Equilibrium Unemployment: The matching models

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Slides based on Chap. 9 “Equilibrium Unemployment” of Cahuc, Carcillo and Zylberberg (2014) (henceforth CCZ).

Empirical analyses reveal *very intense* job and worker reallocations (i.e. flows)... even in “sluggish” labor markets

“Sluggish” $\Leftrightarrow$ markets with small variations in the (un)employment rate (i.e. stocks)

$\Rightarrow$ This chapter is an introduction to *dynamic* models that explicitly recognize these flows...

> ... To yield a theory of (inefficient or efficient) unemployment in equilibrium

The simple version of the models covered here is a useful starting point for thinking at the main factors affecting flows out of unemployment.

Outline

1. Introduction
   - Flows on the labor market
   - Beveridge curve
2. The matching process
3. Wage-setting assumptions
4. Random matching
   - Static model
   - Dynamic model
   - Efficiency
   - Dynamics
   - Critiques of and Extensions to the Random Matching
5. Directed search
6. Note on the literature

Introduction: Some Definitions and Facts

Job flows (Davis, Haltiwanger and Schuh, 1996)

Imagine a large data set of production units (firms or plants).
In an country, a region or a sector, and for a given unit of time (often a year),

- Job creations (JC) = Σ of job gains in new units or in growing ones
- Job destructions (JD) = Σ of job losses due to closing or contractions
- Net employment changes = JC- JD
- Job reallocation (JR) = JC + JD
- Excess job reallocation
  = Job reallocation - | Net employment changes |
  = JC + JD - | JC - JD |
  = 0 if JC $\cdot$ JD = 0
  = 2 $\cdot$ JD if JC > JD > 0
  = 2 $\cdot$ JC if 0 < JC < JD.

Orders of Magnitude

Country (period)	JC	JD	JC+JD	Net Empl.	Excess job reall.
F* (99-00)	12.0	8.3	20.3	3.7	16.6
G* (77-99)	8.4	7.1	15.5	1.3	14.2
I* (86-94)	12.3	10.2	22.5	2.1	20.4
U.K.*(80-98)	11.5	12.6	24.2	-1.1	23.1
U.S. (88-97)	12.5	10.0	22.5	2.5	20.0

Table: Job creation and destruction flows: Average values.
Annual average rate as a percentage of total employment, all sectors of the economy.
Note: * F=France, G =West Germany, I = Italy; Manufacturing only for the U.K. Mostly private sector for Germany and the U.S. Source: data from Haltiwanger et al. (2010); except for France where data come from Picart (2008, Table 2)

Job flows

Some conclusions

- Net employment growth is the (relatively small) difference between two large rates.
- Excess job reallocation $>$ | Net employment changes |.
- Properties not illustrated above: At the national level, job reallocation is to a large extent a *within*-sector phenomenon. Holds true even within narrowly defined sectors!

Note: Job creation rates and job destruction rates are underestimated (job reallocations within firm are for instance ignored)

Orders of magnitude in levels:
Every day in France, *about* 10,000 jobs are created and 10,000 are destroyed.
In the US, 8 millions new jobs were created in the first quarter of 2000 and 7 millions were destroyed (Laing, 2011, p. 809).

Worker flows

Imagine now a data set of movements of workers into jobs (hirings) and out of jobs (separations) over a specified period of time.

Turnover: it measures the gross number of labor market transitions *during a period of time*
= full counting of all events (i.e. every time a worker is hired or separates during the period) during that period;

(Gross) worker reallocation: it measures the number of persons who participate to transitions *between two discrete points in time*, say a month;
$\Rightarrow$ e.g. hirings equal the number of workers who are with the firm at time t, but were not with that employer at time t-1;
= a more limited counting than turnover;

If job creations & destructions, hirings H & separations S are all measured by a comparison
- between the same two points in time;
- on the same geographical area;
- and with coherent data sources;

Then:

- Net employment changes = JC - JD = H - S
- Excess worker reallocation or "churn"
  = H + S - | Net employment changes |
  = H + S - | H - S |

Beware

- The quantification of job and worker flows raises many methodological issues and measurement issues (not detailed here).
- These job and workers flows have been measured in a large number of countries ...
- ... on the basis of non homogeneous definitions and country-specific data sets
  $\Rightarrow$ lack of comparability is an issue.

Those interested by the cyclicality of these flows are referred to CCZ.

Worker Flows

Country	Entry rate (hirings)	Exit rate (separations)
Belgium	14	12
France	14	14
Germany	16	14
Italy	11	10
U.K.	15	14
U.S.	19	22
EU 15	15	14

Table: Annual employment inflows and outflows in percentages: Year 2011.

Source: OECD Labor Force Statistics Database. Note : 2010 for the United States. The entry rate is calculated as the ratio of persons employed for less than one year to the average stock of employment in t and $t-1$ and the exit rate as the difference between the entry rate and the employment growth rate.

Worker Flows

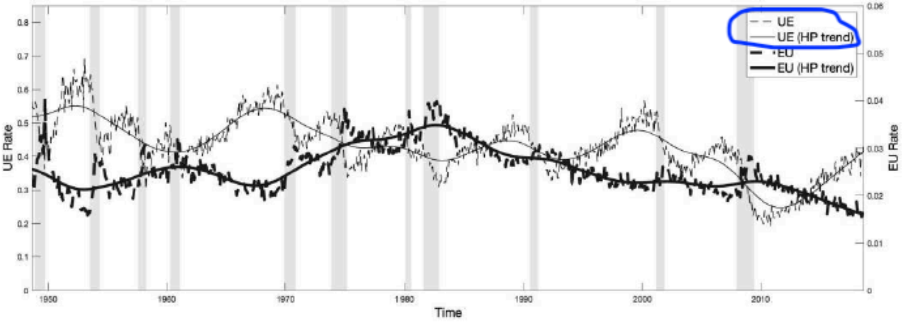


Figure 3: UE and EU Rates US: 1948-2018

Figure: UE hazard rate (unemployed workers become employed) and EU rate (employed workers become unemployed): USA 1948-2018. Grey bands are recessions. Source: Martellini and Menzio (2020)

Worker Flows

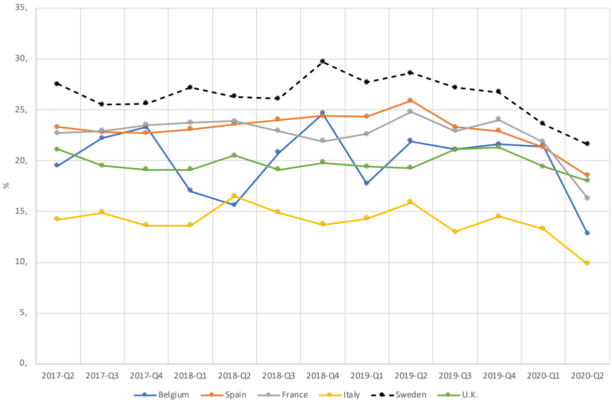


Figure: Seasonally Adjusted Quarterly UE Rates (unemployed workers become employed). Source: Eurostat (lfsi_long_q)

Beware: Based on small samples in most countries!

