On the optimality of search matching equilibrium when workers are risk averse *

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Abstract

This paper revisits the normative properties of search-matching economies when homogeneous workers have concave utility functions and wages are bargained over. The optimal allocation of resources is characterized first when information is perfect and second when search effort is not observable. In the former case, employees should be unable to extract a rent. The optimal marginal tax rate is then 100%. As search effort becomes unobservable, an appropriate positive rent is needed and the optimal marginal tax rate is lower. Moreover the pre-tax wage is lower in order to boost labor demand. Finally, in both cases, non-linear income taxation is a key complement to unemployment insurance.

Keywords: Unemployment, Non-linear Taxation, Unemployment Benefits, Moral Hazard, Search, Matching.

JEL codes: J64, J65, J68, H21, D82

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1 Introduction

Models of frictional unemployment in which workers and firms bargain over wages have become popular to discuss labor market policies (see Mortensen and Pissarides 1999 and Pissarides 2000). In the policy debate, unemployment is typically not only perceived as a waste of resources but also as a major source of risk for workers' income. How risk-averse workers should be insured in a frictional economy is a central question that we here address. With endogenous wages, this question cannot be fruitfully addressed without considering the financing of unemployment insurance. Income taxation affects wage formation. In particular, more progressive taxes moderate wages in these models. Hence, taxation affects job-search intensity, the number of vacancies and thereby the intensity of the unemployment risk. In the present paper, we derive the optimal combination of unemployment insurance and non-linear taxation in a matching model with endogenous search intensity and Nash bargaining over wages.

We contrast two polar informational settings. In the first-best case, search intensity is observable while in the second-best, it is not. We show that, compared to the first-best optimum, the second-best one is characterized by *i*) incomplete insurance; *ii*) positive rents for employed workers through a lower marginal tax rate; *iii*) a lower pre-tax wage to increase job arrival rates, thereby stimulating job-search intensity and relaxing the moral-hazard incentive constraint; *iv*) a lower job search intensity. From these results, we deduce that a better control over job-search effort should not only lead to a better insurance against the unemployment risk but should also affect tax progressivity. More precisely, if the relationship between the extent to which job search is observable and the optimal marginal tax rate is monotone, this relationship is increasing.

This paper also contributes to the literature about the desirability of progressive labor taxes in wage bargaining models (e.g. Malcomson and Sator 1987; Lockwood and Manning 1993; Sorensen 1999, Pissarides 1998 and Boone and Bovenberg 2002). For a given level of taxes, the negotiated wage is a decreasing function of the marginal tax rate. Accordingly, a more progressive labor tax schedule should reduce unemployment.

However, the desirability of progressive labor income taxes has been recently questioned by papers that introduce in-work effort (Hansen 1999, Fuest and Huber 2000) or training decisions (Boone and de Mooij 2003 and Hungerbühler 2006). A more progressive tax schedule can reduce productivity per capita so that the total effect on output becomes ambiguous. We put forward another unfavorable effect of tax progressivity. Through a reduction in the rent extracted by employees, tax progressivity decreases the incentives unemployed people have to search (see also Pissarides 2000, page 221).

We also contribute to the literature on optimal unemployment insurance. The seminal article of Baily (1977) formulates the search for optimal unemployment insurance as a moral hazard problem. Including firms' behavior and the negotiation of wages enriches the analysis since labor demand influences the unemployed workers' incentive constraint. We highlight that non-linear taxation and unemployment insurance are complementary instruments. However, we do not pay attention to the profile of unemployment benefits as a way of improving the trade-off between insurance and efficiency under imperfect information (Shavell and Weiss, 1979; Hopenhayn and Nicolini, 1997; Cahuc and Lehmann, 2000; Fredriksson and Holmlund, 2001). Nor do we pay attention to sanctions, that is the reduction or the withdrawal of unemployment benefits if search effort is judged insufficient (e.g. Boadway and Cuff, 1999, Boone and van Ours, 2000, Boone et al, 2001).

The paper is organized as follows. Section 2 describes the structure of the economy. Section 3 is devoted to the equilibrium, Section 4 to the first-best optimum, Section 5 to the second-best optimum. Section 6 discusses the result. Section 7 concludes the paper.

2 Assumptions and Notations

We consider a labor market which is made of a continuum of infinitely-lived, risk-averse, and homogeneous workers. There are no financial markets. Workers are either employed or unemployed. Jobs are either filled or vacant. Matching unemployed workers with vacant jobs is a time-consuming and costly process. Time is continuous. The flow of hires M is a function $M(\Sigma, \mathcal{V})$ of the number of job-seekers measured in efficiency units

Σ and of the number of vacancies $\mathcal{V}$. Denoting by e the average search intensity and by u the mass of unemployed workers, one has $\Sigma = e \cdot u$. As usual, the matching function is increasing and concave in both arguments (with $M(0, \mathcal{V}) = M(\Sigma, 0) = 0$) and returns to scale are constant (see e.g. Petrongolo and Pissarides, 2001). Let $\theta \equiv \mathcal{V}/\Sigma$ be tightness on the labor market (measured in efficiency units). The rate at which a vacant job is filled is $m(\theta)$ with $m(\theta) \equiv M(1/\theta, 1) = \frac{M(\Sigma,\mathcal{V})}{\mathcal{V}}$, and $m'(.) < 0$. An unemployed with search intensity $e_i \geq 0$ flows out of unemployment at the endogenous rate $e_i \cdot \alpha(\theta) = \frac{e_i}{e} \cdot \frac{M(e \cdot u, \mathcal{V})}{u}$, with $\alpha(\theta) \equiv M(1, \theta) = \theta \cdot m(\theta) = \frac{M(\Sigma,\mathcal{V})}{\Sigma}$ and $\alpha'(\theta) > 0$, $\alpha''(.) < 0$. Job matches end at the exogenous rate q.

We normalize the size of the labor force to 1. In steady state, equality between entries and exits yields the "Beveridge curve" equation:

$$e \cdot \alpha(\theta) \cdot u = q(1-u) \qquad \Leftrightarrow \qquad u = \frac{q}{q + e \cdot \alpha(\theta)} \tag{1}$$

that negatively links the unemployment rate to tightness θ.

