemployment benefits have the usual positive effect on wages.

III.4 Steady state Equilibrium

Conditional on z, T, T_m, a steady-state equilibrium (θ, w, e) is a solution of the system:

$$w = \phi(\theta) \qquad S(\theta, w, e) = 0 \qquad WS(\theta, w, e) = 0 \tag{18}$$

where $\phi(.)$, $S(., ., .)$ and $WS(., ., .)$ are respectively defined in (10), (14) and (17). Equation (1) then gives the unemployment rate u and consequently the rate of vacant jobs v. Finally, equation (3) sets the level of the transfer to the representative firm-owner χ.

The equilibrium can be characterized in a more simple way by defining functions $\mathbb{S}(\theta, e) \equiv S(\theta, \phi(\theta), e)$ and $\mathbb{W}(\theta, e) \equiv WS(\theta, \phi(\theta), e)$. Since equation (10) depends neither on search intensity e nor on policy variables (z, T, T_m), one gets $\mathbb{S}'_e(\theta, e) = S'_e(\theta, \phi(\theta), e) < 0$, $\mathbb{S}'_x(\theta, e) = S'_x(\theta, \phi(\theta), e)$ for $x = z$, T, T_m [6]. Similarly, one has $\mathbb{W}'_\theta(\theta, e) < 0$, $\mathbb{W}'_e(\theta, e) = W'_e(\theta, \phi(\theta), e) = -\frac{\mathbb{S}(\theta,e)}{r+q+e\cdot\alpha(\theta)}$ and $\mathbb{W}'_x(\theta, e) = W'_x(\theta, \phi(\theta), e)$ for $x = z, T, T_m$. Appendix 2 then shows that system $\mathbb{S}(\theta, e) = \mathbb{W}(\theta, e) = 0$ admits at most one solution, so the equilibrium if any is unique.

[6] $\mathbb{S}'_\theta(\theta, e) = S'_\theta(\theta, \phi(\theta), e) + S'_w(\theta, \phi(\theta), e) \cdot \phi'(\phi(\theta))$ can not be signed.

Exploiting the property that in equilibrium $\mathbb{W}'_e = 0$, Appendix 2 shows that:

$$d\theta = -\frac{\mathbb{W}'_z}{\mathbb{W}'_\theta}dz - \frac{\mathbb{W}'_T}{\mathbb{W}'_\theta}dT - \frac{\mathbb{W}'_{T_m}}{\mathbb{W}'_\theta}dT_m$$

Since $\mathbb{W}'_\theta < 0$, $\mathbb{W}'_z < 0$, $\mathbb{W}'_T < 0$, $\mathbb{W}'_{T_m} > 0$ one has $d\theta/dz < 0$, $d\theta/dT < 0$ and $d\theta/dT_m > 0$. These results are consistent with the literature (see Pissarides (1998, 2000) and Fredriksson and Holmlund (2001)). Since $\phi(\theta)$ is not directly influenced by z, T or T_m, the marginal effects of these policy parameters on the equilibrium value of tightness are actually directly determined by their partial effects on the wage setting curve $WS(.,.,.)$. However, Appendix 2 explains why the marginal effect of z, T, T_m on e cannot be signed. Hence, in general, we cannot conclude about the net effects of these policy parameters on unemployment.

Proposition 1 *There is (at most) a single steady-state equilibrium in this economy. At the equilibrium, tightness θ (respectively, the gross wage w) decreases (resp. increases) with the levels of unemployment benefits z and tax T and increases (resp. decreases) with the marginal tax rate T_m.*

IV The First-Best optimum

In this section, we look at the optimal allocation of resources that a benevolent social planner would implement if he could perfectly control search intensity.

IV.1 The central planner problem

Consider a benevolent planner in charge of the unemployment insurance and the redistribution systems. For the reasons given in the introduction, we assume that the planner maximizes the following social welfare function Ω:

$$\Omega = (1-u)\,V + u \cdot V^u + \eta \cdot \mathbb{P}$$

The objective function Ω is a utilitarian criterion that adds two components. The first one, $(1-u)\,V + u \cdot V^u$, is the sum of the inter-temporal utilities of employed and unemployed workers weighted by their numbers. The second component is the inter-temporal profit of the representative firm-owner weighted by a parameter $\eta > 0$. This parameter is therefore the (constant) social marginal value of profits. The first component of Ω can also be reinterpreted in an *ex-ante* perspective as the expected utility of a representative worker who is aware that she will be unemployed with a probability [7] u and employed

[7] The reinterpretation of u as a probability is made possible by our normalization to 1 of the (exogenous) size of the labor force.

with probability $1-u$. In what follows, we privilege the insurance interpretation and the expression "redistribution" will designate the way net income is shared between the firm-owner and the workers taken as a whole.

Appendix 3 then shows that:

$$\Omega = \int e^{-rt}\left\{(1-u)\,v(\omega) + u \cdot v(z - d(e)) + \eta \cdot \Pi\right\}\, dt$$

Hence, at the steady state, maximizing Ω is equivalent to maximizing $(1-u)\,v(\omega) + u \cdot v(z-d(e)) + \eta \cdot \Pi$. For simplicity, it is standard to ignore the transitional dynamics and to consider that $r \to 0$. The social planner therefore maximizes $(1-u)\,v(\omega) + u \cdot v(z-d(e)) + \eta \cdot \Pi$. It is reasonable to impose a nonnegative [8] value for Π. However, a constraint $\Pi \geq 0$ will not explicitly be imposed in the following maximization problems. It is implicitly assumed that the exogenous value of η is sufficiently high so that Π is nonnegative at the optimum. Taking the resource constraint (4) and the flow equilibrium equation (1) into account and remembering that $v = e \cdot \theta \cdot u$, the planner's program then consists in:

$$\max_{\theta,\omega,u,z,e} \quad (1-u) \cdot v(\omega) + u \cdot v(z - d(e)) + \eta \cdot [(1-u)(y-\omega) - (z + c \cdot e \cdot \theta)\,u] \quad (19)$$

$$s.t.: e \cdot \alpha(\theta) \cdot u = q\,(1-u) \quad (\delta_1)$$

Introducing subscript 1 to denote the first-best optimum and denoting δ_1 the Lagrange multiplier, the first order conditions are:

$$0 = (1-u_1)\left[v'(\omega_1) - \eta\right] \quad (20)$$

$$0 = u_1\left[v'(z_1 - d(e_1)) - \eta\right] \quad (21)$$

$$0 = u_1\left[-v'(z_1 - d(e_1)) \cdot d'(e_1) - \eta \cdot c \cdot \theta_1 + \delta_1 \cdot \alpha(\theta_1)\right] \quad (22)$$

$$0 = v(z - d(e_1)) - v(\omega_1) + \eta\,(\omega_1 - y - z_1 - c \cdot e_1 \cdot \theta_1) + \delta_1\,(e_1 \cdot \alpha(\theta_1) + q) \quad (23)$$

$$0 = -c \cdot e_1 \cdot \eta \cdot u_1 + \delta_1 \cdot e_1 \cdot \alpha'(\theta_1) \cdot u_1 \quad (24)$$

Equations (20) and (21) imply that:

$$v'(\omega_1) = v'(z_1 - d(e_1)) = \eta \quad (25)$$

Hence:

$$\omega_1 = z_1 - d(e_1)$$

Under perfect information, according to the *ex-ante* perspective, the social planner can perfectly insure workers against the unemployment risk. With the instantaneous utility functions chosen above, this is achieved by making workers indifferent between the

[8] When $r \to 0$, it can be checked that $\Pi \to \chi$.

unemployment and employment states [9]. This is an extension in a general equilibrium context of a very standard result about the optimal unemployment insurance in partial equilibrium (e.g. Shavell and Weiss (1979) or Hopenhayn and Nicolini (1997)). Under perfect information, the trade-off between allocative efficiency and the insurance and redistribution dimensions does not hold. Perfect insurance can be achieved together with efficient outcomes. The marginal utilities of the workers equal the social marginal value of profits, η. Taking this into account, conditions (22) (23) and (24) can be respectively rewritten as:

$$\left(\frac{\delta_1}{\eta}=\right)\frac{d'(e_1)+c\cdot\theta_1}{\alpha(\theta_1)}=\frac{y+d(e_1)+e_1\cdot c\cdot\theta_1}{e_1\cdot\alpha(\theta_1)+q}=\frac{c}{\alpha'(\theta_1)} \tag{26}$$

Consequently, the optimal levels of tightness and search intensity are defined by a system of two equations that are independent of η. Put another way, the optimal level of θ and e can be chosen irrespectively of the way net output is shared between the workers and the representative firm-owner.

From equalities in (26), we get that the social optimum is determined by either $F(\theta_1,e_1)=G(\theta_1,e_1)=0$, or $F(\theta_1,e_1)=H(\theta_1,e_1)=0$ or $G(\theta_1,e_1)=H(\theta_1,e_1)=0$, where:

$$\begin{aligned}F(\theta,e) &\equiv \alpha'(\theta)\left(d'(e)+c\cdot\theta\right)-c\cdot\alpha(\theta)\\ G(\theta,e) &\equiv \alpha(\theta)(y+d(e)+c\cdot\theta\cdot e)-\left(c\cdot\theta+d'(e)\right)(e\cdot\alpha(\theta)+q)\\ H(\theta,e) &\equiv \alpha'(\theta)(y+d(e)+e\cdot c\cdot\theta)-c(e\cdot\alpha(\theta)+q)\end{aligned}$$

Conditional upon the optimal conditions (25), function $H(\theta,e)$ implicitly defines the optimal level of tightness as a function of search intensity whereas function $G(\theta,e)$ implicitly defines the optimal level of search intensity as a function of tightness.

Appendix 4 first shows that $F'_\theta<0$ and $F'_e>0$. Consequently, function $F(.,.)$ is upward-sloping in the (θ,e) plane (see figure 1). Second, we show that $G'_e<0$ and $G'_\theta=\frac{q}{\alpha(\theta)}F(\theta,e)$. Consequently, in the (θ,e) plane function $G(.,.)$ is upward-sloping (respectively downward-sloping) at the left (respectively at the right) of function $F(.,.)$ and intersects function $F(.,.)$ horizontally (see figure 1). Third, we show that $H'_\theta<0$ and $H'_e=F(\theta,e)$. Hence, in the (θ,e) space, function $H(.,.)$ is upward-sloping (respectively downward-sloping) above (respectively below) function $F(.,.)$ and intersects function $F(.,.)$ vertically (see Figure 1). This configuration guarantees the unicity of a solution to the system (26) [10].