Worker Flows

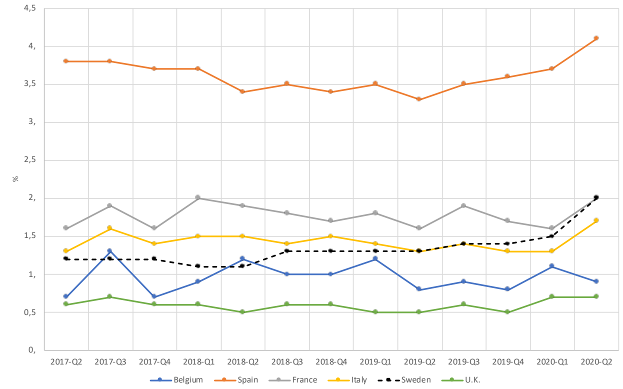


Figure: Seasonally Adjusted Quarterly EU Rates (employed workers become unemployed). Source: Eurostat (lfsi_long_q)

Beware: Based on small samples in most countries!

Exits from employment are the sum of

1. Quits,
2. The ending of short-term contracts,[1]
3. Retirements,
4. Firing for cause,
5. Job loss through no fault of the employee = “Job displacement”.

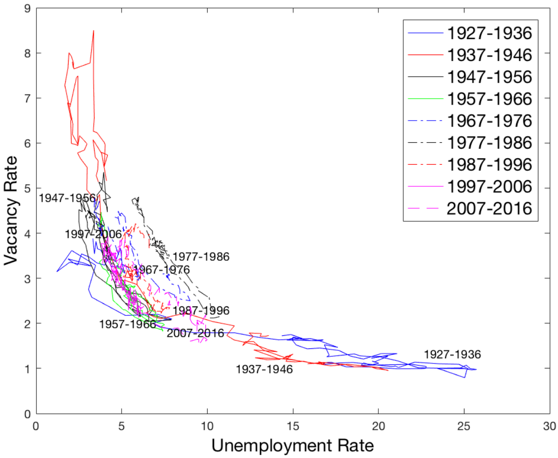
[1] A big share of exits in countries where employment protection legislation is strong and coexists with a segment of the labor market which is much less protected

Coexistence of vacant jobs and job-seekers

- v: vacancies-workforce ratio at a point in time.
- u: unemployment rate at a point in time.
- The (descriptive) "Beveridge curve" = a plot of (u, v) pairs measured at different points in time.
- Two following slides contain examples of Beveridge curves.
- In many countries, data about vacancies are rather poor because they are based on information collected by Public Employment Agencies (however, many vacant position are ignored by these agencies).
  Since the end of 2000, JOLTS data in the US are of good quality. Since 2008, European countries are developing specific surveys.

Beveridge curve in the US

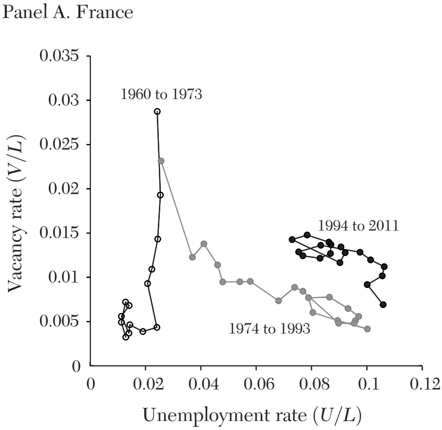
1926-2018

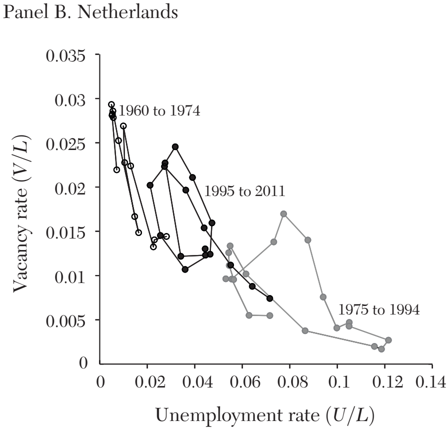


Source: Martellini and Menzio (2020).

Shifts of the "Beveridge curve"

Source: Elsby, Michaels and Ratner (2015), *L* designates the size of the workforce



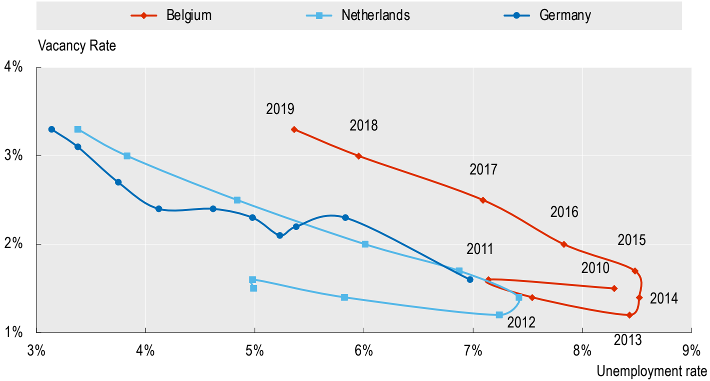


Note on Statistics in the EU

The EU commission and its statistical institute (EUROSTAT) propose "Beveridge curves" where the vertical axis does not display the vacancies-labor force ratio, but different measures such as:

- The "vacancy rate" understood as the number of job openings over the sum of employment and job openings. See `http://ec.europa.eu/eurostat/web/labour-market/job-vacancies`.
- The "labour shortage indicator", i.e. the proportion of firms reporting labour shortage as a factor limiting production.

The “Beveridge curve” 2010-2019 with homogeneous Eurostat data. Source: OECD (2020)



Beveridge curve: Summary of stylized facts

- "First, at cyclical frequencies unemployment and vacancies move in opposite directions, tracing out a negatively inclined Beveridge locus."
- "Second, the position of this locus has shifted periodically in many developed economies, most notably during the persistent rise in European unemployment in the 1980s, and more recently in the wake of the Great Recession in the United States." (Elsby, Michaels and Ratner, 2015, p. 572)

The Matching Process

- **Central idea**:
  Trade in the labor marked is decentralized, uncoordinated, time-consuming and costly for both firms and workers (Pissarides, 2000, p.3)
- Why? Because of heterogeneities, imperfect information and lack of coordination, that are generating "**search and matching frictions**" on the labor market.[2]
  - Jobs and workers are heterogeneous (in skills, location,...).
  - Imperfection information about job and worker characteristics is prevalent.
  - When deciding where to apply, job-seekers do not coordinate their choices.