Let $r \geq 0$ be the discount rate common to workers and firms. Later, we will consider the limit case where r tends to 0. The after-tax (pre-tax) wage is denoted x (w). Taxation is a differentiable function of the pre-tax wage $\tau(w)$. Since we consider a single segment of the labor market, only the levels of tax $T = \tau(w)$ and of the marginal tax rate $T_m = \tau'(w)$ matter. By definition, $x = w - T$. The utility function $v(.)$ is increasing and concave. An employed worker has an instantaneous utility $v(x)$. An unemployed worker has an instantaneous utility $v(z - d(e))$ where z denotes her untaxed unemployment benefits. Function $d(e)$ denotes the monetary cost of job-search activities and the money value of home production or of informal activities. We assume that function $d(.)$ is non-negative, increasing and convex (with $\lim_{e \to 0} d'(e) = 0$ and $\lim_{e \to \infty} d'(e) = +\infty$)[1].

The model is developed in steady state. Let V and V^u denote the expected lifetime utility of respectively an employed and an unemployed worker. V solves:

$$r \cdot V = v(x) + q(V^u - V) \tag{2}$$

[1] An alternative specification for the utility function will be considered in Section 6.

Two cases will be considered. The one where search intensity is observable will be introduced later. When search cannot be observed, V^u solves:

$$r \cdot V^u = \max_{e_i} \left\{ v\left(z - d\left(e_i\right)\right) + e_i \cdot \alpha\left(\theta\right)\left(V - V^u\right) \right\} \tag{3}$$

Each firm is made of a unique filled or vacant job. Each filled (vacant) job produces (costs) a flow of y (c) units of output. w denotes the gross wage (or equivalently the wage cost). Let J denote the intertemporal expected value of a filled vacancy and $J^{\mathcal{V}}$ the expected value of an open vacancy. J and $J^{\mathcal{V}}$ solve:

$$r \cdot J = y - w + q\left(J^{\mathcal{V}} - J\right) \tag{4}$$

$$r \cdot J^{\mathcal{V}} = -c + m\left(\theta\right)\left(J - J^{\mathcal{V}}\right) \tag{5}$$

We assume that the budgetary surplus χ of the Unemployment Insurance system (henceforth, UI) should at least be equal to an exogenous lower bound $\overline{\chi}$. The UI system has typically to be balanced. Then, $\overline{\chi} = 0$. For the purpose of generality, $\overline{\chi}$ could represent exogenous public expenses and take any value. So,

$$\chi = T\left(1 - u\right) - u \cdot z \geq \overline{\chi} \tag{6}$$

Following e.g. Fredriksson and Holmlund (2001), we consider a Utilitarian criterion:

$$\Omega = \left(1 - u\right) r \cdot V + u \cdot r \cdot V^u + \left(1 - u\right) r \cdot J + \mathcal{V} \cdot r \cdot J^{\mathcal{V}}$$

Further, we ignore the transitional dynamics by assuming that r tends to 0. Hence, from equations (1) to (5), as well as $e \cdot \alpha\left(\theta\right) \cdot u = m\left(\theta\right) \cdot \mathcal{V}$, the social planner's objective is:

$$\Omega = \left(1 - u\right) v\left(x\right) + u \cdot v\left(z - d\left(e\right)\right) + \left(1 - u\right)\left(y - w\right) - \mathcal{V} \cdot c \tag{7}$$

3 The Market equilibrium

3.1 Free entry

Assuming free entry of vacancies, a steady-state equilibrium should be characterized by $J^{\mathcal{V}} = 0$. Hence, in such an equilibrium:

$$J = \frac{c}{m\left(\theta\right)} = \frac{y - w}{q} \qquad \Rightarrow \qquad x + T = \phi\left(\theta\right) \equiv y - \frac{c \cdot q}{m\left(\theta\right)} \tag{8}$$

This relationship between the after tax wage x and tightness θ is downward-sloping. As the wage or the tax level increases, the value of a filled job J declines and so do the number of vacancies and tightness θ. Since θ is measured in efficiency units, one should note that this relation does not depend on search intensity e.

Given the flow equilibrium equation (1) and the free-entry condition (8), one gets $(1-u)(y-w) = c \cdot \mathcal{V}$, so the budget surplus χ defined in (6) can be rewritten as:

$$\chi = Y - (1-u)\, x - u\,(z - d\,(e)) \tag{9}$$

where

$$Y \equiv (1-u)\, y - u \cdot d\,(e) - c \cdot \mathcal{V} \tag{10}$$

stands for total output net of search and vacancy costs. As it is often done in the equilibrium search-matching literature, "efficiency" means here the maximization of Y. Finally, using again $(1-u)(y-w) = c \cdot \mathcal{V}$, the social planner's objective (7) is equal to workers' expected utility:

$$\Omega = (1-u)\, v\,(x) + u \cdot v\,(z - d\,(e)) \tag{11}$$

3.2 Search Behavior

Search intensity solves (3) where V, V^u and θ are taken as given. The first-order condition of this problem is:

$$d'\,(e) \cdot v'\,(z - d\,(e)) = \alpha\,(\theta)\,(V - V^u) \tag{12}$$

The left-hand side measures the marginal cost of search effort. On the right-hand side, the marginal gain increases with tightness and the rent extracted by employed workers $V - V^u$. Substituting (2) and (3) into this rent, Equation (12) implicitly defines the optimal search level e according to $S\,(\theta, x, e) = 0$ defined in Appendix A.1. There, we show that a higher wage x and a tighter labor market θ raise search intensity e because the marginal gain of search is then raised.

3.3 The Wage Bargain

A match generates a surplus that is shared between the matched worker and firm. Let γ be the exogenous bargaining power of the worker, with $0 < \gamma < 1$. The gross wage rate maximizes the following Nash product:

$$\max_{w} \quad (V - V^u)^{\gamma} (J - J^{\mathcal{V}})^{1-\gamma}$$

The wage setters realize that a marginal rise of the gross wage of an amount Δw changes the level of taxes by $T_m \cdot \Delta w$, where T_m denotes the marginal tax rate. Taking this relationship and θ as given, the first-order condition is:

$$V - V^u = \frac{\gamma (1 - T_m)}{1 - \gamma} v'(x) \left(J - J^{\mathcal{V}}\right) \tag{13}$$

Let $\hat{\gamma}$ be such that:

$$\frac{\hat{\gamma}}{1 - \hat{\gamma}} = \frac{\gamma (1 - T_m)}{1 - \gamma} \tag{14}$$

$\hat{\gamma}$ denotes the workers' *actual* bargaining power taking into account the negative effect of the marginal tax rate on their effective bargaining strength. For given tightness θ, search intensity e, bargaining power γ and level of taxes T, a higher marginal tax rate T_m lowers the change in the after tax wage resulting from a given increase in the negotiated gross wage. This lowers the employees' rent $V - V^u$ and eventually moderates wages (see e.g. Malcomson and Sator, 1987; Lockwood and Manning, 1993). Substituting $V - V^u$ from (2) and (3) and using (8), condition (13) becomes an implicit wage-setting equation $WS(\theta, x, e) = 0$ defined in Appendix A.1. There, we show that conditional on e, the wage-setting equation implies an increasing relation between θ and x. For given levels of taxes T, tightness θ and search intensity e, a more progressive tax schedule puts a downward pressure on the negotiated wage. A rise in unemployment benefits z has the usual positive effect on wages.