[9] If unemployment workers' instantaneous utility was $v(z)-d(e)$, the first best would equalize instantaneous incomes instead of utilities.

[10] The proof of the unicity of the solution to $F(\theta,e)=H(\theta,e)=0$ is similar to the proof of the unicity of equilibrium. One simply has to replace $\mathbb{S}$ by $-F$ and $\mathbb{W}$ by H.

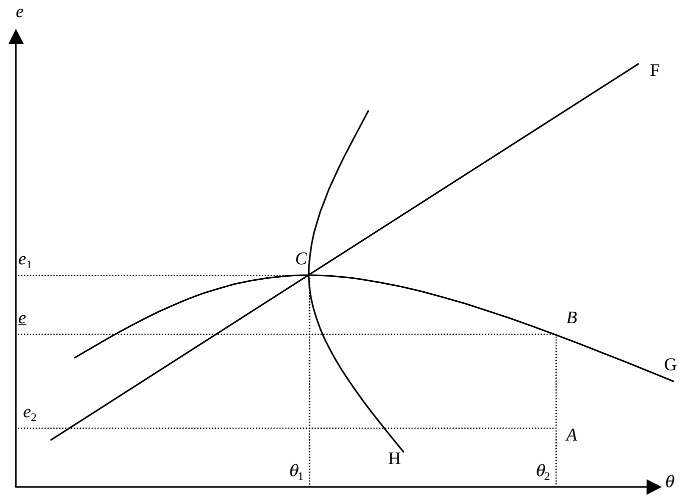

Figure 1: The first-best choice of (θ,e)

To end this section, one can compare conditions (26) to the first-order conditions of the program that maximizes total output net of costs Y subject to the flow equilibrium condition (1), that is:

$$\begin{aligned}\max_{e,u,\theta}\quad Y &= (1-u)y-u\cdot d(e)-c\cdot e\cdot\theta\cdot u \\ s.t.\quad &: \; e\cdot\alpha(\theta)\cdot u=q(1-u)\end{aligned} \tag{27}$$

Denoting δ^Y the Lagrange multiplier, the first-order conditions are:

$$\left(\delta^Y=\right)\frac{d'(e)+c\cdot\theta}{\alpha(\theta)}=\frac{y+d(e)+e\cdot c\cdot\theta}{e\cdot\alpha(\theta)+q}=\frac{c}{\alpha'(\theta)}$$

Comparing these expressions with (26), it is obvious that the solution (e,θ) to problem (27) is exactly the first-best optimum (e_1,θ_1). Hence, the unemployment rate u is equal to u_1. At the first-best optimum, total output net of costs is therefore maximized. Consequently, efficiency and equity goals can be achieved separately. The following proposition summarizes the principal results of this section:

Proposition 2 *Under perfect information, the central planner is able to deal separately with allocative efficiency and with the risk-sharing and redistributive dimensions. The first-best levels of search effort and tightness maximize total output net of costs. Given* *these optimal levels, the first-best income levels guarantee a constant level of utility whether workers are employed or not. This level is higher the lower the social weight attributed to profits.*

IV.2 Decentralization of the First-Best optimum

In this section, we describe the policies required to implement the first-best optimum in a decentralized economy where the State perfectly monitors the level of search intensity e. Given the required unemployment benefits z, tax level T, marginal tax rate T_m and search intensity e, the equilibrium solves $w = \phi(\theta)$ and $WS(\theta, w, e) = 0$. The steady-state level of unemployment u is then given by equation (1).

To implement the first-best optimum, the State has to decentralize an equilibrium in which workers are indifferent between employment and unemployment. According to (17), implementing such an equilibrium requires $T_{m,1} = 1$. Such a high marginal tax rates is unavoidable to prevent insiders from extracting a rent $V - V^u > 0$ though wage bargaining [11]. The State can fix the level of unemployment benefits to z_1. By assumption, it is also able to impose a search intensity e_1. Therefore, employed workers earn a net income equal to the first-best one: $\omega_1 = z_1 - d(e_1)$.

Knowing the optimal value θ_1, let then the level of tax be given by $T_1 = \phi(\theta_1) - \omega_1$. Since, $T_{m,1} = 1$, $z = z_1$ and $e = e_1$, the wage bargaining process implies $w = w_1 = \omega_1 + T_1$ according to (17). Given this gross wage, the representative firm chooses its optimal level of vacancies until the free-entry condition (9) is met. The equivalence between the free-entry condition (9) and equation (10) guarantees that θ solves equation $w_1 = \phi(\theta)$, which has a unique solution, namely θ_1.

Proposition 3 *Under perfect information, the state can always decentralize the first-best optimum with a marginal tax rate of* 100%.

Two important observations have to be emphasized. First, in a setting where marginal utilities are not constant and equal to η, the optimum is no more characterized by an Hosios-type condition on the bargaining power of the worker. Whatever this power, one has instead to implement an appropriate marginal tax rate (100%) [12]. Second, the decentralization of the first-best optimum is impossible without a progressive income tax schedule. For, the above explanation shows that one needs two tax instruments to decentralize the first best: The marginal tax rate and the level of taxes (or an average tax rate).

[11] Since the workers' bargaining power verifies $\gamma > 0$.

[12] See also Boone and de Mooij (2000) and Boone and Bovenberg (2001a) who characterize the tax system that allows to decentalize the optimal allocation when the Hosios condition is not fulfilled. These papers deal with risk-neutral workers however.

Linear income taxation without a non zero intercept cannot decentralize the first best. Linear taxation with an intercept (that can be interpreted as a basic income scheme) is sufficient however to decentralize the first best. This is obviously so because agents are homogeneous. With heterogeneity among workers, a more complex non linear tax schedule would be unavoidable to decentralize the first best.

To end this section let us explain how the State can in practice obtain at the equilibrium the first-best search level e_1. Since the search effort is perfectly observed, the State can offer a schedule of unemployment benefits conditional on this search intensity. Let $z(e)$ denote this schedule. A very simple schedule that guarantees that each unemployed worker chooses the first-best level of search intensity is:

$$z(e) = \begin{cases} z_1 \\ z_1 - d(e_1) - \varepsilon \end{cases} \quad \text{if} \quad e \begin{cases} \geq e_1 \\ < e_1 \end{cases}$$

with $\varepsilon > 0$. $d(e_1) - \varepsilon$ then corresponds to a sanction imposed on any worker with a search intensity e below e_1.

Under this scheme, the individual choice of search intensity can be described as follows: First, whether $0 \leq e < e_1$ or $e_1 \leq e$; second, within these intervals, the choice of search intensities. $T_{m,1} = 1$ implies that workers are indifferent between employment and unemployment for workers ($V = V^u$) at equilibrium. The marginal gain of search is thus nil. So unemployed workers never choose to search more than the search intensity that insures them a given income level. Therefore, one either has $e = e_1$ or $e = 0$. Moreover, any unemployed worker enjoys a higher utility by searching e_1 than by not searching[13]. Therefore, the sanction is *ex ante* a threat that is never *ex post* effective.

The schedule $z(e)$ is actually equivalent to a perfect sanction scheme [14]. By "perfect" we mean that the penalty is applied without error. No unemployed who develops a search effort below e_1 would be eligible to the higher level of unemployment benefits z_1. Every unemployed worker who at least devotes e_1 to search intensity receives the level z_1 of unemployed benefits. Such a perfect mechanism is highly implausible but is unavoidable to implement the first best optimum. In the following, we therefore consider the opposite case where the level of search cannot be observed by the State.

V The second-Best optimum

V.1 The central planner problem

In this section, we consider the more realistic case where search intensity is not observed by the State. For reasons of tractability, we keep the assumption of a replacement ratio

[13] If $e = e_1$, the instantaneous utility level is $v(z_1 - d(e_1))$. If $e = 0$, the utility amounts to $v(z_1 - d(e_1) - \varepsilon)$.

[14] There is a recent interest in the literature on ...

independent of the length of an unemployment spell [15]. As in the first best, the tax system and (the constant level of) unemployment benefits are the instruments used to promote efficiency and equity. But now the State faces a moral hazard problem. An incentive constraint concerning unemployed workers' search behavior should now be taken onto account by the State. Since r tends to 0, this behavior is described at the steady state by:

$$d'(e) \cdot v'(z - d(e))(q + e \cdot \alpha(\theta)) = \alpha(\theta)(v(\omega) - v(z - d(e))) \tag{28}$$

According to equations (11), (12) (13), $\omega = w - T$. As in the first-best case, we implicitly assume that η is chosen in such a way that $\Pi \geq 0$ at the optimum. The second-best optimization consists in problem (19) extended to include this incentive constraint (28), namely:

$$\max_{\theta,\omega,u,z,e} \quad (1-u) \cdot v(\omega) + u \cdot v(z - d(e)) + \eta[(1-u)(y-\omega) - z \cdot u - c \cdot e \cdot \theta \cdot u]$$

$$0 = e \cdot \alpha(\theta) \cdot u - q(1-u) \tag{δ_2}$$

$$0 = \alpha(\theta)(v(\omega) - v(z - d(e))) - d'(e) \cdot v'(z - d(e))(q + e \cdot \alpha(\theta)) \tag{ψ_2}$$

Introducing subscript 2 to denote the second-best optimum, let δ_2 (respectively ψ_2) denote the Lagrange multiplier associated with the flow equilibrium (respectively the incentive constraint). The first order conditions are:

$$0 = (1-u_2)\left[v'(\omega_2) - \eta\right] + \psi_2 \cdot \alpha(\theta_2) \cdot v'(\omega_2) \tag{29}$$

$$\begin{aligned} 0 = {} & u_2\left[v'(z_2 - d(e_2)) - \eta\right] \\ & -\psi_2\left\{\alpha(\theta_2) \cdot v'(z_2 - d(e_2)) + v''(z_2 - d(e_2)) \cdot d'(e_2) \cdot (q + e_2 \cdot \alpha(\theta_2))\right\} \end{aligned} \tag{30}$$

$$\begin{aligned} 0 = {} & u_2\left[-d'(e_2) \cdot v'(z_2 - d(e_2)) - \eta \cdot c \cdot \theta_2 + \delta_2 \cdot \alpha(\theta_2)\right] \\ & +\psi_2\left\{-d''(e_2) \cdot v'(z_2 - d(e_2)) + \left(d'(e_2)\right)^2 \cdot v''(z_2 - d(e_2))\right\}(q + e_2 \cdot \alpha(\theta_2)) \end{aligned} \tag{31}$$

$$0 = v(z_2 - d(e_2)) - v(\omega_2) + \eta(\omega_2 - y - z_2 - c \cdot e_2 \cdot \theta_2) + \delta_2(e_2 \cdot \alpha(\theta_2) + q) \tag{32}$$

$$\begin{aligned} 0 = {} & -c \cdot e_2 \cdot \eta \cdot u_2 + \delta_2 \cdot \alpha'(\theta_2) \cdot e_2 \cdot u_2 + \\ & \psi_2 \cdot \alpha'(\theta_2) \cdot \left\{v(\omega_2) - v(z_2 - d(e_2)) - e_2 \cdot d'(e_2) \cdot v'(z_2 - d(e_2))\right\} \end{aligned} \tag{33}$$