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[2]Other relevant frictions on the labor market affecting (un)employment dynamics are adjustment costs such as hiring and firing costs.

Two opposite views

1. “**Undirected Search**” or “**Random Matching**”
Job seekers meet all vacancies randomly.[3]

2. “**Directed search**”
Job seekers have here perfect information about the different wages[4] offered by different jobs before they decide where to look for work.
The origin of frictions is then a lack of coordination among job-seekers because *trade is decentralized*.
The importance of directed search in the literature is growing steadily.

[3] When types of workers and/or jobs are explicit, one needs to specify who meets (randomly) with whom. When everybody is formally identical, one more job-seeker of any type adds frictions that matter for *all* other unemployed and *all* positions.

[4] More generally, the working conditions and job (dis)amenities.

- Like the production function, the "matching function" is

  > *"a modeling device that captures the implications of the costly trading process without the need to make the heterogeneities and other features that give rise to it explicit." (Pissarides, 2000, p.4)*

- So, formally, the agents who participate to trade in the labor market are *homogeneous*. However, the fact that their "meeting" is time-consuming is due to the above-mentioned "frictions".
  - So, the basic framework intends to study "frictional" unemployment.
  - A very sensible extension: Introduce explicit heterogeneities in the type of jobs (productivity, risk of job destruction,...) and/or in the type of workers (ability, skill,...).[5]

[5] The literature proposes different approaches here (see e.g. Mortensen and Pissarides, 2003, Shimer, 2007, Lise and Robin, 2017, and the assignment literature mentioned at the end of this chapter). Hall and Schulhofer-Wohl (2018) estimate a matching function for the US with heterogeneous job-seekers.

- In the basic matching model there is full specialization in either trade or production:

Jobs	Filled	Production
Jobs	Vacant	Search for applicants $\rightarrow$ Matching
Workers	Employed	Production
Workers	Unemployed	Search for vacant jobs $\rightarrow$ Matching

  $\Rightarrow$ No on-the-job search ! (See extensions later however).

- Micro-foundation of the matching process: often an “urn-ball model” (See CCZ p. 584).

The matching function

Continuous-time setting

The *instantaneous* flow of hires, H, is assumed to be a function of the number (stock) of job-seekers, U, and the number (stock) of vacancies, V. This matching is assumed to be random process (*all* vacancies and *all* job-seekers meet randomly[6]).
The matching function is defined as

$$H = M(V, U) \tag{1}$$

As it is standard now, let us assume that $M(V, U)$ is increasing, concave and homogeneous of degree 1.
Moreover: $M(0, U) = M(V, 0) = 0$.

[6] An interesting alternative is the so-called "stock-flow matching"; see e.g. Coles and Petrongolo (2008).

Remarks:

- More generally, the function $H = M(V, U)$ describes the number of *contacts* between vacancies and job seekers: A contact does not necessarily lead to a new hire.
- Below, U will designate the number of unemployed.
  - This number could be weighted to account for
    - Search/recruitment effort: $H = M(e^V \cdot V, e^U \cdot U)$ with e^V, e^U the respective (endogenous) effort levels (see Davis, Faberman and Haltiwanger, 2013 for a generalization),
    - Heterogeneity in the unemployment pool: $H = M(V, c \cdot U)$ with c being a time-dependent (exogenous) parameter, say, affected by the share of long-term unemployed,...
  - With on-the-job search (case not considered here), employed job-seekers should be included as an argument of the matching function.
    Let $\mathcal{E}$ designate the number of employed job seekers and s their matching efficiency relative to the one of the unemployed. Then the matching function could be written as: $H = M(V, U + s \cdot \mathcal{E})$.

The matching function

Empirical support

- "The usefulness of the matching function depends on its empirical viability". (Pissarides, 2000, p.4).
- See Petrongolo and Pissarides (2001) for a survey. Many studies support the above assumptions. In particular, constant returns to scale. Nevertheless, increasing returns to scale are sometimes advocated: (i) in Diamond (1982) IRS are key to reach multiple equilibria; (ii) in Economic Geography.
- The Cobb-Douglas specification is often not rejected:

$$H_t = A_t\ V_t^{1-\eta}\ U_t^{\eta} \tag{2}$$

An order of magnitude for η being $[0.4; 0.7]$.

Tightness and various rates

Definition: Tightness on the labor market

$$\theta \equiv \frac{V}{U}$$

The rate (= the probability per unit of time) at which a vacant job is filled is:

$$\frac{M(V,U)}{V} = M(1, U/V) \equiv m(\theta)$$

Differentiating $M(1, U/V) \equiv m(\theta)$ w.r.to U yields:

$$\frac{\partial M(1, U/V)}{\partial U}\frac{1}{V} = m'(\theta)\left(-\frac{V}{U^2}\right) \quad \Rightarrow \quad m'(\theta) < 0$$

The following “Inada conditions” are useful to guarantee the existence of an equilibrium:

$$\lim_{\theta \to 0} m(\theta) = +\infty \qquad \text{and} \qquad \lim_{\theta \to +\infty} m(\theta) = 0$$

The rate at which an unemployed finds a job:

$$\frac{M(V,U)}{U} = \frac{V}{U}\frac{M(V,U)}{V} = \theta m(\theta)$$

Differentiating $M(V,U)/U = \theta m(\theta)$ w. r. to V yields:

$$[\theta m(\theta)]' > 0$$

The following “Inada conditions” are useful to guarantee some properties of the model:

$$\lim_{\theta \to 0} \theta m(\theta) = 0 \text{ and } \lim_{\theta \to +\infty} \theta m(\theta) = +\infty \tag{3}$$

Search or congestion externalities

An additional vacancy

reduces the rate at which job vacancies are filled
= the "congestion external effect"
increases the exit rate out of unemployment
= the "thick market (beneficial) externality"

Similarly for an additional unemployed in the queue for jobs:

"A worker deciding to join a queue or stay in one considers the probabilities of getting a job, but not the effects of his decision on the probabilities that others face..." (Tobin, 1972)

Key question: (addressed later on)

Will decentralized decisions internalize those externalities?

Links between the matching function and the job-search literature

- The mechanisms underlying the matching function are closely related to those at the root of the *job search model* considered earlier.
- General equilibrium considerations clearly suggest that the job arrival rate and the level (or the distribution) of wages should be endogenous.
- Both *Equilibrium Search* (previous chapter) and *Matching Models* put emphasis
  1. on the role of employers (the demand side of the labor market)
  2. on wage formation.