3.4 Equilibrium

Conditional on the exogenous variables (z, T, T_m, γ), a steady-state equilibrium (θ, x, e, u) is a solution of the system $x + T = \phi(\theta)$, $S(\theta, x, e) = 0$, $WS(\theta, x, e) = 0$ and (1) that satisfies (6). This equilibrium is solved recursively as follows. First, Appendix A.2 proves that there exists at most one solution (θ, x, e) to the system $x + T = \phi(\theta)$, $S(\theta, x, e) = 0$, $WS(\theta, x, e) = 0$. Then, Equation (1) gives a unique unemployment rate u. Finally, from z, T and the obtained value of u, we compute the budget surplus χ (see 6), and verifies whether it satisfies the requirement $\chi \geq \overline{\chi}$. An equilibrium does not exist when this last inequality is not satisfied. When it exists, the equilibrium is unique. This property will be useful to decentralize social optima. Furthermore, it is shown in Appendix A.2 that equilibrium tightness (the pre-tax wage w) decreases (increases) with the levels of taxes and unemployment benefits and increases (decreases) with the marginal tax rate. These properties are well-known when workers are risk neutral (see e.g. Pissarides, 2000).

4 The first-best optimum

This section first looks at the optimal allocation of resources that a benevolent social planner would implement if he could perfectly control search intensity. The decentralization of the optimum is considered afterwards. The central planner controls tightness, the level of effort, the unemployment rate, net income and the unemployment benefit. He maximizes workers' expected utility Ω subject to the budget constraint (6) and the flow equilibrium equation (1). Remembering that $\mathcal{V} = e \cdot \theta \cdot u$ and Equations (9) and (10), the planner's program is[2]:

$$\begin{aligned} \max_{\theta,x,u,z,e} & \quad (1-u)\, v(x) + u \cdot v(z - d(e)) \qquad (15) \\ s.t.: \overline{\chi} & \leq (1-u)(y-x) - u(z + c \cdot e \cdot \theta) \\ 0 & = e \cdot \alpha(\theta) \cdot u - q(1-u) \end{aligned}$$

Subscript 1 denotes the first-best optimum. Let η_1 be the Lagrange multiplier of the budget constraint. Appendix B.2 shows that the social planner perfectly insures workers against the unemployment risk:

$$v'(x_1) = v'(z_1 - d(e_1)) = \eta_1 \quad \Leftrightarrow \quad x_1 = z_1 - d(e_1) = (v')^{-1}(\eta_1) \qquad (16)$$

[2]Formally, one should maximize Ω with $r > 0$ under the dynamical constraint $\dot{u} = q(1-u) - e \cdot \alpha(\theta) \cdot u$, derive the first-order and envelope conditions and take the limits of those conditions for $r \to 0$. It can be verified that this method and the maximization of the following problem give the same results for $r \to 0$.

Perfect insurance means here that the level of utility is independent of the position occupied in the labor market. Finally (e_1, θ_1, u_1) is the value of (e, θ, u) that maximize output net of search cost Y. It should be noticed that (e_1, θ_1, u_1) does not depend on the endogenous Lagrange multiplier η_1, nor on the exogenous budget surplus requirement $\overline{\chi}$.

The first-best setting with perfect monitoring of job search intensity is clearly a highly idealized case. However, looking briefly at the decentralization of this optimum highlights the complementarity between non-linear taxation and unemployment insurance. The State has to decentralize an equilibrium in which workers are perfectly insured against the unemployment risk. Substituting (16) in (2) and (3), employed workers then extract no rent from a match, so $V - V^u = 0$. From the wage bargain condition (13), the *actual* bargaining power $\hat{\gamma}_1$ has to be equal to zero. Whenever the workers' bargaining power γ is positive, this can only be achieved with a marginal tax rate $T_{m,1} = 100\%$ (see Equation (14)). So, as soon as workers have some bargaining power, the decentralization of the first-best optimum cannot be achieved without an "extremely" progressive income tax schedule.

One may wonder why the decentralization with risk averse workers differ so much from the one under "linear" preferences (i.e. with $v''(.) \equiv 0$). In the latter case, the social planner is only concerned with total output net of search costs, Y, independently of the way this output is shared between the employed and the unemployed workers. There is therefore a multiplicity of first-best optima. Any combination of x and z leading to the same total output Y_1 and the same budget surplus $\chi = \overline{\chi}$ is actually a first-best optimum in this case. The *laissez faire* economy (without taxes and unemployment insurance) only corresponds to one of these optima if the Hosios (1990) condition is fulfilled. This condition requires that workers' bargaining power be equal to the elasticity of the matching function with respect to unemployment measured in efficiency units $\Sigma = e \cdot U$. Employed workers should receive a certain share of the rent generated by a match in order to compensate them for the cost inherent to job search activities and to prevent the creation of too many vacancies in equilibrium[3]. When preferences tend to the "linear" case, our decentralization

[3]When the bargaining power does not fulfill the Hosios condition, Boone and Bovenberg (2002) show

(with $\hat{\gamma}_1 = 0$) leads to another efficient optimum with perfect unemployment insurance.

5 The second-Best optimum

In this section, we consider the polar case where search intensity is not observed by the State. As in the first best, the tax system and the level of unemployment benefits are the instruments used to promote efficiency and to insure workers. Since search effort is now chosen by the unemployed, the State faces a moral hazard problem. The incentive constraint $S(\theta, x, e) = 0$ has therefore to be included in the planner's problem. Let subscript 2 denote the second-best optimum. The following results are shown in Appendix B.3.

First, the incentive constraint implies that workers are now imperfectly insured against the unemployment risk. Therefore, $x_2 > z_2 - d(e_2)$ and employed workers extract a positive rent from a match. The optimum is necessarily decentralized thanks to an optimal marginal tax rate lower than 1 [4]. In particular, the optimal marginal tax is lower at the second best than at the first best: $T_{m,2} < T_{m,1} = 1$.