If $\psi_2 = 0$, these conditions would be exactly those found in the first-best case. However, the incentive constraint is binding. Otherwise, workers would be perfectly insured against the unemployment risk and the unemployed workers would then have no incentive to search. Consequently, the second-best optimum requires $\omega_2 > z_2 - d(e_2)$. Appendix 5

[15] Hence, we leave aside the question of the optimal profile of unemployment benefits as a function of unemployment duration (see e.g. Shavel and Weiss (1979), Wang and Williamson (1996), Hopenhayn and Nicolini (1997), Cahuc and Lehmann (2000), Fredriksson and Holmlund (2001)).

explains why $\psi_2 > 0$. Moreover, Appendix 5 shows that $G(\theta_2, e_2) > 0$ and $H(\theta_2, e_2) < 0$. Remembering the properties of functions G and H, these properties immediately imply the following inequalities:

$$e_2 < e_1 \qquad \text{and} \qquad \theta_2 > \theta_1$$

The intuition behind these properties is the following. Because of moral hazard, there is now a trade-off between efficiency in insurance against the unemployment risk, equity between the firm-owner and workers and allocative efficiency. To induce search effort, employed workers necessarily enjoy higher utility levels than unemployed ones at the second best. Keeping search effort at its first-best level would however require a difference in utilities between employed and unemployed workers that would be too detrimental to the objective of insurance among workers. This explains why $e_2 < e_1$. Compared to the first best, the social planner integrates the beneficial effect of a more tight labor market on search effort. An increase in tightness allows to relax the incentive constraint. This intuitively explains why $\theta_2 > \theta_1$.This implies that the expected cost of filling a vacancy $c/m(\theta)$ is too high at the second best. Since tightness is measured in efficiency units, these properties have no clear implications on the comparison between v_2/u_2 and v_1/u_1.

Comparing conditions (29) with (20) yields $\omega_1 < \omega_2$. In the most plausible case where higher unemployment benefits have a negative impact on search effort, i.e. when $S'_z < 0$, condition (30) and (21) leads to $z_2 - d(e_2) < z_1 - d(e_1)$. Since, $e_2 < e_1$, these results imply $z_2 < z_1$.

The intuition behind $z_2 - d(e_2) < z_1 - d(e_1)$ and $\omega_1 < \omega_2$ is similar to the one underlying $\theta_1 < \theta_2$. Compared to the first best, the social planner integrates the beneficial effect of a higher (respectively a lower) utility level for employed (respectively unemployed) worker on search effort. Furthermore, since $e_2 < e_1$, one has $d(e_2) < d(e_1)$. This property and the inequality $z_2 - d(e_2) < z_1 - d(e_1)$ imply that the level of unemployment benefits is lower in the second than in the first best.

One can compare the total output net of cost Y at the first-best and at the second-best optima. We have shown that the first-best levels of tightness and search intensity θ_1, e_1 maximize the total output net of costs. Search intensity and tightness differ at the second best compared to their first best values. Hence one has $Y_1 > Y_2$.

The following proposition summarizes our main results.

Proposition 4 *When search effort in unobservable, allocative efficiency, risk-sharing efficiency and redistributive objectives cannot be achieved separately anymore. Compared to the first best, the second best is characterized by lower search effort e and total net output Y and higher tightness θ and net income in employment ω.*

To end this section, let us briefly emphasize how this general equilibrium analysis enriches partial equilibrium studies such as Baily (1977). In a partial equilibrium framework, both θ and w are exogenous. So, any conclusion is necessarily contingent on the values taken by θ and w. Let us assume that they are fixed at their second best optimal value [16]. Recall that $G(\theta, e) = 0$ defines the level of search intensity that maximizes net output Y conditional on θ. In a partial equilibrium perspective, the loss in efficiency when search effort is unobservable can be captured by the distance between the second-best optimum e_2 and the solution $\underline{e}$ to equation $G(\theta_2, e) = 0$. This can be illustrated in Figure 1 where this distance is A-B. In a general equilibrium framework, the loss due to the unobservability of search effort is larger because now the second-best outcome, A in Figure 1, has to be compared to the first-best C where search effort e_1 is higher than $\underline{e}$ and less vacancies have to be posted per unemployed worker (measured in efficiency units): $\theta_1 < \theta_2$.

V.2 Decentralization of the second best

In this section, we describe the policies required to implement the second-best optimum in a decentralized economy where the State cannot observe the level of search intensity e. In the particular case where neither insurance nor redistribution are an issue (marginal utilities are constant and equal to η), the State should not intervene provided that the Hosios condition holds (see Hosios (1990), Pissarides (2000) or Appendix 6). In this case, adjusting a single parameter, namely the bargaining power γ, allows to optimize the behaviors of firms, unemployed workers and wage-setters. We now turn to the case where insurance and redistribution matter.

To implement the second-best optimum, the State has to decentralize an equilibrium in which employed workers are better off than unemployed people. According to Equation $WS(\theta, w, e) = 0$, implementing such an equilibrium requires a marginal tax rate:

$$T_{m,2} < 1 = T_{m,1}$$

A lower marginal tax than in the first best is required in order to induce an appropriate search effort. Now, insiders extract some rent from a match ($V - V^u > 0$), so as to give unemployed workers an incentive to search. The initial papers about tax progressivity and imperfect competition on the labor market (Malcomson and Sator (1987), Lockwood and Manning (1993)) did not identify limits to progressivity. More recently, the introduction of in-work effort (hours) and of on-the-job training has highlighted the limits of tax progressivity (see e.g. Hansen (1999), Sorensen (1999) and Fuest and Huber (1998)). In this paper, another mechanism is identified (namely, search effort) that counteracts the favorable influence of tax progressivity through wage formation.

[16] The second-best value of the wage rate will be made precise in the following sub-section.

Now, consider that the State selects z equal to z_2, T equal to $T_2 = \phi(\theta_2) - \omega_2$ and the value of T_{m2} that solves $WS(\theta_2, \phi(\theta_2), e_2) = 0$ for $z = z_2$. It should then be verified that the equilibrium (θ, w, e) matches the second best optimum. Actually, one should only verify that $\theta = \theta_2$, $w = \omega_2 + T_2$ and $e = e_2$ solve system (18) for $T = T_2$, $T_m = T_{m2}$ and $z = z_2$. Since the incentive constraint is binding at the second best, one has, $S(\theta_2, \omega_2 + T_2, e_2) = 0$. Moreover, the choice of the level of taxes was such that $\omega_2 + T_2 = \phi(\theta_2)$. Finally, the choice of the marginal tax rate was such that $WS(\theta_2, \omega_2 + T_2, e_2) = 0$.

Finally, since $\theta_2 > \theta_1$ and $\omega_2 > \omega_1$, one has

$$T_2 < T_1$$

The level of tax required to decentralize the second-best optimum is lower than the first-best one. This follows from two effects. First, employed workers' income has to be higher at the second best. Second, the inequality $\theta_2 > \theta_1$ implies that the gross wage is lower at the second best.

Knowing how the second-best optimum can be decentralized, it is now possible to characterize the tax schedule in a more precise way. First, from $WS(\theta_2, \phi(\theta_2), e_2) = 0$, it is immediately seen that T_m increases with γ. Second, combining (10), (16) with the first equality in (9), it can be checked that for $r = 0$:

$$\frac{1 - T_m}{1 - (T/w)} = \frac{1-\gamma}{\gamma} \frac{v(\omega) - v(z - d(e))}{\omega \cdot v'(\omega)} \frac{m(\theta) \cdot y - c \cdot q}{c(q + e \cdot \alpha(\theta))} \tag{34}$$

where $(1 - T_m)/(1 - (T/w))$ is the so-called coefficient of residual income progression CRIP (i.e. $d \ln(\omega) / d \ln(w)$). Consequently, for $(\theta, w, e) = (\theta_2, \phi(\theta_2), e_2)$, the CRIP would be equal to 1 if $\gamma = \tilde{\gamma}$, with:

$$\frac{\tilde{\gamma}}{1 - \tilde{\gamma}} = \frac{v(\omega_2) - v(z_2 - d(e_2))}{\omega_2 \cdot v'(\omega_2)} \frac{m(\theta_2) \cdot y - c \cdot q}{c(q + e_2 \cdot \alpha(\theta_2))} \tag{35}$$

For this particular value of γ, the tax schedule needed to decentralize the second-best optimum would actually be linear [17]. Otherwise, a non linear tax schedule is required. From (34) and (35), the latter will be regressive (i.e. CRIP > 1) for $\gamma < \tilde{\gamma}$ and progressive (i.e. CRIP < 1) for $\gamma > \tilde{\gamma}$.

The following proposition summarizes our results.

Proposition 5 *To decentralize the second-best optimum, the marginal income tax rate has to be lower than 100%. The marginal tax rate is an increasing function of the bargaining power of the workers. There exists a threshold value of this bargaining power, $\tilde{\gamma}$, below which taxation is regressive and above which it is progressive. The level of taxes is lower than at the first-best optimum.*

[17] From Equation (35), $\tilde{\gamma}$ lies in the $(0, 1)$ interval.

VI Simulations

The previous sections gave some qualitative indications about the implications of imperfect information on the optimal policies. When search intensity is not observable, in comparison with the first-best, search intensity is too low, tightness is too high, and incomes are too high for employed workers and too low for unemployed ones. This implies lower levels for the unemployment benefits, for taxes and for the marginal tax rate. However, these results are only qualitative. So computing order of magnitudes of these effects is a first motivation for the following simulations.