Wage-Setting Assumptions

Wage-setting assumptions

There are two basic wage formation approaches:

Wage posting models. Take-it-or-leave-it wage offers are set ("posted") by employers in a non-cooperative setting while

- = Assumption of the Equilibrium Search Model
- = Also an assumption introduced later in this chapter.

Wage bargaining. The **surplus** created when a job-seeker and a vacancy meet leads to a negotiation over the wage. This is the standard assumption in the case of the matching model with **undirected search.** Seminal papers: Diamond (1981), Diamond (1982), and Mortensen (1982).

Relative importance of the two wage-settings

Surveys by Hall and Krueger (2012) and Brenzel, Gartner and Schnabel (2014) find similar shares.
Quoting the latter:

> *Both modes of wage determination coexist in the German labor market, with more than one-third of hirings being characterized by individual wage negotiations. Wage bargaining is more likely for more-educated applicants, for jobs with special requirements, and in tight regional labor markets. Wage posting (in the sense of a fixed offer) dominates in the public sector, in larger firms, in firms covered by collective bargaining agreements, and in jobs involving part-time and fixed-term contracts.*

Undirected Search or Random matching

A static version of the model

Not included in CCZ

Assumptions:

- Risk-neutral workers and firm owners.
- At the beginning of the period, an exogenous number $N = U$ of individuals are jobless and search randomly for a job.
- Each firm is made of a single vacant or filled job.
- If a vacancy is filled, a given (real) amount of output y is produced. The worker receives a (real) wage w and the firm's (real) profit level is $y - w - \kappa$, where κ is a fixed cost incurred to open a vacancy (create a job slot).
- If the job remains vacant, the profit is equal to $-\kappa$.

A static version of the model

Assumptions:

- $M(V, N) < \min(V, N) \Rightarrow m(\theta) < \min(1, \theta^{-1})$ where $\theta \equiv V/N$.
- Wages are negotiated *after* efforts have been made on both sides of the labor market to create vacancies and search for a partner (sunk costs)
- Throughout this chapter, one neglects credit market frictions: Entrepreneurs have no problem financing their creation of vacancies and search of an applicant.

Questions that are raised:

1. How many vacancies are created?
2. How is the surplus of the match splitted?
3. How many are (un)employed?

A static version of the model

labor demand

There is free entry of vacancies. Firms open vacancies as long as the expected return is nonnegative.
In equilibrium:

$$m(\theta)(y - w - \kappa) + (1 - m(\theta))(-\kappa) = 0 \text{ or} \tag{4}$$

$$m(\theta) = \frac{\kappa}{y - w} \tag{5}$$

Since $m'(\theta) < 0$, the higher y (the lower the wage and the cost of opening a vacancy), the higher θ: This means a 'thick' labor market with many vacancies per job-seeker.
Equation (5): A downward-sloping labor demand equation in a (θ, w) space.
Notice that $0 < \kappa/(y - w) < 1$ is required, hence $w < y$.

A static version of the model

The wage bargain

When an individual meets a vacancy, the total surplus created if they form a match is y
(since κ is sunk + no income for the unemployed).

Assume the following non-cooperative two-stage game:

Stage 1. The firm-owner and the worker propose a wage contract.

Stage 2. It one of the two players (or both) does not accept the contract in stage 1, then

- with probability $\gamma, 0 < \gamma < 1$, the worker makes a take-it-or-leave-it offer
- with probability $1 - \gamma$ the firm owner makes such an offer.
- If the offer is rejected, the job is destroyed.

A static version of the model

Characterization of the subgame perfect equilibrium

In stage 2 if the worker makes the offer, the pay-off for the employer is zero and the total surplus, y, accrues to the worker. The opposite holds if the firm owner makes the offer.

In stage 1 $\Diamond$ the worker knows that at the end of stage 2, his expected income will be $\gamma \cdot y + (1-\gamma)0$.
$\Diamond$ the employer knows that at the end of stage 2, his expected pay-off will be equal to $\gamma \cdot 0 + (1-\gamma)y$.
Therefore, in stage 1, indifference between
(i) signing a contract at stage 1 giving an expected income to the worker $\gamma \cdot y$ and an expected profit $(1-\gamma)y$ and
(ii) waiting until stage 2.

A static version of the model

The subgame perfect equilibrium

Assume a small cost of going to stage 2.
The unique equilibrium consists in immediately signing a contract that *shares the surplus* by paying the wage:

$$w = \gamma \cdot y.$$

Equivalently, one could maximize the following (asymmetric) Nash product with respect to w:

$$[w]^{\gamma}\,[y - w]^{1-\gamma} \tag{6}$$

The first-order condition is: $w = \gamma y$.

A static version of the model

Characterization of the equilibrium

The equilibrium is the triple (w^*, θ^*, V^*) characterized by:

$$w^* = \gamma y, \tag{7}$$

$$m(\theta^*) = \frac{\kappa}{(1-\gamma)y} \Rightarrow V^* = N \cdot m^{-1}\left(\frac{\kappa}{(1-\gamma)y}\right) \tag{8}$$

The higher γ or κ (the lower y), the smaller is θ^*.
The expected number of unemployed (and, by the law of large numbers, their actual number) is:

$$N(1 - \theta^* m(\theta^*)) < N \tag{9}$$

It is decreasing in θ^*.

A static version of the model

Message

- Whatever the levels of the parameters (γ, κ, y, N), there is some unemployment due to frictions (i.e. the exogenous function $m(\cdot)$).
- This does not imply that the wage level does not matter: The intensity of the unemployment problem varies with the way total surplus y is splitted.

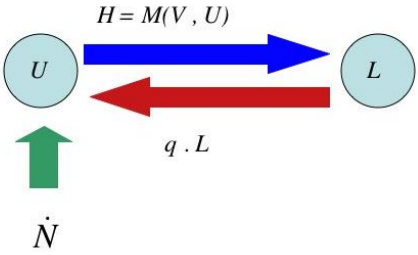
The mechanisms we have identified here hold true in the following dynamic setting.

Such a static model has e.g. been used by Hungerbühler, Lehmann, Parmentier and Van der Linden (2006) and Landais, Michaillat and Saez (2018a).

Equilibrium of flows and the Beveridge Curve

In a dynamic setting

$N =$ the size of the labor force. N is very large (continuum of workers).
$\dot{N} = \frac{dN}{dt}$ (= exogenous!).
Assume that those who enter the labor force begin by looking for a job.
Assume an exogenous separation rate q (extensions: endogenous q).



Equilibrium of flows and the Beveridge Curve

$$\dot{U} = \dot{N} + qL - \theta m(\theta) U \text{ (law of large numbers)} \tag{10}$$

$$\dot{u} = q + n - [q + n + \theta m(\theta)]\, u \text{ where } u = U/N, n = \dot{N}/N \tag{11}$$

In a steady state $\dot{u} = 0$:
entries in = exit from unemployment.