Second, the social planner is now aware that a higher tightness on the labor market increases the marginal gain of job search, thereby relaxing the incentive constraint. So, $\theta_2 > \theta_1$. According to the free-entry condition (8), this requires a lower pre-tax wage at the second best $w_2 < w_1$. Put differently, the inobservability of search effort has to be compensated by a stimulation of labor demand through the tax schedule.

Third, search effort is lower at the second best $e_2 < e_1$. Keeping search effort at its first-best level would require a difference in utilities between employed and unemployed workers that would be too detrimental to the insurance objective. Although highly plausible, this property is less standard than often believed. For instance, Mas-Colell et al (1995, Exercise 14.B.4) provide a counter-example of a moral hazard problem where effort level is higher at the second best than at the first best.

how non lump-sum income taxation can be used to decentralize the optimum. Taxation can restore efficiency because a positive marginal tax rate (resp. a negative one) decreases (resp. increases) the share of the surplus that accrues to the workers.

[4] It also requires that workers' bargaining power γ be positive.

Now, it should be emphasized that the property $\theta_2 > \theta_1$ does not simply follow from the property $e_2 < e_1$ since the $\mathcal{V}/u$ ratio is endogenous. We cannot in general compare first- and second-best values of unemployment rates and of $\mathcal{V}/u$ ratios. Unreported simulation results often lead to $u_2 > u_1$ and $\mathcal{V}_2/u_2 < \mathcal{V}_1/u_1$. Nevertheless, in the limiting case where effort can take only two values, the lowest being zero, the first- and second-best optimal level of search are equal. Then, the property $\theta_2 > \theta_1$ implies $u_2 < u_1$ and $\mathcal{V}_2/u_2 > \mathcal{V}_1/u_1$.

Finally, the bargained wage has to verify the wage-setting condition (13). Since the values of $V - V^u$, J and x are given by the second-best optimum, this condition yields the second-best value of the actual bargaining power $\hat{\gamma}_2$. For each value of the bargaining power γ, there is a unique optimal marginal tax rate $T_{m,2}$. To offset the wage-push effect of a higher workers' bargaining power, the optimal marginal tax has to increase. Furthermore, this optimal marginal tax rates varies from $-\infty$ to 1 when the bargaining power varies between 0 and 1. Therefore, depending on the value of γ, the optimal marginal tax rate can be negative or positive and can be smaller or larger than the average tax rate. Consequently, except for very particular values for the bargaining power, the complementarity between non-linear taxation and unemployment insurance remains central in the second-best setting.

6 Discussion of the results

This section discusses whether our results are robust to the choice of the utility function and to the introduction of endogenous working hours.

The specification of the utility function we have chosen $v\left(z - d\left(e\right)\right)$ has the advantage of leading more directly and more clearly to new results. Among the possible specifications, the separable form $v(z) - d(e)$ is a natural alternative. Then, at the first best, perfect insurance equalizes income levels and not utility levels in employment and unemployment [5]. Therefore, the optimum is decentralized with a replacement ratio z_1/x_1 equal to 1 but a marginal tax rate $T_{m,1}$ below 1. Consequently, first- and second-best

[5] This echoes the sensitivity of insurance contracts to specific assumptions about the utility function (see e.g. Rosen 1985, p 1158).

values of the replacement ratios can be easily compared but this is no longer the case for marginal tax rates. However the economics behind this comparison remains unchanged. Employed workers have to extract more surplus at the second best than at the first best. Since higher marginal tax rates reduces the employed workers' rent, it is highly plausible, yet much more difficult to prove, that the optimal marginal tax rate is higher when the moral hazard problem disappears.

Introducing endogenous working hours h would lead to different optima whether the planner observes hours or not. In the former case, taxation is a function $\tau(w, h)$ of both total earnings w and hours h. If job-search effort is in addition perfectly observed, complete insurance can be achieved with an optimal marginal tax on earnings $\partial\tau/\partial w = 100\%$. Furthermore, an additional marginal tax rate on hours, $\partial\tau/\partial h \neq 0$, is used to decentralize the optimal level of hours. If job search is unobserved, hours of work remaining observed, insurance becomes incomplete and the marginal tax rate on earnings decreases. In sum, our results remain unchanged. Conversely, if hours are unobserved, the planner faces an additional incentive constraint. Even when job search is observable, a 100% marginal tax rate implies zero working hours which cannot be optimal. Hansen (1999) shows in such a context that lower tax progression increases hours but also unemployment.

7 Conclusion

When job-search effort cannot be observed, it is well known that optimal unemployment insurance solves a moral hazard problem. This paper enriches the literature by considering endogenous job arrival rates and wage bargaining. We show how to use income taxation as a complementary tool to relax the moral hazard incentive constraint. This is first achieved by decreasing the pre-tax wage. This raises the job arrival rate, thereby increasing the marginal gain of search for unemployed worker. This conclusion can be related to other arguments in favor of a stimulation of the labor demand (see e.g. Drèze and Malinvaud 1994 and Phelps 1997). Second, a lower marginal tax rate is needed. International institutions such as the OECD and the academic profession (see Boadway and Cuff, 1999, Boone and van Ours 2000, or Boone et al 2001) work more and more on the effects of monitoring search effort. From our paper, it can be concluded that a better control of job-search effort should not only lead to a better insurance against the unemployment risk (conditional on the income tax schedule). To be optimal, such reforms should also be accompanied by a tax reform. We have not shown that the marginal tax rate rises monotonically as one moves from unobservable to partially observable search effort. However, from our paper, one can conclude that if this relation is monotonic, it is necessarily increasing.

This paper could be extended in different ways. First, one should consider intermediate levels of the ability to observe job search effort. From a normative point of view, this extension should take the cost of monitoring search into account. Second, one could search for the optimal profile of unemployment benefits over the unemployment spell rather than for a single level of unemployment benefits. Third, we could consider heterogeneity in workers productivity as in Mirrlees (1971), Saez (2002), Boone and Bovenberg (2004) and Hungerbühler *et al* (2006). Finally, it would be interesting to analyze the same issues from a political economy viewpoint instead of a normative one.

References

[1] **Baily, M. N. (1977)** 'Unemployment insurance as insurance for workers', *Industrial and Labor Relations Review*, **30**, 495-504.

[2] **Boadway, R. and Cuff, K. (1999)** 'Monitoring job search as an instrument for targeting transfers', *International Tax and Public Finance*, **6**, 317-37.