In addition, we were not able to analytically compare unemployment rates, tightness measured in gross units, expected lifetime utilities of workers and of the representative firm-owner. Numerical comparisons between the first and second best are then required for these variables. This is the second motivation for this section [18].

VI.1 Calibration

Taking the year as the unit of time, the parameters are adjusted in such a way that the observed French economy is at a second-best optimum. We assume a Cobb-Douglas matching function $m_0 (e \cdot u)^{0.5} v^{0.5}$, an iso-elastic utility function $v(x) = \frac{x^{1-\sigma}}{1-\sigma}$ [19], an iso-elastic disutility function of search effort $d(e) = e^{\beta}$. This economy is characterized by a separation rate of $q = 0.15$, an average net wage of 18000 Euros/year, an after-tax replacement ratio of 0.7 (consistent with Martin (1996)). For our benchmark economy, the relative risk aversion σ is put to 1 and the bargaining power $\gamma = 0.5$. Hence, the Hosios condition would be verified if workers were risk neutral and taxes were put to zero.

To reproduce a microeconomic elasticity of unemployment duration with respect to the level of unemployment benefits of 0.5 (consistent with estimations surveyed by Layard *et al* (1991) and Holmlund (1998)), β is set to 2.8 with a benchmark value of search intensity $e_0 = 14.55$ chosen by unemployed workers (according to equation (13)). Then, to match an unemployment rate around 0.1 together with an average duration of openness of vacancies of 0.1 year (see Maillard (1997)), m_0 is fixed to 0.96 with a benchmark value of tightness of $\theta_0 = 0.0093$ (hence, $v_0/u_0 = 0.13$). The remaining parameters are such that the observed equilibrium is a second-best optimum. This leads to $y = 22959$, $\eta = \eta_0 = 0.00006$ and $c = 171876$.

In the following subsections, four parameters will be allowed to vary, namely risk aversion σ, the weight given to the employer in the social welfare function η, the marginal effect of search effort on utility β and the bargaining power of the workers γ. As γ will

[18] The mathematica 4.0 program is available upon request.

[19] If $\sigma = 1$, we take $v(x) = \ln(x)$.

be allowed to vary, it will be interesting to contrast the second-best outcomes with those accessible through a linear tax schedule. Most of the simulation results will be summarized by way of a panel showing twelve figures. Each panel will have the same structure and will use the same conventions. Solid lines will correspond to the first best, dotted lines to the second best and dashed lines to what could be called "the third best", i.e. the best allocation that can be achieved under imperfect information and with linear taxation (see Appendix 7 for a formal presentation of this optimization problem). Each panel will display tightness θ, search intensity e, the unemployment rate u, the net income of employed workers ω, the unemployment benefits z, the after-tax replacement ratio $\rho = z/\omega$, the average tax rate T/w, the marginal tax rate T_m, profits Π, the v/u ratio, the utility flows V (expressed in certainty equivalent), and the net output Y. The figure related to the V's will show three dotted curves for the second best. The upper (respectively, the lower, the intermediate) curve will represent the employed workers' (respectively the unemployed workers', the average) instantaneous utility expressed in certainty equivalent.

VI.2 General results

It is first useful to report on the magnitude of the distance between equilibria A, B and C in Figure 1. For the benchmark economy, the difference between the second-best search effort, e_2, and the first-best solution in partial equilibrium, $\underline{e}$, is very large ($e_2 = 14.55, \underline{e} = 27.49$). On the contrary, the difference between the first-best search effort in general equilibrium, e_1, and the partial-equilibrium counterpart, $\underline{e}$, is small (since $e_1 = 27.52$). Therefore, $G(\theta, e) = 0$ defines a rather flat relationship between e and θ over a wide range of values of θ (namely, between $\theta_1 = 0.0066$ and $\theta_2 = 0.0093$). To illustrate that the losses due to the unobservability of search effort should be evaluated by comparing equilibria A and C instead of A and B, notice that the latter comparison underestimates the loss in net output by 10% and the loss in welfare (i.e. $(1-u)\, v(\omega) + u \cdot v(z - d(e)) + \eta \cdot \Pi$) by 7.6% and overestimates the increase in unemployment by 27%.

Some other interesting properties of the model are suggested by the numerical simulations reported in Figures 2, 3, 4 and 6. Even if they could not be proved analytically, these properties appear to be robust since they were systematically found in all (reported and unreported) simulations First, the v/u ratio is always higher and the unemployment rate u is always lower at the first best than at the second best. Recall that unemployed workers search less in the second best ($e_2 < e_1$) but that the labor market is then also more tight ($\theta_2 > \theta_1$). Due to these two opposite effects, the net impact on the v/u ratio and the unemployment rate was theoretically unclear. In all the simulations where the tax schedule is allowed to be non linear, the increase in tightness θ is much less important than the decrease in search intensity $e_2 < e_1$. In what follows, we will say that an equilibrium becomes "less efficient" when the following outcomes are simultaneously observed: Tightness θ and the unemployment rate u increase and search intensity e, the v/u ratio and total output net of cost Y decrease.

Another robust property is that the expected utility of the workers $(1-u)\,v(\omega) + u \cdot v(z - d(e))$ is higher at the second best than at the first best. This is counter-intuitive for two reasons. First, it has been shown in proposition 4 that total net output is lower at the second best ($Y_2 < Y_1$). So there are less resources to be shared between workers and the firm owner. Second, it has been shown that unemployment insurance is imperfect at the second-best optimum. So, for a given level of average income, workers' expected utility should be lower at the second best compared to the first best. Since the expected utility of the workers is higher at the second best, the share of Y accruing to them has to increase when search effort becomes unobservable and this increase has to outweigh the two previous effects. In other words, the unobservability of search intensity not only leads to imperfect unemployment insurance and lower search intensity (as emphasized in the literature on optimal unemployment insurance in partial search equilibrium frameworks) or to higher tightness (as shown in proposition 4). It also modifies the distribution of utility levels between workers and the firm owner in favor of the formers. Moreover, this shift is large enough to compensate the negative influences of the decrease in total net output and of the incompleteness of unemployment insurance on the expected utility of workers at the second best. The intuition behind this shift goes as follows. Proposition 4 has shown that the level of taxes is lower at the second best than at the first best [20]. Moreover, the unemployment rate is much higher at the second best, suggesting that unemployment benefits expenditures are probably higher too. Hence, the lump-sum transfer to the capital owner (χ, with $\Pi = \chi$ when $r = 0$) should be lower at the second best.

VI.3 Risk aversion (σ)

Figure 2 displays first- and second-best optima and their implementation in a decentralized economy for different values of the relative risk aversion parameter ($\sigma \in [0.05, 3]$). As σ increases, the weight given to insurance (or redistribution among workers) increases. However, if η was left unchanged as σ varies, the weight given to the employer (relative to workers taken as a whole) would also change. To avoid this, we normalize the value of η so that first-best values of incomes ω and z remain unchanged when σ varies [21]. So

[20] This is due to the higher level of after-tax income for employed workers that is required to increases the incentives to search, and to the lower level of pre-tax wage required to guarantee a higher level of tightness.

[21] That is we impose $\eta = (\eta_0)^\sigma$.

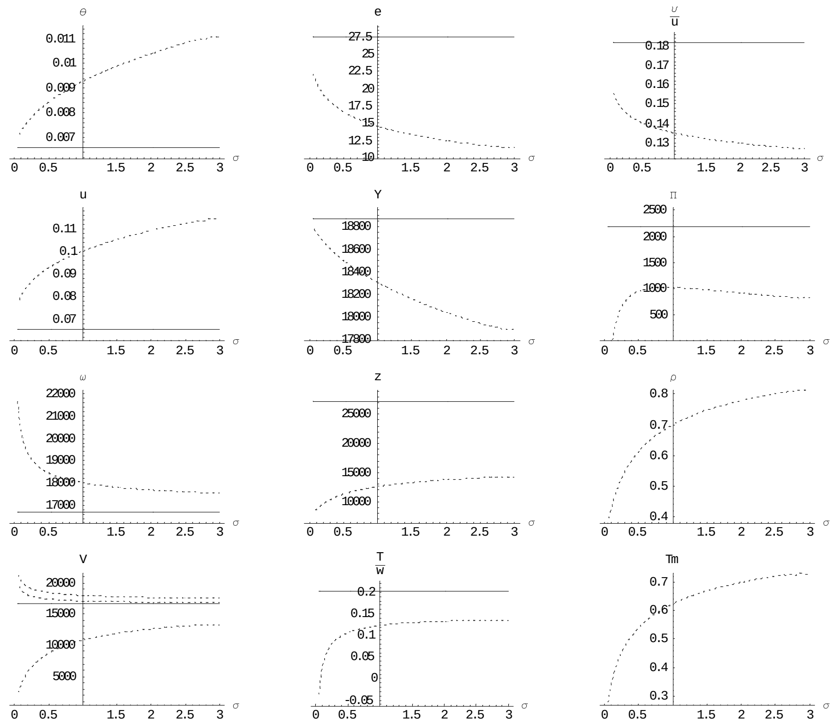

Figure 2: Simulations with respect to σ

doing, changes in σ do not affect the relative weight attributed to the firm owner at the first-best. As a consequence, all the simulated variables become independent of σ in the first best.

When σ increases, the social planner values insurance more. To some extent, the central planner is then ready to sacrifice allocative efficiency in the second best. Put another way, the central planner will put less incentive on search effort. Search intensity e, the v/u ratio and net output Y decrease while the unemployment rate u increases with σ. Search intensity e decreases however less rapidly than the v/u ratio. So, tightness θ is an increasing function of σ. While the previous variables move away from their first-best counterpart as σ increases, employed and unemployed workers' net incomes and utility levels converge to their first best values. This was expected as insurance matters more and more. Consequently, as workers become more risk averse, the after-tax replacement ratio and the marginal tax rate increase a lot, respectively from 0.38 to around 0.8 and from 0.26 to 0.7. It should be noticed that these quite high lower bounds are found for a very small degree of relative risk aversion (namely $\sigma = 0.05$). The order of magnitude for the after-tax replacement ratio observed in Continental Europe (namely, between 0.7 and 0.8) is obtained for a wide range of relative risk aversion values (namely for $\sigma \in [1, 3]$). As expenditures for unemployed workers $z \cdot u$ rises, the level of taxes increases and so does the average tax rate T/w. As far as the pattern of profits Π is concerned, equation (5) can be rewritten as $\Pi_2 = Y_2 - (1 - u_2)\omega_2 - u_2 (z_2 - d(e_2))$. Conflicting effects are at work. First, the decrease in total net output Y_2 tends to lower Π_2. Second, as σ increases, the average consumption level is modified in opposite directions : ω_2 decreases, while $z_2 - d(e_2)$ and u_2 increases.