The previous equation leads to

$$u = \frac{q+n}{q+n+\theta m(\theta)}, \tag{12}$$

which is strictly positive as soon as either q or n is > 0 and $\theta m(\theta) < +\infty$.

Equilibrium of flows and the Beveridge Curve

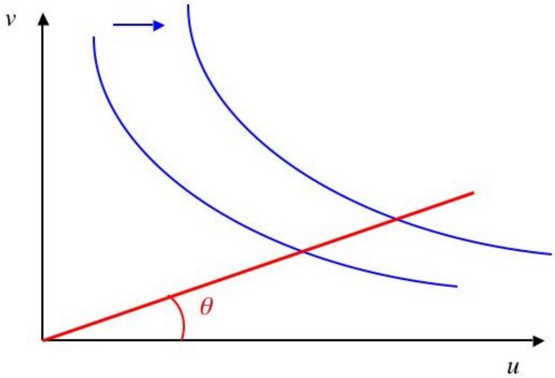
Or, since the "vacancy rate" $v = V/N$,

$$u = \frac{q+n}{q+n+(\frac{v}{u})m(\frac{v}{u})}$$

= an implicit relationship between u and v that defines the "Beveridge curve" understood as
The set of pairs (u, v) such that the unemployment rate remains constant.
To be distinguished from the plot of (u, v) realizations seen before (except for years such that $\dot{u} = 0$).

It can be shown that given the assumptions made before the "Beveridge curve" is decreasing and convex.

The Beveridge Curve as an equilibrium relationship



If $H = A\ V^{1-\eta}\ U^{\eta}$, then $u = \frac{q+n}{q+n+A\left(\frac{v}{u}\right)^{1-\eta}}$

Equilibrium of flows and the Beveridge Curve

A rightward shift of the "Beveridge curve" can be the consequence of

- An increase in q or in n (the rate of growth of N!)
- A deterioration in the "efficiency of the matching process" ($A \searrow$)

If we knew the equilibrium value of θ in steady state, say $\hat{\theta}$, then

- We could draw a straight line $v = \hat{\theta} \cdot u$.
- Next, the steady-state equilibrium u and v would be determined by the intersection between the Beveridge curve and this line.

At this stage however, we ignore the equilibrium value of θ.
The following slides are concerned by the determination of this equilibrium value in steady state.

A dynamic model with ex-post wage bargaining

Assumptions

- There are two goods: a produced good (the *numeraire* sold in a competitive market) and labor (the unique explicit input).
- Two types of agents only: Private firms and the labor force.
- Each firm is made of a single vacant or filled job.
- Infinitely lived agents with perfect foresight + risk neutrality.
- h is the (exogenous and deterministic) cost per vacant job and *per unit of time*. h is typically assumed to capture the cost involved in posting a vacancy, searching for applicants and selecting them. A broader interpretation is proposed by Pissarides (1985) (p. 679-680). See also Muehlemann and Leiser (2018).
- r is the exogenous discount rate common to all agents. Firms can borrow and lend from perfect capital markets at the rate r.

Expected profits

- One Euro invested at time t yields $1 + r\,dt$ at time $t + dt$. So, the discount factor will be $\frac{1}{1+r\,dt}$ on any interval of length dt.
- During any short time period dt, $(y - w)\,dt$ measures the current flow of return.
  - y measures (exogenous) real output (sold),
  - w is the real wage assumed to be lower than y,
  - working time is exogenous and normalized to 1. Hence, aggregate labor supply is $N \cdot 1$ if $w >$ the reservation wage defined later on.
- The worker and the firm separates with probability $q\,dt$.
- At any time t, a firm's real discounted *expected* return from an occupied job is denoted $\Pi_e(t)$.
  A firm's real discounted *expected* return from a vacant job is denoted $\Pi_v(t)$.

Expected return from an occupied job

With a perfect capital market and an infinite horizon, $\Pi_e(t)$ satisfies the following Bellman equation:

$$\Pi_e(t) = \frac{(y - w)dt + qdt\,\Pi_v(t + dt) + (1 - qdt)\Pi_e(t + dt)}{1 + rdt} \tag{13}$$

As CCZ, one here assumes that with probability $q\,dt$ the job becomes vacant.
Pissarides (2000) considers that at a rate q the job is destroyed: Its value is then zero.
Under free-entry (see below), both approaches lead to the same conclusions.

Expected return from an occupied job

Multiply both sides of (13) by $1 + r\,dt$. Then divide both sides by dt and take the limit for $dt \to 0$. This yields

$$r\Pi_e(t) - \frac{d\Pi_e(t)}{dt} = y - w + q(\Pi_v(t) - \Pi_e(t)). \tag{14}$$

In a steady state, this Bellman equation becomes:

$$r\Pi_e = y - w + q(\Pi_v - \Pi_e). \tag{15}$$

A filled vacancy can be seen as an asset owned by the firm.

- $r\Pi_e$ is the (instantaneous) rate of return on this asset.
- At any time t, this rate of return is the sum of
    - instantaneous profits $y - w$
    - and the expected net return due to a change of state is $q(\Pi_v - \Pi_e)$, which is actually negative.

Expected return from from a vacant job

The same reasoning leads to the following Bellman equation for $\Pi_v(t)$. In an infinitesimal time interval, the probability of meeting more than one job-seeker can be neglected. *Implicit assumption: any contact with a job seeker leads to a match:* $\Pi_e(t) > \Pi_v(t)$ and as all workers are identical the first one which is met is recruited.

One ends up with:

$$r\Pi_v(t) - \frac{d\Pi_v(t)}{dt} = -h + m(\theta(t))(\Pi_e(t) - \Pi_v(t)) \tag{16}$$

or in a steady state

$$r\Pi_v = -h + m(\theta)(\Pi_e - \Pi_v). \tag{17}$$

An unfilled job can also be seen as an asset owned by the firm (the interpretation is similar to the one on the previous slide).

The rest of this section is developed in a steady state.