[3] **Boone, J. and Bovenberg, L. A. (2002)** 'Optimal labour taxation and search', *Journal of Public Economics*, **85**, 53-97.

[4] **Boone, J. and Bovenberg, L. A. (2004)** 'The optimal taxation of unskilled labour with job search and social assistance', *Journal of Public Economics*, **88**,. 2227-58.

[5] **Boone, J. and de Mooij, R. (2003)** 'Tax policy in a model of search with training', *Oxford Economic Papers,* **55**, 121-47.

[6] **Boone, J., Fredriksson, P., Holmlund, B. and van Ours, J. (2001)** 'Optimal Unemployment Insurance with Monitoring and Sanctions', *Discussion Paper 2001-85 Center for Economic Research*, Tilburg University.

[7] **Boone, J. and van Ours, J. (2000)** 'Modeling financial incentives to get unemployed back to work', *Journal of Institutional and Theoretical Economics*, Forthcoming.

[8] **Cahuc, P. and Lehmann, E.(2000)** 'Should unemployment benefits decrease with the unemployment spell?' *Journal of Public Economics*, **77**, 135-43.

[9] **Drèze, J. H. and Malinvaud, E. (1994)** 'Growth and employment: The scope for a European initiative', *European Economic Review,* **38**, 489-504.

[10] **Fredriksson, P. and Holmlund, B. (2001)** 'Optimal unemployment insurance in search equilibrium', *Journal of Labor Economics,* **19**, 370-99.

[11] **Fuest, C. and Huber, B. (2000)** 'Is tax progression really good for employment ? A model with endogenous hours of work', *Labour Economics,* **7***,* 79-93.

[12] **Hansen, C. T.(1999)** 'Lower tax progression, longer hours and higher wages', *Scandinavian Journal of Economics,* **101**, 49-65.

[13] **Hosios, A. (1990)** 'On the efficiency of matching and related models of search and unemployment', *Review of Economic Studies,* **57**, 279-98.

[14] **Hopenhayn, H. and Nicolini, J. (1997)** 'Optimal unemployment insurance', *Journal of Political Economy*, **105**, 412-38.

[15] **Hungerbühler, M., (2006)** 'Tax progression and Training in a Matching Framework', *Labour Economics*, forthcoming.

[16] **Hungerbühler, M., Lehmann, E., Parmentier, A. and Van der Linden, B. (2006) '**Optimal Redistributive Taxation in a Search Equilibrium Model', *Review of Economic Studies*, Forthcoming.

[17] **Lockwood, B. and Manning, A (1993)** 'Wage setting and the Tax system, Theory and evidence for the United Kingdom', *Journal of Public Economics, **52**, 1-29.*

[18] **Malcomson J. and Sator, N (1987) '**Tax push inflation in a unionized labour market', *European Economic Review, **31**, 1581-96.*

[19] **Mas-Colell, A., Whinston, M. and Green, .J.(1995)** *Microeconomic Theory*, Oxford University Press.

[20] **Mirrlees, J., (1971)**, An exploration in a Theory of Optimal Income Taxation, *Review of Economic Studies*, **38**, 175-208.

[21] **Mortensen, D. and Pissarides, C. (1999)**, 'New developments in models of search in the Labor Market', in O. Ashenfelter and D. Card (eds.), *Handbook of Labor Economics*, vol 3B (North-Holland, Amsterdam).

[22] **Petrongolo, B. and Pissarides C. A. (2001)** 'Looking into the blackbox: A survey of the matching function', *Journal of Economic Literature*, **39**, 390-431.

[23] **Phelps, E. (1997)** *Rewarding work: How to restore participation and self-support to free enterprise*, Harvard University Press, Cambridge, MA.

[24] **Pissarides, C. A.(1998)** 'The Impact of employment tax cuts on unemployment and wages: The role of Unemployment Benefits and Tax structure', *European Economic Review*, **42**, 155-83.

[25] **Pissarides, C. A. (2000)** *Equilibrium Unemployment Theory*, 2^{nd} edition, MIT Press, Cambridge, MA.

[26] **Rosen, S. (1985)** 'Implicit contracts: A survey', *Journal of Economic Literature*, **23**, 1144-75.

[27] **Saez, E. (2002)** 'Optimal income transfer programs: Intensive versus Extensive labor supply responses', *Quarterly Journal of Economics*, **117**, 1039-73.

[28] **Shavell, S. and Weiss, L.(1979)** 'The optimal payment of unemployment insurance benefits over time', *Journal of Political Economy, **87**, 1347-62.*

[29] **Sorensen, P. B. (1999)** 'Optimal tax progressivity in imperfect labour markets', *Labour economics, **6**, 435-52.*

A Equilibrium

Deducting (3) from (2) and letting r tend to 0,

$$(q + e \cdot \alpha(\theta))(V - V^u) = v(x) - v(z - d(e)) \tag{17}$$

A.1 Partial Derivatives

The search intensity is implicitly defined by Eqs (12) and (17) through $S(\theta, x, e) = 0$ where:

$$\begin{aligned} S(.) &\equiv \alpha(\theta)(v(x) - v(z - d(e))) - d'(e)\, v'(z - d(e))(q + e \cdot \alpha(\theta)) \quad (18) \\ S'_e &= \Big(-d''(e) \cdot v'(z - d(e)) + [d'(e)]^2 v"(z - d(e))\Big)(q + e \cdot \alpha(\theta)) \\ S'_x &= \alpha(\theta) \cdot v'(x) > 0 \\ S'_\theta &= \alpha'(\theta)[v(x) - v(z - d(e)) - e \cdot d'(e) \cdot v'(z - d(e))] \end{aligned}$$

$v''(.) < 0$ and $d''(.) \geq 0$ imply $S'_e < 0$. Equation $S(.) = 0$ leads to:

$$v(x) - v(z - d(e)) = \frac{q + e \cdot \alpha(\theta)}{\alpha(\theta)} d'(e) \cdot v'(z - d(e))$$

Therefore, using again (12) and (17):

$$S'_\theta = \frac{\alpha'(\theta)}{\alpha(\theta)} \cdot q \cdot d'(e) \cdot v'(z - d(e)) > 0$$

Finally, one has: $S'_{T_m} = S'_T = 0$ and

$$S'_z = -\alpha(\theta) \cdot v'(z - d(e)) - d'(e)(q + e \cdot \alpha(\theta))\, v''(z - d(e))$$

Hence, search intensity increases with the wage x and the tightness θ.