VI.4 The social marginal value of profits (η)

In Figure 3, η is allowed to vary within the interval $[0.95\ \eta_0, 3\ \eta_0]$, where η_0 was fixed at the calibration stage. As expected, as η/η_0 increases, the net income and utilities of employed and unemployed workers shrink whereas profits rise. Simultaneously, the financing of unemployment benefits rely more heavily on workers (income taxes are higher). These tendencies are observed at the first- and second-best optima. The decrease in z turns out to be stronger than the one in ω. Therefore, the after-tax replacement ratio decreases from 0.72 to 0.58. The marginal tax rate increases from 0.615 to 0.775.

As η/η_0 increases, the second best turns out to become less efficient: search effort, the v/u ratio and net output Y decrease. As in the simulations for σ, search intensity e decreases less rapidly than the v/u ratio. Consequently, tightness θ rises with η. So does the unemployment rate. An intuition behind this decreasing efficiency can be expressed as follows. As previously suggested, the unobservability of search effort tends to shift the distribution of total output at the expense of the firm-owner. As η increases, the social value of profits becomes higher and the social planner prefers a reduction in efficiency. Otherwise, the more and more valued position of the firm-owner would be too much reduced.

VI.5 The elasticity of disutility of effort (β)

In Figure 4, β is allowed to vary between 1.5 and 4. As β increases, the marginal disutility of search rises. Therefore, both the first-best and the second-best levels of search effort sharply decrease with β. The first and second-best values of the v/u ratio and of total net output Y decreases, too. As above, the net effect is an increase in tightness θ and in

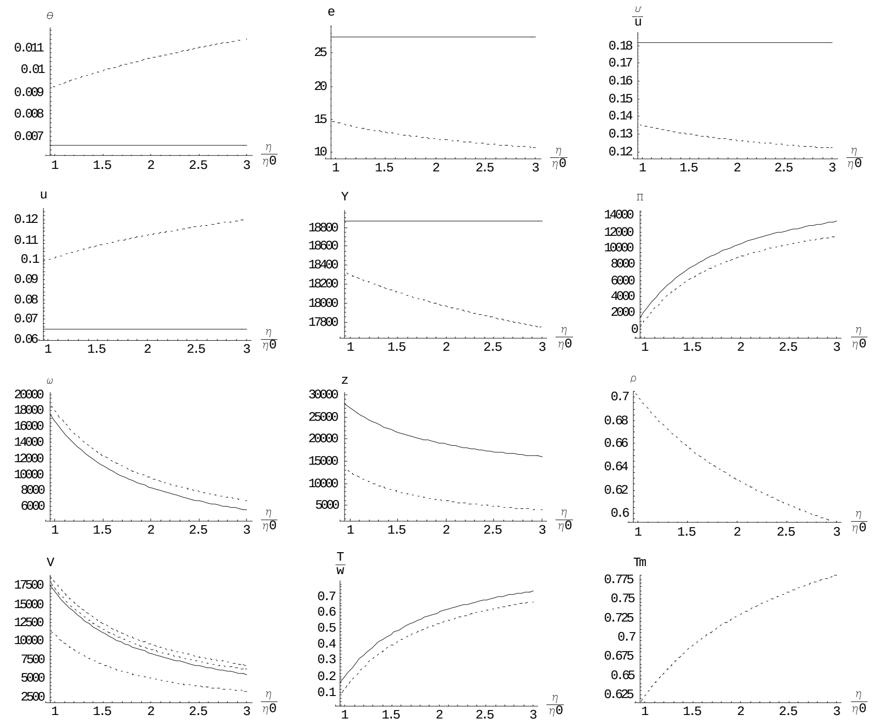

Figure 3: Simulations with respect to η

the unemployment rate u. Notice that the changes in θ and in u are here much larger than in previous figures. Since inducing effort becomes more costly, the trade-off between allocative efficiency and insurance (redistribution) is altered in favor of more insurance at the second best. This explains the convergence of the workers' utility and the rise of the marginal tax rate.

As far as the replacement ratio is concerned, two main effects can be identified. Let us decompose the replacement ratio as

$$\rho = \frac{z - d(e)}{\omega} \cdot \frac{z}{z - d(e)}$$

The first term stands for a replacement ratio in 'utility units' whereas the second essentially depends on the patterns of $d(e)$. As β increases, the social planner values insurance

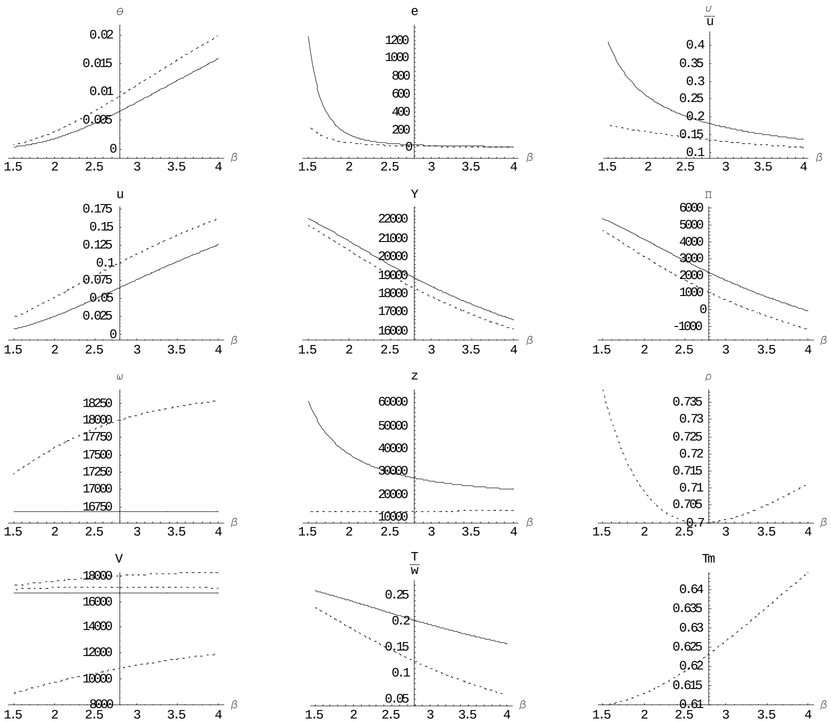

Figure 4: Simulations with respect to β

more. So, the first term increases. However, as β increases, search intensity decreases so dramatically that $d(e)$ decreases (despite the rise of β). So, the second term decreases. These two conflicting effects are consistent with the U-shaped profile of the second-best replacement ratio observed in Figure 4.

VI.6 Non linear vs linear income taxation

The bargaining power γ influences neither the social objective Ω nor the flow equilibrium (1), nor the incentive constraint (28). Changes in γ affect neither the first nor the second-best solutions. They only influence the decentralization of the second best. As γ increases, insiders get more bargaining power. Hence, the marginal tax rate should increase to prevent insiders from extracting too large a rent. The horizontal line in Figure 5 displays

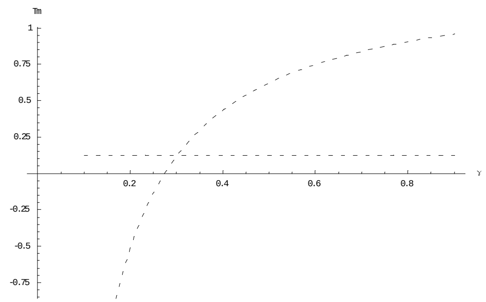

Figure 5: Simulations with respect to γ

the average tax rate and the upward-sloping curve plots the marginal tax rate at the second best. Figure 5 shows how sensible the marginal tax rate is to changes in γ. Two cases are of particular interest. First, when γ verifies the usual Hosios condition (i.e. for $\gamma = 0.5$), the optimal marginal tax is much larger than the average tax rate (around 0.62). We are far from the conclusions reached when workers are risk neutral. So, taking insurance (redistributive) issues into account deeply affect the normative conclusion about the role of taxation in economies with frictions. Second, for values of the bargaining power lower than 0.5, the optimal marginal tax decreases a lot. It can even become very negative when the bargaining power of the workers is low.

Finally, it is worthwhile to compare the non linear and linear income taxation cases. From theory, we know that restricting taxes to be linear will typically not allow to decentralize the second-best optimum (the problem is then formally stated in Appendix 7). Here, we provide some evidence on what is lost if taxes have to be linear. For our benchmark economy, the bargaining power $\tilde{\gamma}$ such that linear taxation can decentralize the second-best optimum amounts to 0.3.

For $\gamma < \tilde{\gamma}$, insiders have not enough bargaining power to negotiate wages that give the appropriate incentive to search. A non linear tax schedule has then to be regressive in order to guarantee an optimal trade-off between insurance and efficiency. This is not possible with linear taxes. In that case, tightness and the replacement ratio are too high and search efficiency is too low (see Figure 6). One observes that changes in θ now dominates those in e, so that the unemployment rate is too low. For $\gamma > \tilde{\gamma}$, income taxation should be progressive to compensate for the excessive bargaining power of the workers. As a matter of consequence, search effort is now too high and the replacement ratio is too low. As it turns out, the difference made by the impossibility of non linear taxes can be substantial.