Labor demand

The system of Bellman equations (15) and (17) can be solved to yield the following expression for Π_v:

$$\Pi_v = \frac{-(r+q)h + m(\theta)(y-w)}{r(r+q+m(\theta))},$$

which decreases with tightness θ. Given the above-mentioned Inada conditions (3):

$$\lim_{\theta \to 0} \Pi_v = (y-w)/r > 0 \text{ if } y > w \text{ and } \lim_{\theta \to +\infty} \Pi_v = -h/r < 0.$$

Whatever the unemployment level, the inflow of vacancies ends when the profit expected from an additional vacant job becomes zero:

$$\Pi_v = 0 \quad \text{i.e. The key “Free-Entry” condition}$$

Labor demand

Under *free entry* of vacancies, set $\Pi_v = 0$ in:

(17) $\Rightarrow \Pi_e = \frac{h}{m(\theta)}$ $h \times$ the expected length of time $1/m(\theta)$

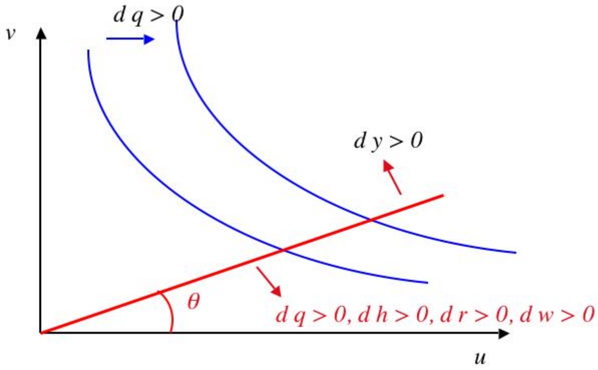
(15) $\Rightarrow \Pi_e = \frac{y-w}{r+q}$ q is added to r to discount $y - w$

So, the labor demand (or "*vacancy-supply curve*") is the following downward-sloping relationship between w and θ:

$$\boxed{\frac{h}{m(\theta)} = \frac{y-w}{r+q}} \Leftrightarrow \boxed{w = y - \frac{(r+q)h}{m(\theta)}} \tag{18}$$

Labor demand

If the wage level is exogenous ($w < y$), this demand side of the model leads to the following comparative statics (recall that $m'(\theta) < 0$)



Note

The absence of physical capital

The absence of physical capital is noteworthy.
An alternative, developed e.g. by Acemoglu (2001), consists in assuming that
(i) vacancy costs h are negligible (actually, zero) and
(ii) an equipment is required to produce, the acquisition cost being incurred before meeting applicants.
Now, if $\tilde{k}$ designate a fixed cost of equipment per job, the free entry condition introduced above becomes

$$\Pi_v = \tilde{k}$$

This does not change the properties of the model in steady-state.

So, I stick to the standard presentation which emphasizes the role of vacancy costs.

The Behavior of workers

Keeping the assumption that workers are risk neutral, one could again look at a small interval of time dt and then take the limit $dt \to 0$.

Let V_e and V_u be the (steady-state) real discounted expected value of the income stream respectively in employment and an unemployment. The model is written under the *implicit assumption that* $V_e > V_u$, which turns out to be true under Nash bargaining.

Utility = Instantaneous income in employment: w
(implicitly: hand-to-mouth agents; full-time job; leisure time ignored).
At an exogenous rate q, the job is lost.

The expected utility of an employed person satisfies:

$$rV_e = w + q(V_u - V_e) \tag{19}$$

The Behavior of workers

An unemployed worker is always in search of a job (Job-search effort is here fixed and normalized to 1).

At each instant, this search procures him or her a net gain denoted by z (by assumption $z < y$):

- the value of time (leisure, home production) minus whatever disutility, if any, comes from not having a job (stigmatization);
- $\oplus$ benefits linked to being unemployed (unemployment insurance or social welfare transfers, if any)
- $\ominus$ the various costs attached to searching for a job (commuting to the public employment agency, posting applications,... costs that are presumably shrinking thanks to the Internet).

The Behavior of workers

Since the exit rate from unemployment is $\theta m(\theta)$, the expected utility of an unemployed person satisfies in steady-state:[7]

$$rV_u = z + \theta m(\theta)(V_e - V_u) \tag{20}$$

(the unemployed will not turn down job opportunities since $V_e > V_u$ under Nash bargaining)

Subtracting (20) from (19) implies that workers have no incentive to quit if $w > z$:

$$V_e - V_u = \frac{w - z}{r + q + \theta m(\theta)}. \tag{21}$$

[7] Looking again at an interval of time of infinitesimal length, the probability of receiving more than one offer can be neglected.

Wage formation

The timing of events is essential here:

1. The (unemployed) workers and firms engage in a costly (time-consuming) search process (e.g. the firm incurs cost h during a period of time in order to create a vacancy and recruit a worker);
2. Once they have sunk this cost, they bargain over the wage (*ex-post individual* bargaining over wages).

This leads to a situation of bilateral monopoly.
Implications can be phrased in two actually equivalent ways:
Once the two partners have met,

1. There is a range of wages at which both partners prefer to match rather than breakup or
2. There is a "match-specific **surplus**" (a rent) that has to be shared.

The surplus of a match

The (total) *surplus of a match* = the sum of the rents (of the firm and of the worker) that a filled job procures.

Rent = gain from the contractual relationship minus outside option
= $\Pi_e - \Pi_v$ in case of the employer
= $V_e - V_u$ in case of the employee

$\Rightarrow$ (total) surplus $S = V_e - V_u + \Pi_e - \Pi_v$.

As V_e can be rewritten as $(w + q\, V_u)/(r + q)$ and Π_e as $(y - w + q\,\Pi_v)/(r + q)$,[8]

$$S = \frac{y - r\,(V_u + \Pi_v)}{r + q}$$

[8] Note that both V_e and Π_e are linear in w. This is the so-called "transferable utility case". See L'Haridon, Malherbet and Pérez-Duarte (2013).

How is the total surplus split?

One can define a worker's reservation wage, $\underline{w}$ characterized by the indifference condition:

$$V_e(\underline{w}) - V_u = 0 \qquad \Leftrightarrow \qquad \frac{\underline{w} - rV_u}{r+q} = 0$$

Similarly, remembering that the cost of vacancy creation is sunk, the employer's reservation wage, $\overline{w}$, is such that:

$$\Pi_e(\overline{w}) - \Pi_v = 0 \qquad \Leftrightarrow \qquad \frac{y - \overline{w} - r\Pi_v}{r+q} = 0$$

So, the boundaries of the bargaining set in which the negotiated wage has to be is

$$[rV_u, y - r\Pi_v]$$

Any wage within the bargaining set could be an outcome of the bargain. So, there is an indeterminacy in matching models!

How is a wage chosen in $[rV_u, y - r\Pi_v]$?

$\Diamond$ The literature typically selects a specific wage in the bargaining set through either

- An axiomatic Nash bargaining solution (Nash, 1953);
  See next slides.
- Or a strategic bargaining game approach (Rubinstein, 1982):
  Two players alternate offers over many periods with an (in)finite horizon (An example of such a game has been introduced in the static model above).
  The precise environment of the game matters.

Under some assumptions both approaches lead to the same outcome. CCZ discuss all this on p.415-422. If needed, see Chapter 16 of Osborne (2004) and L'Haridon, Malherbet and Pérez-Duarte (2013).