The negotiated wage is implicitly defined by eqs (2), (3) (8) and (13) through: $WS(\theta, x, e) = 0$ with:

$$\begin{aligned} WS(.) &\equiv v(x) - v(z - d(e)) - \frac{c \cdot \gamma (1 - T_m)}{1 - \gamma} v'(x) \frac{q + e \cdot \alpha(\theta)}{m(\theta)} \qquad (19) \\ WS'_\theta &= -\frac{c \cdot \gamma}{1 - \gamma} \cdot \frac{(1 - T_m) v'(x)}{m(\theta)} \left(e \cdot \alpha'(\theta) - \frac{m'(\theta)(q + e \cdot \alpha(\theta))}{m(\theta)} \right) < 0 \\ WS'_x &= v'(x) - \frac{c \cdot \gamma}{1 - \gamma} (1 - T_m) \frac{q + e \cdot \alpha(\theta)}{m(\theta)} \cdot v''(x) > 0 \\ WS'_e &= d'(e) \cdot v'(z - d(e)) - \frac{c \cdot \gamma}{1 - \gamma} (1 - T_m) \frac{\alpha(\theta)}{m(\theta)} \cdot v'(x) \end{aligned}$$

After some manipulations, $WS(.) = 0$ implies:

$$\frac{c \cdot \gamma}{1 - \gamma} \cdot \frac{(1 - T_m) \cdot v'(x)}{m(\theta)} = \frac{v(x) - v(z - d(e))}{q + e \cdot \alpha(\theta)}$$

Taking this equality into account leads to:

$$WS'_e = d'(e) \cdot v'(z - d(e)) - \alpha(\theta) \frac{v(x) - v(z - d(e))}{q + e \cdot \alpha(\theta)} = -\frac{S(\theta, x, e)}{q + e \cdot \alpha(\theta)} \qquad (20)$$

Hence, WS'_e is equal to zero in equilibrium. Finally, one has: $WS'_T = 0$, $WS'_{T_m} = \frac{c \cdot \gamma}{1 - \gamma} \frac{q + e \cdot \alpha(\theta)}{m(\theta)} \cdot v'(x) > 0$ and $WS'_z = -v'(z - d(e)) < 0$. Hence, the wage increases with tightness and unemployment benefits. It decreases with the marginal tax rate and is independent of search intensity.

A.2 Uniqueness of the equilibrium and comparative statics

Conditionally on (z, T_m, T, γ), we define $\mathbb{W}(\theta, e) \equiv WS(\theta, \phi(\theta) - T, e)$ and $\mathbb{S}(\theta, e) \equiv S(\theta, \phi(\theta) - T, e)$ and show that the system $\mathbb{S}(\theta, e) = \mathbb{W}(\theta, e) = 0$ admits at most one solution. One gets:

$$\begin{aligned} \mathbb{S}'_e &= S'_e < 0 \qquad \mathbb{S}'_\theta = S'_\theta + \phi'_\theta \cdot S'_x \qquad \mathbb{S}'_T = -S'_T < 0 \\ \mathbb{S}'_z &= S'_z \qquad \text{and} \qquad \mathbb{S}'_{T_m} = S'_{T_m} = 0 \end{aligned}$$

Similarly, one has:

$$\begin{aligned} \mathbb{W}'_e &= WS'_e = 0 \qquad \mathbb{W}'_\theta = WS'_\theta + \phi'_\theta \cdot WS'_x < 0 \\ \mathbb{W}'_T &= -WS'_x \qquad \mathbb{W}'_z = WS'_z \qquad \text{and} \qquad \mathbb{W}'_{T_m} = WS'_{T_m} \end{aligned}$$

First, we prove uniqueness. Since $\mathbb{S}'_e(\theta, e) < 0$, for any given θ, the equation $\mathbb{S}(\theta, e) = 0$ admits at most one solution in e. Call this solution $\mathbb{E}(\theta)$ if it exists. The implicit function theorem insures that function $\mathbb{E}(\theta)$ is differentiable wherever it is defined. Now, let $\mathcal{W}(\theta) \equiv \mathbb{W}(\theta, \mathbb{E}(\theta))$. An equilibrium necessarily solves $\mathcal{W}(\theta) = 0$. Differentiating function $\mathcal{W}(.)$ yields $\mathcal{W}'(\theta) = \mathbb{W}'_\theta(\theta, \mathbb{E}(\theta)) + \mathbb{E}'(\theta) \cdot \mathbb{W}'_e(\theta, \mathbb{E}(\theta))$. Since $\mathbb{E}(\theta)$ solves $\mathbb{S}(\theta, \mathbb{E}(\theta)) = 0$, one has $\mathbb{W}'_e(\theta, \mathbb{E}(\theta)) = 0$ from Eq. (20). Hence, $\mathcal{W}'(\theta) = \mathbb{W}'_\theta(\theta, \mathbb{E}(\theta)) < 0$. So, Eq. $\mathcal{W}(\theta) = 0$ admits at most one solution.

Second, we look at the comparative statics of the equilibrium. Differentiating $\mathbb{S}(\theta, e) = \mathbb{W}(\theta, e) = 0$, taking into account that $\mathbb{W}'_e = 0$ at the equilibrium, one has:

$$d\theta = -\frac{WS'_z}{\mathbb{W}'_\theta}dz + \frac{WS'_x}{\mathbb{W}'_\theta}dT - \frac{WS'_{T_m}}{\mathbb{W}'_\theta}dT_m$$

Since $\mathbb{W}'_\theta < 0$, $WS'_z < 0$, $WS'_x > 0$, $WS'_{T_m} > 0$ one has $d\theta/dz < 0$, $d\theta/dT < 0$ and $d\theta/dT_m > 0$. Since $w = \phi(\theta)$ and $\phi'(.) < 0$, one further has $dw/dz > 0$, $dw/dT > 0$ and $dw/dT_m < 0$.

B Optima

B.1 The first-order conditions

The second-best program solves:

$$\max_{\theta,x,u,z,e} \quad (1-u)\,v(x) + v(z - d(e))\,u$$
$$s.t: \overline{\chi} \leq (1-u)(y-x) - z \cdot u - c \cdot e \cdot \theta \cdot u$$
$$0 = e \cdot \alpha \cdot (\theta)\,u - q(1-u)$$
$$0 = S(\theta, x, e)$$

Let η, δ and ψ be the Lagrange multipliers for respectively the budget constraint, the flow equilibrium equation and the incentive constraint. The first-best problem is obtained by eliminating the incentive constraint (so $\psi = 0$ in the following first-order conditions).