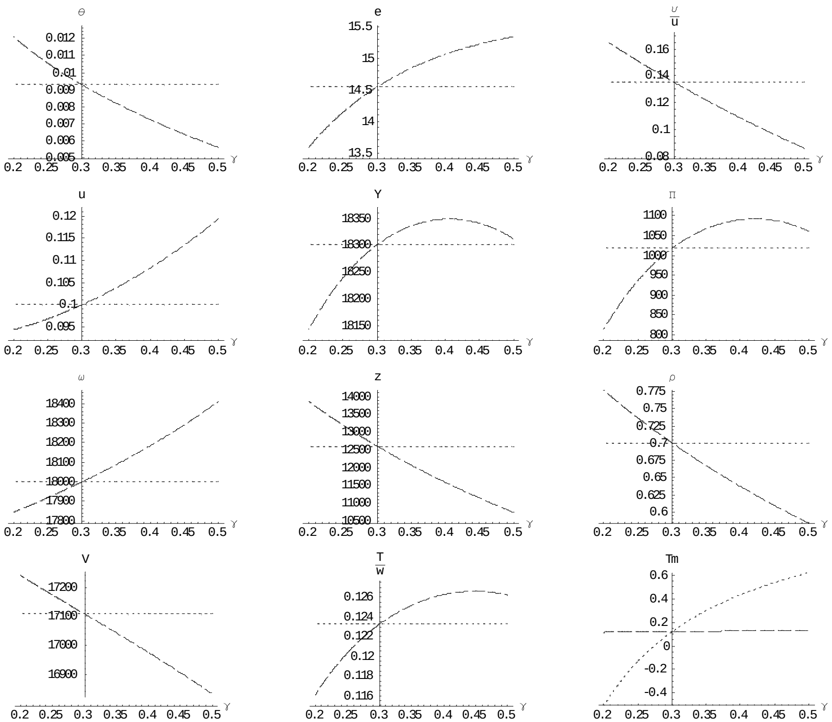

Figure 6: Proportionnal versus non linear taxations

For instance in the benchmark case with $\gamma = 0.5$, the replacement ratio is equal to 0.58 when taxation has to be linear while it equals 0.7 when taxation is allowed to be non linear.

VII Conclusion

It is widely accepted that unemployment benefits improve risk-sharing on the labor market but have negative effects on search intensity and labor demand. By integrating the impact of the shape of the tax schedule on wage formation, this paper has contributed to the literature on optimal unemployment benefits. For this purpose, this paper has developed a matching framework with risk-averse workers, endogenous job-search and wage

bargaining. We have contrasted a first-best optimum where the State can perfectly monitor unemployed workers' search intensity and a second-best optimum with moral hazard. In the first best, the efficiency objectives can be achieved independently of the redistributive issues and the State can perfectly insure workers against the unemployment risk. The implementation of the first best requires a 100% marginal tax rate as well as a perfect mechanism of sanctions. Therefore, the decentralization of the optimum does no more require an Hosios-type condition on the bargaining power of the worker. Whatever this power, one has instead to implement an appropriate marginal tax rate (100%). Moreover, the decentralization of the first-best optimum is impossible without a progressive income tax schedule.

We have then shown that search intensity and total net output are lower and tightness (measured in efficiency units) are higher in the second-best optimum than in the first-best one. It has also been shown that in comparison with the decentralization of the first-best allocation, the one of the second-best optimum requires a lower marginal tax rate, a lower level of taxes on labor and typically a lower level of unemployment benefits. Except for one particular level of workers' bargaining power, a non linear tax schedule is again necessary to decentralize the second-best optimum.

In addition, through numerical simulations for a wide range of values of the parameters, it appears that the unemployment rate and the workers' expected utility are higher at the second best than at the first best. In sum, the unobservability of the effort of the unemployed leads to imperfect unemployment insurance, creates large inefficiencies and shifts the redistribution of output in favor of workers and at the expense of the firm owner. Numerical simulations have also pointed out that the optimal second-best policies are quite sensitive to the values of parameters. In particular, a higher degree of risk aversion dramatically increases unemployment benefits and the average and marginal tax rates. A rise in the social weight of the firm-owner's welfare increases the average tax rate and decreases the level of unemployment benefits. A more elastic disutility of search decreases unemployment benefits and the average tax rate. Furthermore, the optimal marginal tax rate strongly depends on the value of the bargaining power. Everything else equal, it lies between −150% and +90% when the bargaining power varies between 0.1 and 0.9. Finally, the simulation exercises indicate that in comparison with proportional taxes, non linear income taxation allow to improve the allocation of resources and risk-sharing substantially.

This paper could be extended in different ways. First, different papers have been concerned with the optimal profile of unemployment benefits over the unemployment spell rather than a single level of unemployment benefits (Shavell and Weiss (1979), Wang and Williamson (1996), Hopenhayn and Nicolini (1997), Cahuc and Lehmann (2000) or Fredriksson and Holmlund (2001). Introducing, say, two levels of unemployment benefits in our analytical framework would introduce additional instruments to share risks and to redistribute income between firms and workers. Second, with respect to the monitoring of search effort, we have only considered two polar cases (namely job-search decisions were either perfectly observed or not observed at all). However, in a more realistic framework, the State can imperfectly observe search behavior and therefore introduce an imperfect sanction mechanism, just as in Boone and van Ours (2000), or Boone et al (2001). Third, we have assumed a homogenous labor force. However, workers actually have different levels of productivity in employment (as in the optimal taxation literature following Mirrlees (1971) and in Boone and Bovenberg (2001b)) and the disutility of search can be heterogeneous too. Introducing such features would clearly enrich the analysis but are expected to complicate the model a lot.

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Appendix

Appendix 1: Partial Derivatives

From

$$S(\theta,w,e) \equiv \alpha(\theta)\left(v(w-T)-v(z-d(e))\right)-d'(e)\cdot v'(z-d(e))\left(r+q+e\cdot\alpha(\theta)\right)=0$$

one has:

$$\begin{aligned}
S'_e &= \alpha(\theta)\cdot d'(e)\cdot v'(z-d(e))-\alpha(\theta)\cdot d'(e)\cdot v'(z-d(e))+ \\
&\quad \left(-d''(e)\cdot v'(z-d(e))+\left[d'(e)\right]^2 v''(z-d(e))\right)\left(r+q+e\cdot\alpha(\theta)\right) \\
S'_w &= \alpha(\theta)\cdot v'(w-T)>0 \\
S'_\theta &= \alpha'(\theta)\left[v(w-T)-v(z-d(e))-e\cdot d'(e)\cdot v'(z-d(e))\right]
\end{aligned}$$

Since, $v''(.)<0$ and $d''(.)\geq 0$, it is easily checked that $S'_e<0$. Equation $S(.,.,.)=0$, can be rearranged to yield:

$$v(w-T)-v(z-d(e))=\frac{r+q+e\cdot\alpha(\theta)}{\alpha(\theta)}d'(e)\cdot v'(z-d(e))$$

Therefore,

$$S'_\theta=\frac{\alpha'(\theta)}{\alpha(\theta)}(r+q)\cdot d'(e)\cdot v'(z-d(e))>0$$

Finally, one has

$$\begin{aligned}
S'_{T_m} &= 0 \\
S'_T &= -\alpha(\theta)\cdot v'(w-T(w))=-S'_w<0 \\
S'_z &= -\alpha(\theta)\cdot v'(z-d(e))-d'(e)\left(r+q+e\cdot\alpha(\theta)\right)v''(z-d(e))
\end{aligned}$$

From:

$$WS(\theta,w,e)\equiv v(w-T)-v(z-d(e))-\frac{c\cdot\gamma}{1-\gamma}(1-T_m)\frac{r+q+e\cdot\alpha(\theta)}{m(\theta)}\cdot v'(w-T)=0$$

the following partial derivatives can be computed:

$$\begin{aligned}
WS'_\theta &= -\frac{c\cdot\gamma}{1-\gamma}\cdot\frac{1-T_m}{m(\theta)}\left(e\cdot\alpha'(\theta)-\frac{m'(\theta)\left(r+q+e\cdot\alpha(\theta)\right)}{m(\theta)}\right)<0 \\
WS'_w &= v'(w-T)-\frac{c\cdot\gamma}{1-\gamma}(1-T_m)\frac{r+q+e\cdot\alpha(\theta)}{m(\theta)}\cdot v''(w-T)>0 \\
WS'_e &= d'(e)\cdot v'(z-d(e))-\frac{c\cdot\gamma}{1-\gamma}(1-T_m)\frac{\alpha(\theta)}{m(\theta)}\cdot v'(w-T)
\end{aligned}$$

After some manipulation, $WS(.,.,.)=0$ becomes:

$$\frac{c\cdot\gamma}{1-\gamma}\cdot\frac{(1-T_m)\cdot v'(w-T)}{m(\theta)}=\frac{v(w-T)-v(z-d(e))}{r+q+e\cdot\alpha(\theta)}$$

Taking this equality into account leads to:

$$WS'_e=d'(e)\cdot v'(z-d(e))-\alpha(\theta)\frac{v(w-T)-v(z-d(e))}{r+q+e\cdot\alpha(\theta)}=-\frac{S(\theta,w,e)}{r+q+e\cdot\alpha(\theta)}$$

Hence, WS'_e is equal to zero in equilibrium. Finally, one has:

$$\begin{aligned}
WS'_T &= -v'(w-T)+\frac{c\cdot\gamma}{1-\gamma}(1-T_m)\frac{r+q+e\cdot\alpha(\theta)}{m(\theta)}\cdot v''(w-T)<0 \\
WS'_{T_m} &= \frac{c\cdot\gamma}{1-\gamma}\frac{r+q+e\cdot\alpha(\theta)}{m(\theta)}\cdot v'(w-T)>0 \\
WS'_z &= -v'(z-d(e))<0
\end{aligned}$$

Appendix 2: Unicity of the equilibrium, comparative statics and dynamic properties

First we show that the system $\mathbb{S}(\theta,e)=\mathbb{W}(\theta,e)=0$ has at most one solution. Since $\mathbb{S}'_e(\theta,e)<0$, for any θ, the equation $\mathbb{S}(\theta,e)=0$ admits at most one solution. Call this solution $\mathbb{E}(\theta)$ if it exists. The implicit function theorem insures that function $\mathbb{E}(\theta)$ is continuous and differentiable wherever it is defined. Now, let $\mathcal{W}(\theta)\equiv\mathbb{W}(\theta,\mathbb{E}(\theta))$. An equilibrium necessarily solves $\mathcal{W}(\theta)=0$. Differentiating function $\mathcal{W}(.)$ yields $\mathcal{W}'(\theta)=\mathbb{W}'_\theta(\theta,\mathbb{E}(\theta))+\mathbb{E}'(\theta)\cdot\mathbb{W}'_e(\theta,\mathbb{E}(\theta))$. Since $\mathbb{E}(\theta)$ solves $\mathbb{S}(\theta,\mathbb{E}(\theta))=0$, one has:

$$\mathbb{W}'_e(\theta,\mathbb{E}(\theta))=-\mathbb{S}(\theta,\mathbb{E}(\theta))/\left(r+q+\mathbb{E}(\theta)\cdot\theta\right)=0$$

Hence, $\mathcal{W}'(\theta)=\mathbb{W}'_\theta(\theta,\mathbb{E}(\theta))<0$. So equation $\mathcal{W}(\theta)=0$ admits at most one solution. The equilibrium, if any, is therefore unique.