$\Diamond$ Alternative views are developed e.g. by Hall (2005) and Farmer (2011), who deal with this indeterminacy in matching models.

Wage bargaining

Surplus sharing

Wages are renegotiated continuously.

When they bargain over the *current* wage, the players take V_u and Π_v as well as tightness on the labor market as given.

The value of the wage negotiated at each moment is the solution of the maximization of the following "Nash product":

$$\underset{w}{\text{Max}}\ (V_e - V_u)^{\gamma}(\Pi_e - \Pi_v)^{1-\gamma},\ 0 \leq \gamma \leq 1 \tag{22}$$

The first-order condition of this problem can be written as:

$$V_e - V_u = \gamma S \quad \text{and} \quad \Pi_e - \Pi_v = (1-\gamma)S \tag{23}$$

So, the total surplus S is split according to the shares γ and $1-\gamma$.

To derive a wage equation from

$$V_e - V_u = \gamma\, S = \gamma\, \frac{y - r\,(V_u + \Pi_v)}{r+q}$$

we do NOT exploit (21) but

$$V_e - V_u = \frac{w - rV_u}{r+q}$$

$$\Rightarrow w = rV_u + \gamma(y - r(V_u + \underbrace{\Pi_v}_{\text{0 by free entry}})) = rV_u + \gamma(y - rV_u) \qquad (24)$$

This expression has an intuitive interpretation:

$\gamma = 0$ — $w = \underline{w} = rV_u$ (workers get no rent)

as $\gamma \nearrow$ — an increasing share of the difference ($y - rV_u$) accrues to the worker

Note: If $\gamma = 1$, Π_v is negative (see (15) and (17)).
So, firms do not open vacancies at all because they cannot recoup the sunk cost h.

Wage bargaining

Getting rid of rV_u

To complete the analysis, one would like to relate the negotiated wage w to θ and to the parameters of the model. So we need to reformulate rV_u as a function of θ.

This can be done in various ways.

1) The book follows one approach that leads to:

$$rV_u = \frac{z(r+q) + \gamma y \theta m(\theta)}{r+q+\gamma\theta m(\theta)}$$

Substituting this expression of rV_u, they get this ‘wage curve”:

$$w = z + (y-z)\Gamma(\theta) \quad \text{with} \quad \Gamma(\theta) = \frac{\gamma[r+q+\theta m(\theta)]}{r+q+\gamma\theta m(\theta)},\ \Gamma' > 0 \qquad (25)$$

Wage bargaining

Getting rid of rV_u

2) Consider now the approach of Pissarides (2000). The starting point is again

$$w = rV_u + \gamma(y - rV_u) = (1-\gamma)rV_u + \gamma \cdot y, \tag{26}$$

where V_u solves

$$rV_u = z + \theta m(\theta)(V_e - V_u)$$

Under free entry, the solution to the game $V_e - V_u = \gamma S$ can be rewritten:

$$(1-\gamma)(V_e - V_u) = \gamma \Pi_e$$

where $\Pi_e = h/m(\theta)$. So,

$$V_e - V_u = \frac{\gamma}{1-\gamma}\frac{h}{m(\theta)} \Rightarrow rV_u = z + \theta m(\theta)\frac{\gamma}{1-\gamma}\frac{h}{m(\theta)}$$

Wage bargaining

Plugging this expression in (26) yields this "wage curve" (*WC*)

$$\boxed{w = (1-\gamma)z + \theta\gamma h + \gamma y} \tag{27}$$

or, if h is proportional to y ($h = k \cdot y$):

$$\boxed{w = (1-\gamma)z + \gamma y\,(1+\theta k)} \tag{28}$$

Whether (25) or (27) is chosen, the bargained wage reacts to θ (i.e. wages are 'flexible').

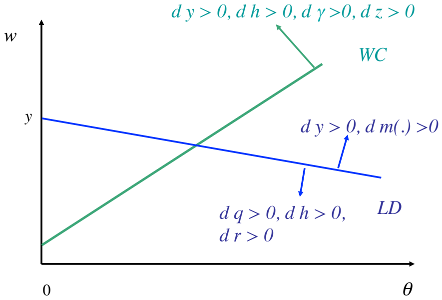
Note: For any value of θ, the equilibrium wage is unique.
Whether this holds true with on the job search is briefly discussed at the end (under the heading "extensions").

The labor market equilibrium

$WC : w = (1-\gamma)z + \theta\gamma h + \gamma y$ $\quad$ $LD : w = y - \frac{(r+q)h}{m(\theta)}$

Because the matching function has C.R.S., equilibrium $(\theta, w) \perp (u, v)$.

Combining the wage curve (27) and the labor demand (18) it can be shown that the equilibrium (θ, w) pair is unique



The labor market equilibrium

Getting rid of w yields an implicit equation in (equilibrium) tightness:

$$\boxed{\frac{(1-\gamma)(y-z)}{r+q+\gamma\theta m(\theta)} = \frac{h}{m(\theta)}} \tag{29}$$

where the LHS is decreasing and the RHS is increasing in θ (uniqueness thanks to the Inada conditions (3)).

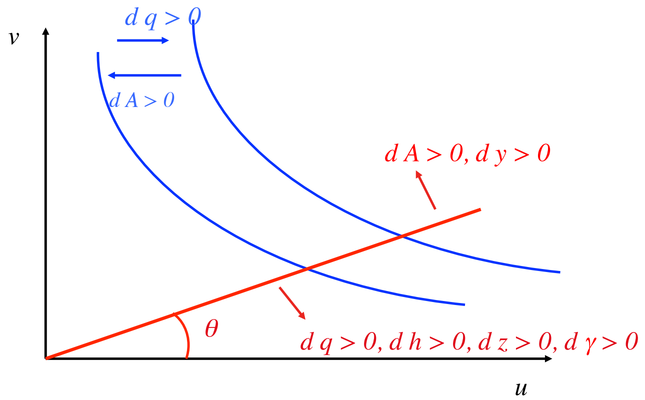
$$\text{If } h = k \cdot y, \text{ then } \frac{(1-\gamma)(y-z)}{r+q+\gamma\theta m(\theta)} = \frac{k \cdot y}{m(\theta)} \tag{30}$$

which can be made independent of y if one assumes constant "replacement ratios" z/w (check it; hint: Return to (28) $\Rightarrow w = \tilde{\rho} y$).

With the Cobb-Douglas matching function (2), Condition (29) becomes

$$\frac{(1-\gamma)(y-z)}{r+q+\gamma\, A\, \theta^{1-\eta}} = \frac{h}{A\, \theta^{-\eta}}$$

Comparative statics: Last expression with the Cobb-Douglas



Note on the role of unemployment benefits

Strong property: More generous benefit levels bring about higher unemployment.
For this, the way the Nash product (22) is written is essential!