These conditions are:

$$
\begin{aligned}
0 &= (1-u)\left[v'(x)-\eta\right]+\psi\cdot S'_x && (21)\\
0 &= u\left[v'(z-d(e))-\eta\right]+\psi\cdot S'_z && (22)\\
0 &= u\left[-d'(e)\cdot v'(z-d(e))-\eta\cdot c\cdot\theta+\delta\cdot\alpha(\theta)\right]+\psi\cdot S'_e && (23)\\
0 &= v(z-d(e))-v(x)+\eta(x-y-z-c\cdot e\cdot\theta)+\delta(e\cdot\alpha(\theta)+q) && (24)\\
0 &= -c\cdot e\cdot\eta\cdot u+\delta\cdot\alpha'(\theta)\cdot e\cdot u+\psi\cdot S'_\theta && (25)
\end{aligned}
$$

as well as the all the constraints in the above program and the Kuhn and Tucker conditions

$$
\begin{aligned}
0 &\leq \eta\\
0 &= \eta\left[(1-u)(y-x)-z\cdot u-c\cdot e\cdot\theta\cdot u-\overline{\chi}\right]
\end{aligned}
$$

B.2 The first-best optimum

At the first best, $\psi_1=0$, so eqs (21) and (22) lead to equalities (16). Conditions (23) (24) and (25) are respectively rewritten as:

$$
\left(\frac{\delta_1}{\eta_1}=\right)\frac{d'(e_1)+c\cdot\theta_1}{\alpha(\theta_1)}=\frac{y+d(e_1)+e_1\cdot c\cdot\theta_1}{e_1\cdot\alpha(\theta_1)+q}=\frac{c}{\alpha'(\theta_1)} \tag{26}
$$

From the equalities in (26), we get that the social optimum is determined by either $F(\theta_1,e_1)=G(\theta_1,e_1)=0$, or $F(\theta_1,e_1)=H(\theta_1,e_1)=0$ or $G(\theta_1,e_1)=H(\theta_1,e_1)=0$, where:

$$
\begin{aligned}
F(\theta,e) &\equiv \alpha'(\theta)(d'(e)+c\cdot\theta)-c\cdot\alpha(\theta)\\
G(\theta,e) &\equiv \alpha(\theta)(y+d(e)+c\cdot\theta\cdot e)-(c\cdot\theta+d'(e))(e\cdot\alpha(\theta)+q)\\
H(\theta,e) &\equiv \alpha'(\theta)(y+d(e)+e\cdot c\cdot\theta)-c(e\cdot\alpha(\theta)+q)
\end{aligned}
$$

Function $G(.)$ $(H(.))$defines the optimal level of search intensity (of tightness) as a function of tightness (search intensity).

The partial derivatives of F have unambiguous signs:

$$
F'_\theta=\alpha''(\theta)(d'(e)+c\cdot\theta)<0 \qquad F'_e=\alpha'(\theta)\cdot d''(e)>0
$$

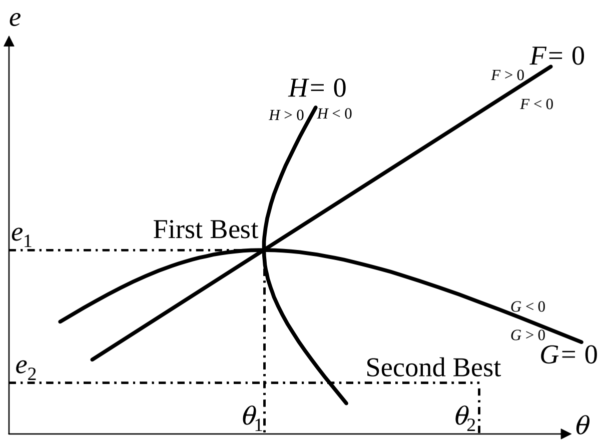

Figure 1: The first-best choice of (θ, e)

Consequently, the locus $F = 0$ is upward-sloping in the (θ, e) plane (see Fig. 1). Second,

$$G'_e = -d''(e)\,(e \cdot \alpha(\theta) + q) < 0 \qquad G'_\theta = \alpha'(\theta)\,(y + d(e) - e \cdot d'(e)) - c \cdot q$$

However, along $G(\theta, e) = 0$, one has:

$$y + d(e) = \frac{c \cdot \theta \cdot q + d'(e)\,(e \cdot \alpha(\theta) + q)}{\alpha(\theta)} = \frac{q}{\alpha(\theta)} \cdot (c \cdot \theta + d'(e)) + e \cdot d'(e)$$

Therefore,

$$G'_\theta = \alpha'(\theta) \cdot \frac{q}{\alpha(\theta)} \cdot (c \cdot \theta + d'(e)) - c \cdot q = \frac{q}{\alpha(\theta)} F(\theta, e)$$

Consequently, in the (θ, e) plane, the locus $G = 0$ is upward-sloping (respectively downward-sloping) at the left (respectively at the right) of the curve $F = 0$. Moreover, the locus $G = 0$ intersects this curve horizontally (see Figure 1). Third,

$$H'_\theta = \alpha''(\theta)\,(y + d(e) + e \cdot c \cdot \theta) < 0 \qquad H'_e = F(\theta, e)$$

Hence, in the (θ, e) space, the locus $H = 0$ is upward-sloping (respectively downward-sloping) above (respectively below) the curve $F = 0$. In addition, the locus $H = 0$ intersects this curve vertically (see Figure 1). This configuration guarantees the uniqueness of a solution to the system (26) [6].

[6] The proof is similar to the one of the uniqueness of equilibrium. One simply has to replace $\mathbb{S}$ by $-F$ and $\mathbb{W}$ by H.

B.3 The second-best optimum

Let us first show that $\psi_2 > 0$. The incentive constraint $S(.) = 0$ evaluated at the second best encompasses two constraints, namely $S(.) \geq 0$ and $S(.) \leq 0$. The former (respectively the latter) requires that the chosen search intensity e be at least (at most) equal to e_2. According to Kuhn and Tucker conditions, the former (latter) constraint is associated with a Lagrange multiplier $\psi_2^+ \geq 0$ ($\psi_2^- \leq 0$). One has $\psi_2 = \psi_2^+ + \psi_2^-$ and either $\psi_2^- = 0$ or $\psi_2^+ = 0$. From an economic viewpoint, only the constraint $e \geq e_2$, matters. For, the constraint $e \geq e_2$ ($e \leq e_2$) means that the net earnings x_2 should be sufficiently (not too) high compared to $z_2 - d(e_2)$, which is detrimental (beneficial) to the insurance objective. So $\psi_2^- = 0$ and $\psi_2 = \psi_2^+ \geq 0$. Finally, the incentive constraint $S = 0$ implies that $x_2 > z_2 - d(e_2)$, thereby $v'(x_2) < v'(z_2 - d(e_2))$. Therefore, $\psi_2 \neq 0$ according to (21) and (22). Consequently, $\psi_2 > 0$.