Second, we look at the comparative statics of the equilibrium. Differentiating $\mathbb{S}(\theta,e)=\mathbb{W}(\theta,e)=0$ yields:

$$\begin{pmatrix}\mathbb{W}'_\theta & \mathbb{W}'_e \\ \mathbb{S}'_\theta & \mathbb{S}'_e\end{pmatrix}\begin{pmatrix}d\theta \\ de\end{pmatrix}=-\begin{pmatrix}\mathbb{W}'_z & \mathbb{W}'_T & \mathbb{W}'_{T_m} \\ \mathbb{S}'_z & \mathbb{S}'_T & \mathbb{S}'_{T_m}\end{pmatrix}\begin{pmatrix}dz \\ dT \\ dT_m\end{pmatrix}$$

Since around the equilibrium $\mathbb{W}'_e=0$, one has:

$$\begin{pmatrix}d\theta \\ de\end{pmatrix}=\begin{pmatrix}-\frac{1}{\mathbb{W}'_\theta} & 0 \\ \frac{\mathbb{S}'_\theta}{\mathbb{S}'_e\cdot\mathbb{W}'_\theta} & -\frac{1}{\mathbb{S}'_e}\end{pmatrix}\begin{pmatrix}\mathbb{W}'_z\cdot dz & \mathbb{W}'_T\cdot dT & \mathbb{W}'_{T_m}\cdot dT_m \\ \mathbb{S}'_z\cdot dz & \mathbb{S}'_T\cdot dT & \mathbb{S}'_{T_m}\cdot dT_m\end{pmatrix}$$

Hence:

$$d\theta=-\frac{\mathbb{W}'_z}{\mathbb{W}'_\theta}dz-\frac{\mathbb{W}'_T}{\mathbb{W}'_\theta}dT-\frac{\mathbb{W}'_{T_m}}{\mathbb{W}'_\theta}dT_m$$

Since $\mathbb{W}'_\theta<0$, $\mathbb{W}'_z<0$, $\mathbb{W}'_T<0$, $\mathbb{W}'_{T_m}>0$ one has $d\theta/dz<0$, $d\theta/dT<0$ and $d\theta/dT_m>0$. Moreover,

$$de=\frac{\mathbb{S}'_\theta\cdot\mathbb{W}'_z-\mathbb{W}'_\theta\cdot\mathbb{S}'_z}{\mathbb{S}'_e\cdot\mathbb{W}'_\theta}dz+\frac{\mathbb{S}'_\theta\cdot\mathbb{W}'_T-\mathbb{W}'_\theta\cdot\mathbb{S}'_T}{\mathbb{S}'_e\cdot\mathbb{W}'_\theta}dT+\frac{\mathbb{S}'_\theta\cdot\mathbb{W}'_{T_m}-\mathbb{W}'_\theta\cdot\mathbb{S}'_{T_m}}{\mathbb{S}'_e\cdot\mathbb{W}'_\theta}dT_m$$

Since $\mathbb{S}'_\theta$ has an ambiguous sign, the marginal effect of z, T, T_m on e cannot be signed.

Third, consider the dynamic properties. Along the transitional dynamics, given z, T and T_m, equations (9), (13) and (16) gives θ, w and e as a function of J and $V - V^u$ according to:

$$\theta = (m)^{-1}\left(\frac{c}{J}\right) \qquad e = \left(d'(e)\cdot v'(z-d(e))\right)^{-1}\left(\alpha(\theta)(V-V^u)\right)$$
$$w = T + (v')^{-1}\left(\frac{1-\gamma}{\gamma}\cdot\frac{V-V^u}{J}\right)$$

Hence, the dynamics of (θ, w, e) depends only on the evolution of J and $V - V^u$. According to equations (7), (11) and (12), one has

$$\dot{J} = (r+q)J - y + w(J, V-V^u)$$
$$\dot{\overline{V - V^u}} = [r+q+e(J,V-V^u)\cdot\alpha(\theta(J,V-V^u))](V-V^u) - v(w(J,V-V^u)-T) + v(z-d(e(J,V-V^u)))$$

So, $X = (J, V - V^u)$ is a vector of state variables that evolves as a function of itself only. Denote D the dynamics so $\dot{X} = D(X)$. A steady state equilibrium solves $D = 0$. Since all the variables in vector X are forward looking, two cases might appear regarding to the eigenvalues of the Jacobian of D in the neighborhood of the steady state. If all the eigenvalues have a strictly positive real part, the transitional dynamics is unique and the state variables instantaneously reach their steady state value. Otherwise, there is a multiplicity of transitional dynamics, one of them being the instantaneous jump of the state variable to their steady-state value. Applying a selection criteria based on simplicity, one can think that this dynamics would be the more realistic. This is exactly what we assume. Hence, we consider that X and therefore θ, e and w are always at their steady-state values. The adjustment of the unemployment rate is then determined by equation $\dot{u} = q(1-u) - e\cdot\alpha(\theta)$ where e and θ always are at their steady state values.

Appendix 3 The Social Welfare Criteria

The social welfare criteria $\Omega = (1-u)V + u\cdot V^u + \eta\cdot\mathbb{P}$ can be differentiated with respect to time. This derivative can then be substracted from $r\cdot\Omega$ to yield :

$$r\cdot\Omega - \dot{\Omega} = (1-u)\left(r\cdot V - \dot{V}\right) + u\cdot\left(r\cdot V^u - \dot{V}^u\right) + \eta\left(r\cdot\mathbb{P} - \dot{\mathbb{P}}\right) + \dot{u}(V-V^u)$$
$$= (1-u)v(\omega) + u\cdot v(z-d(e)) + \eta\left(r\cdot\mathbb{P} - \dot{\mathbb{P}}\right) + (V-V^u)(e\cdot\alpha(\theta)\cdot u - q(1-u) + \dot{u})$$

by definition of V and V^u. Moreover, according to (6), one has:

$$\dot{\mathbb{P}}_t = -\Pi_t + \int_t^{+\infty} r\cdot e^{-r(\tau-t)}\cdot\Pi_\tau\cdot d\tau = -\Pi_t + r\cdot\mathbb{P}_t$$

Since $\dot{u} = q(1-u) - e\cdot\alpha(\theta)\cdot u$ and $r\cdot\mathbb{P} - \dot{\mathbb{P}} = \Pi$ one concludes that:

$$r\cdot\Omega - \dot{\Omega} = (1-u)v(\omega) + u\cdot v(z-d(e)) + \eta\cdot\Pi$$

So,

$$\Omega = \int e^{-rt}\{(1-u)v(\omega) + u\cdot v(z-d(e)) + \eta\cdot\Pi\}\cdot dt$$

Appendix 4: Characterization of the first-best optimum

Since $F(\theta,e) = \alpha'(\theta)(d'(e) + c\cdot\theta) - c\cdot\alpha(\theta)$, the partial derivatives of F have unambiguous signs:

$$F'_\theta = \alpha''(\theta)\left(d'(e) + c\cdot\theta\right) + c\cdot\alpha'(\theta) - c\cdot\alpha'(\theta) < 0$$
$$F'_e = \alpha'(\theta)\cdot d''(e) > 0$$

$G(\theta, e)$ can be rewritten as $\alpha(\theta)(y + d(e)) - c\cdot\theta\cdot q - d'(e)(e\cdot\alpha(\theta) + q)$. Then,

$$G'_e = -d''(e)(e\cdot\alpha(\theta) + q) < 0$$
$$G'_\theta = \alpha'(\theta)\left(y + d(e) - e\cdot d'(e)\right) - c\cdot q$$

However, along $G(\theta, e) = 0$, one has:

$$y + d(e) = \frac{c\cdot\theta\cdot q + d'(e)(e\cdot\alpha(\theta)+q)}{\alpha(\theta)} = \frac{q}{\alpha(\theta)}\cdot\left(c\cdot\theta + d'(e)\right) + e\cdot d'(e)$$

Therefore,

$$G'_\theta = \alpha'(\theta)\cdot\frac{q}{\alpha(\theta)}\cdot\left(c\cdot\theta + d'(e)\right) - c\cdot q$$
$$= q\cdot\alpha'(\theta)\left(\frac{c\cdot\theta + d'(e)}{\alpha(\theta)} - \frac{c}{\alpha'(\theta)}\right)$$
$$= \frac{q}{\alpha(\theta)}\left(\alpha'(\theta)\left(c\cdot\theta + d'(e)\right) - c\cdot\alpha(\theta)\right) = \frac{q}{\alpha(\theta)}F(\theta,e)$$

Finally, differentiating $H(\theta,e) = \alpha'(\theta)(y + d(e) + e\cdot c\cdot\theta) - c(e\cdot\alpha(\theta) + q)$ yields:

$$H'_\theta = \alpha''(\theta)(y + d(e) + e\cdot c\cdot\theta) + e\cdot c\cdot\alpha'(\theta) - e\cdot c\cdot\alpha'(\theta) < 0$$
$$H'_e = \alpha'(\theta)\left(d'(e) + c\cdot\theta\right) - c\cdot\alpha(\theta) = F(\theta,e)$$

Appendix 5: The second-best optimum

Let us first show that $\psi_2 > 0$. From first-order condition (31),

$$\psi_2 = \frac{u_2\left[d'(e_2)\cdot v'(z_2 - d(e_2)) + \eta\cdot c\cdot\theta_2 - \delta_2\cdot\alpha(\theta_2)\right]}{\left\{-d''(e_2)\cdot v'(z_2 - d(e_2)) + (d'(e_2))^2\cdot v''(z_2 - d(e_2))\right\}(q + e_2\cdot\alpha(\theta_2))} \qquad (36)$$

The denominator of the last expression is clearly negative. Therefore, its numerator has to be negative in order to guarantee that $\psi_2 > 0$. To show that this numerator is negative, one has to extract $\delta_2\cdot\alpha(\theta_2)$ from condition (32). Taking (28) into account, this yields:

$$\delta_2\cdot\alpha(\theta_2) = v'(z_2 - d(e_2))\cdot d'(e_2) + \eta\frac{\alpha(\theta_2)}{q_2 + e_2\cdot\alpha(\theta_2)}(y - \omega_2 + z_2 + c\cdot e_2\cdot\theta_2)$$

Substituting this in the numerator of the right hand side of (36), $\psi_2 > 0$ is equivalent to:

$$d'(e_2)\cdot v'(z_2 - d(e_2)) + \eta\cdot c\cdot\theta_2 < v'(z_2 - d(e_2))\cdot d'(e_2) + \eta\frac{\alpha(\theta_2)}{q_2 + e_2\cdot\alpha(\theta_2)}(y - \omega_2 + z_2 + c\cdot e_2\cdot\theta_2)$$

After some simplifications, using (1), $\psi_2 > 0$ is equivalent to

$$c \cdot e_2 \cdot \theta_2 \cdot u_2 < (1-u_2)(y - \omega_2 + z_2)$$

or:

$$(1-u_2)(\omega_2 - z_2) < (1-u_2)\, y - c \cdot v_2$$

But, by (4)

$$(1-u_2)\,\omega_2 + u_2 \cdot z_2 + \Pi_2 = (1-u_2)\, y - c \cdot v_2$$

So $\psi_2 > 0$ is equivalent to:

$$\begin{aligned} (1-u_2)(\omega_2 - z_2) &< (1-u_2)\,\omega_2 + u_2 \cdot z_2 + \Pi_2 \\ -(1-u_2)\, z_2 &< u_2 \cdot z_2 + \Pi_2 \\ -z_2 &< \Pi_2 \end{aligned}$$

This condition is satisfied since, by assumption Π_2 and z_2 are nonnegative.