To obtain the sign of η_2, one gets from (21):

$$\eta_2 = v'(x_2) + \frac{\psi_2}{1 - u_2} S'_x(\theta_2, x_2, e_2) > 0$$

It will now be shown that one has $G(\theta_2, e_2) > 0$ at the second-best optimum. Dividing first-order condition (24) by η_2 and rearranging yields:

$$\begin{aligned} y + d(e_2) + c \cdot e_2 \cdot \theta_2 = &\frac{\delta_2}{\eta_2}(e_2 \cdot \alpha(\theta_2) + q) \\ &- \frac{v(x_2) - v(z_2 - d(e_2))}{\eta_2} + x_2 - z_2 + d(e_2) \end{aligned} \tag{27}$$

Multiplying both sides by $\alpha(\theta_2)$ yields:

$$\begin{aligned} \alpha(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) = &\frac{\delta_2 \cdot \alpha(\theta_2)}{\eta_2}(e_2 \cdot \alpha(\theta_2) + q) \\ &- \frac{\alpha(\theta_2)[v(x_2) - v(z_2 - d(e_2))]}{\eta_2} + \alpha(\theta_2)(x_2 - z_2 + d(e_2)) \end{aligned}$$

Taking the incentive constraint $S(.) = 0$ into account:

$$\begin{aligned} \alpha(\theta_2)(y + d(e_2) + c \cdot e_2 \cdot \theta_2) \quad = \quad &\frac{e_2 \cdot \alpha(\theta_2) + q}{\eta_2}[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2))] \\ &+ \alpha(\theta_2)(x_2 - z_2 + d(e_2)) \end{aligned}$$

The right-hand side of the last equality can be substituted in the definition of function G evaluated at (θ_2, e_2). After some manipulations, this yields:

$$\begin{aligned} G(\theta_2, e_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta_2}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta_2\right] \\ &\quad + \alpha(\theta_2)(x_2 - z_2 + d(e_2)) - (e_2 \cdot \alpha(\theta_2) + q)\, d'(e_2) \end{aligned}$$

Using once again $S(.) = 0$, $G(\theta_2, e_2)$ can be restated as:

$$\begin{aligned} G(\theta_2, e_2) &= \frac{e_2 \cdot \alpha(\theta_2) + q}{\eta_2}\left[\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta_2\right] \\ &\quad + \alpha(\theta_2)\left[(x_2 - z_2 + d(e_2)) - \frac{v(x_2) - v(z_2 - d(e_2))}{v'(z_2 - d(e_2))}\right] \end{aligned} \tag{28}$$

However, the first-order condition (23) insures that:

$$\delta_2 \cdot \alpha(\theta_2) - d'(e_2) \cdot v'(z_2 - d(e_2)) - c \cdot \theta_2 \cdot \eta_2 = -\frac{\psi_2}{u_2} S'_e > 0$$

So, the first term on the right hand side of (28) is positive. In addition, the concavity of $v(.)$ implies that :

$$v(x_2) - v(z_2 - d(e_2)) < v'(z_2 - d(e_2)) \cdot (x_2 - z_2 + d(e_2))$$

by which the second term on the right hand side of (28) is positive too. Therefore, $G(\theta_2, e_2) > 0$. So, $e_2 < e_1$ (see Fig 1).

Next, it will be shown that $H(\theta_2, e_2) < 0$. The first-order condition (25) together with the incentive constraint $S(.) = 0$ gives:

$$c = \frac{\delta_2}{\eta_2} \cdot \alpha'(\theta_2) + \frac{\psi_2}{\eta_2 \cdot e_2 \cdot u_2} \alpha'(\theta_2) \cdot \frac{q}{\alpha(\theta_2)} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2)$$

Substituting the flow equilibrium (1) yields:

$$c = \alpha'(\theta_2)\left\{\frac{\delta_2}{\eta_2} + \frac{\psi_2}{\eta_2} \cdot \frac{1}{1 - u_2} \cdot v'(z_2 - d(e_2)) \cdot d'(e_2)\right\}$$

Substituting this expression and equation (27) into $H(\theta_2, e_2)$ leads to

$$\begin{aligned} H(\theta_2, e_2) &= \alpha'(\theta_2)\left\{x_2 - z_2 + d(e_2) - \frac{v(x_2) - v(z_2 - d(e_2))}{\eta_2}\right. \\ &\quad \left. - \frac{\psi_2}{\eta_2(1 - u_2)} v'(z_2 - d(e_2))\, d'(e_2)(e_2 \cdot \alpha(\theta_2) + q)\right\} \end{aligned}$$

Taking $S(.) = 0$ into account, this expression can be rewritten as:

$$\begin{aligned} H(\theta_2, e_2) &= \alpha'(\theta_2)\Big\{x_2 - z_2 + d(e_2) - \frac{v(x_2) - v(z_2 - d(e_2))}{\eta_2} \\ &\quad - \frac{\psi_2 \cdot \alpha(\theta_2)}{\eta_2(1-u_2)}\left[v(x_2) - v(z_2 - d(e_2))\right]\Big\} \end{aligned}$$

From first-order condition (21)

$$\frac{\psi_2 \cdot \alpha(\theta_2)}{\eta_2(1-u_2)} = \frac{1}{v'(x_2)} - \frac{1}{\eta_2}$$

Consequently,

$$H(\theta_2, e_2) = \alpha'(\theta_2)\left\{x_2 - z_2 + d(e_2) - \frac{v(x_2) - v(z_2 - d(e_2))}{v'(x_2)}\right\}$$

Finally, concavity of function $v(.)$ implies that:

$$v'(x_2)(x_2 - z_2 + d(e_2)) < v(x_2) - v(z_2 - d(e_2))$$

Therefore, function $H(.,.)$ evaluated at the second-best optimum (θ_2, e_2) is always negative. This implies that $\theta_2 > \theta_1$ (see Fig 1).