It will now be shown that one has $G(\theta_2, e_2) > 0$ at the second-best optimum. Dividing first-order condition (32) by η, adding $d(e_2)$ on both sides and rearranging yields:

$$y + d(e_2) + c \cdot e_2 \cdot \theta_2 = \frac{\delta_2}{\eta}(e_2 \cdot \alpha(\theta_2) + q) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{\eta_2} + \omega_2 - z_2 + d(e_2) \qquad (37)$$

Multiplying both sides by $\alpha(\theta_2)$ yields:

$$\begin{aligned} &\alpha(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) = \frac{\delta_2 \cdot \alpha(\theta_2)}{\eta}(e_2 \cdot \alpha(\theta_2) + q) \\ &-\frac{\alpha(\theta_2)[v(\omega_2) - v(z_2 - d(e_2))]}{\eta} + \alpha(\theta_2)(\omega_2 - z_2 + d(e_2)) \end{aligned}$$

Taking the incentive constraint (28) into account, the previous equality can be rewritten in the following way:

$$\begin{aligned} \alpha(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2))\right] \\ &+\alpha(\theta_2)(\omega_2 - z_2 + d(e_2)) \end{aligned}$$

The right-hand side of the last equality can be substituted in the definition of function G evaluated at (θ_2, e_2). After some manipulations, this yields:

$$\begin{aligned} G(\theta_2, e_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta\right] \\ &+\alpha(\theta_2)(\omega_2 - z_2 + d(e_2)) - (e_2 \cdot \alpha(\theta_2) + q)\, d'(e_2) \end{aligned}$$

Using once again the incentive constraint, $G(\theta_2, e_2)$ can be restated as:

$$\begin{aligned} G(\theta_2, e_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta\right] \qquad (38) \\ &+\frac{\alpha(\theta_2)}{v'(z_2 - d(e_2))}\left[v'(z_2 - d(e_2)) \cdot (\omega_2 - z_2 + d(e_2)) - v(\omega_2) + v(z_2 - d(e_2))\right] \end{aligned}$$

However, the first-order condition (31) insures that:

$$\begin{aligned} &\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta \\ = &\frac{\psi_2}{u_2}\left[d''(e_2) \cdot v'(z_2 - d(e_2)) - (d'(e_2))^2 \cdot v''(z_2 - d(e_2))\right](q + e_2 \cdot \alpha(\theta_2)) > 0 \end{aligned}$$

So, the first term on the right hand side of (38) is positive. In addition, the concavity of $v(.)$ implies that :

$$v(\omega_2) - v(z_2 - d(e_2)) < v'(z_2 - d(e_2)) \cdot (\omega_2 - z_2 + d(e_2))$$

by which the second term on the right hand side of (38) is positive too. Therefore, function G evaluated at the second-best optimum is positive while the same function was zero and reached a maximum evaluated at the first-best optimum. So, $e_2 < e_1$.

Next, it will be shown that $H(\theta, e) < 0$ at the second-best optimum. The first-order condition (33) together with the incentive constraint (28) gives:

$$c \cdot e_2 \cdot \eta \cdot u_2 = \delta_2 \cdot \alpha'(\theta_2) \cdot e_2 \cdot u_2 + \psi_2 \cdot \alpha'(\theta_2) \cdot \frac{q}{\alpha(\theta_2)} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2)$$

Substituting the flow equilibrium (1) yields:

$$c = \alpha'(\theta_2)\left\{\frac{\delta_2}{\eta} + \frac{\psi_2}{\eta} \cdot \frac{1}{1-u_2} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2)\right\}$$

Substituting this expression and equation (37) into $H(\theta_2, e_2)$, i.e. in $\alpha'(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) - c(e_2 \cdot \alpha(\theta_2) + q)$ leads to

$$\begin{aligned} H(\theta_2, e_2) &= \alpha'(\theta_2)\left\{\omega_2 - z_2 + d(e_2) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{\eta}\right. \\ &\left. - \frac{\psi_2}{\eta(1-u_2)} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2) \cdot (e_2 \cdot \alpha(\theta_2) + q)\right\} \end{aligned}$$

Taking (28) into account, this expression can be rewritten as:

$$\begin{aligned} H(\theta_2, e_2) = \alpha'(\theta_2) &\left\{\omega_2 - z_2 + d(e_2) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{\eta}\right. \\ &\left. -\frac{\psi_2 \cdot \alpha(\theta_2)}{\eta(1-u_2)}[v(\omega_2) - v(z_2 - d(e_2))]\right\} \end{aligned}$$

From first-order condition (29)

$$\frac{\psi_2 \cdot \alpha(\theta_2)}{\eta(1-u_2)} = \frac{1}{v'(\omega_2)} - \frac{1}{\eta}$$

Therefore

$$H(\theta_2, e_2) = \alpha'(\theta_2)\left\{\omega_2 - z_2 + d(e_2) - \frac{v(\omega_2) - v(z_2 - d(e_2))}{v'(\omega_2)}\right\}$$

Finally, concavity of function $v(.)$ implies that:

$$0 < v'(\omega_2) \cdot (\omega_2 - z_2 + d(e_2)) < v(\omega_2) - v(z_2 - d(e_2)) < v'(z_2 - d(e_2)) \cdot (\omega_2 - z_2 + d(e_2))$$

Therefore, function H evaluated at the second-best optimum (θ_2, e_2) is always negative. This implies that $\theta_2 > \theta_1$.

Appendix 6 : The second-best optimum with risk-neutral workers

In this appendix, we show that the paper of Hosios (1990) can be seen as a particular case of our model. To do so, we check that the decentralized equilibrium is the second-best optimum when search effort is unobservable and workers are risk neutral with $v(x) = \eta \cdot x$. We consider a *laissez-faire* equilibrium with $T = T_m = z = 0$ and we assume the so-called Hosios condition is fulfilled $\gamma = -\frac{\theta \cdot m'(\theta)}{m(\theta)}$. The proof consists in showing that the decentralized equilibrium verifies $F(\theta, e) = H(\theta, e) = 0$.

Assuming $r = 0$ in expressions (11) and (12) measured in steady state, it is easily seen that:

$$\frac{V - V^u}{v'(w - T)} = \frac{V - V^u}{v'(z - d(e))} = \frac{w + d(e)}{q + e \cdot \alpha(\theta)}$$

Substituting this expression in $WS(\theta, w, e) = 0$ (adapted to take the above assumptions into account), one gets:

$$\frac{w + d(e)}{q + e \cdot \alpha(\theta)} = \frac{\gamma}{1 - \gamma} \cdot \frac{c}{m(\theta)} \tag{39}$$

The optimal search effort level still verifies $S(\theta, w, e) = 0$, which can now be rewritten as:

$$\frac{w + d(e)}{q + e \cdot \alpha(\theta)} = \frac{d'(e) \cdot v'(z - d(e))}{\alpha(\theta)} \tag{40}$$

Equating the left-hand sides of the two last equalities leads to:

$$\theta = \frac{d'(e)}{c} \frac{1 - \gamma}{\gamma}$$

which can be rewritten as:

$$\frac{c \cdot \theta}{1 - \gamma} = c \cdot \theta + d'(e)$$

Substituting the Hosios condition, one finally has

$$\frac{c}{\alpha'(\theta)} = \frac{c \cdot \theta + d'(e)}{\alpha(\theta)}$$

which is exactly $F(\theta, e) = 0$.

Adding $d(e)$ on both sides of equality (10) leads to $w + d(e) = y + d(e) - \frac{c \cdot q}{m(\theta)}$. Combining this equality and equation (39) yields after some manipulation:

$$\frac{y + d(e) + e \cdot c \cdot \theta}{q + e \cdot \alpha(\theta)} = \frac{1}{1 - \gamma} \cdot \frac{c}{m(\theta)}$$

Finally, replacing γ by the Hosios condition, one ends with $H(\theta, e) = 0$.

The message is exactly the main one of Hosios (1990). If redistribution (insurance) is not an issue, i.e. with marginal utilities of all agents equal and constant ($\eta = v'(.)$), the decentralized equilibrium under *laissez faire* is a social optimum provided that the bargaining power of the worker verifies the so-called Hosios condition $\gamma = -\theta m'(\theta) / m(\theta)$. This property has been shown under the assumption that search effort is unobservable.

Appendix 7: The "third-best" case

The "third-best" problem can be defined as the maximization of the utilitarian criterion Ω subject to the set of constraints of the second best problem extended to impose that the CRIP be equal to 1 (i.e. imposing linear taxes). Remembering (34) this problem can be formulated as

$$\max_{\theta,\omega,u,z,e} \quad (1 - u) \cdot v(\omega) + u \cdot v(z - d(e)) + \eta \left[(1 - u)(y - \omega) - z \cdot u - c \cdot e \cdot \theta \cdot u\right]$$
$$0 = e \cdot \alpha(\theta) \cdot u - q(1 - u)$$
$$0 = \alpha(\theta)(v(\omega) - v(z - d(e))) - d'(e) \cdot v'(z - d(e))(q + e \cdot \alpha(\theta))$$
$$1 = \frac{1 - \gamma}{\gamma} \frac{v(\omega) - v(z - d(e))}{\omega \cdot v'(\omega)} \frac{m(\theta) \cdot y - c \cdot q}{c(q + e \cdot \alpha(\theta))}$$