- In (22) above, the threat point for bargaining is the payoff of each partner if they separate (resp. V_u and Π_v).
- According to Hall and Milgrom (2008) (H&M), the realistic threat is to extend the bargaining (not to terminate it) $\Rightarrow$ less connection between the wage and outside options (in particular V_u and, hence, z!)

A more nuanced view about the aggregate effects of unemployment benefits in matching models is also proposed by Landais, Michaillat and Saez (2018a, 2018b).

Note

What's going on on the goods market? The case where z is home production

Normalization: the size of the workforce $N = 1$. At each point in time, the *numeraire* is produced either by firms [in (instantaneous) quantity $y(1-u)$] or by individuals through home production [in quantity $z\,u$]. Abstracting from savings, the demand for the good sums

- The aggregate consumption of the unemployed, i.e. what is produced at home: $z\,u$;
- The aggregate consumption of employed individuals, i.e. aggregate earnings: $w(1-u)$;
- The aggregate consumption of entrepreneurs, i.e. their aggregate profits: $\Pi = (y-w)(1-u) - h\,v$;
- The resources invested in the creation of vacancies $h\,v$.

Adding all these consumption terms yields:

$$z\,u + w(1-u) + (y-w)(1-u) - h\,v + h\,v = z\,u + y(1-u),$$

which is equal to aggregate production.

Note

From the one-job-one-firm case to the "large firm" case

- Generalization of the random matching models to a large firm occupying a continuum of workers and possibly using capital:
  1. Under *individual* wage negotiation:
     - If labor and capital can be adjusted instantaneously so that returns to scale are constant, the large firm and the one-job-one-firm setting are equivalent (case discussed by CCZ).
     - When the marginal product of labor is decreasing, one has to think more at wage formation:
       Under bilateral negotiation *without commitment about future wages*, an additional worker depresses the marginal product of labor and hence the wage of all existing workers (under the additional assumption of automatic renegotiation of wages); see Cahuc and Wasmer (2001), Cahuc, Marque and Wasmer (2008), and Elsby and Michaels (2013).
  2. Other mechanisms of wage formation (collective bargaining,...): see Mortensen and Pissarides (1999); Bauer and Lingens (2014).

Is the equilibrium (constrained) efficient?

Section 4 of the book

There are

- *congestion effects* within each category
- and *positive externalities* between the categories.

Are the search externalities internalized by the ex post Nash bargain? This is the question raised in this section.

The decentralized equilibrium is *Constrained* efficient if it is identical to the allocation chosen by a hypothetical **social planner** who maximizes social welfare *given the fundamental frictions*.

The social planner maximizes "**social output**" defined on the next slide. *The social planer ignores distributional issues: (S)he only cares about aggregate net output created in this economy.*

Social output

With risk neutral agents, **social output** at time t (divided by the exogenous size of the labor force N) is denoted $\omega(t)$ and given by:

$$\omega(t) = y(1 - u(t)) + z \cdot u(t) - h \cdot v(t) = y + [z - y - h \cdot \theta(t)]\, u(t) \quad (31)$$

$\omega(t)$ is simply current output + the value of unemployment - the cost of opening vacancies.

One keeps the assumption $z < y$.

z interpreted as home production of the final good.
Note : Since workers are risk-neutral, it would be inefficient to provide UBs financed by distortive taxes.

The choice made by the social planner

What should we expect?

Due to congestions, the impact of additional vacancies on the hiring rate is declining (keeping u fixed).

Moreover, each additional vacancy costs h.

$\Rightarrow$ unlikely that creating more vacancies is always good from the point of view of net output ω.

The choice made by the social planner

Optimal control

Starting from an initial situation $u(t=0)=u_0$, the social planner would solve:

$$\max_{\theta(t),u(t)} \int_0^{+\infty} \omega(t)\cdot e^{-rt}dt$$

subject to the equation of motion (taking $n=0$):

$$\dot{u}(t) = q(1-u(t)) \; - \; \theta(t)\cdot m(\theta(t))\cdot u(t).$$

This *optimal control* approach is followed in Section 4.2.2. in the book (θ is the control variable and u the state variable).

The choice made by the social planner

A simpler approach

Consider the steady-state value of $\omega(t)$ only
(ignoring the adjustment from the initial condition at $t = 0$ to the steady state $\Rightarrow$ the path of the economy is not discounted)
$\Rightarrow$ we can later compare the so-called "social optimum" in a steady state with the steady state emerging from decentralized decisions when $r \rightarrow 0$.

Maximizing (31) with respect to θ with u defined by the Beveridge curve $u = \frac{q}{\theta m(\theta)+q}$ is equivalent to maximizing the following expression with respect to θ only:

$$y + \frac{q}{\theta m(\theta) + q}[z - y - h \cdot \theta]. \tag{32}$$

Interpret!

The choice made by the social planner

The optimal value of tightness has to be such that the sum of two effects becomes nil.

$$\underbrace{\frac{d}{d\theta}\left[\frac{q}{\theta m(\theta)+q}\right]}_{\ominus}\underbrace{[z-y-h\cdot\theta]}_{\ominus} + \frac{q}{\theta m(\theta)+q}\cdot(-h) = 0$$

1. Increasing θ reduces the unemployment rate.
2. Conditional on the level of unemployment, a higher tightness entails more vacancy costs.

Through this maximization, the benevolent planner takes the consequences of his (her) choice of θ on the externalities due to frictions.

Is the equilibrium efficient?

This first-order condition of the *planner's problem* with respect to θ leads to:

$$\frac{(y-z)(1-\eta(\theta))}{q+\theta\, m(\theta)\eta(\theta)} = \frac{h}{m(\theta)}, \tag{33}$$

where

- $\eta(\theta) = \mid d\log\left(m(\theta)\right)/d\log(\theta)\mid$, ; if $H_t = A_t\ V_t^{1-\eta}\ U_t^{\eta}, \eta(\theta) \equiv \eta$
- $1-\eta(\theta) = \mid d\log\left(\theta m(\theta)\right)/d\log(\theta)\mid$

In the *decentralized equilibrium* with ex post Nash bargaining, θ solves (see (29) above when $r \to 0$):

$$\frac{(y-z)(1-\gamma)}{q+\theta\, m(\theta)\gamma} = \frac{h}{m(\theta)} \tag{34}$$

In general, the solution, say θ^e, of (33) and the one, say θ^d, of (34) are different.

Is the equilibrium efficient?

(33) $\Leftrightarrow$ (34) if

- the bargaining power of the worker γ equals $\eta(\theta^e)$, the elasticity of the rate *m* of filling a vacancy with respect to tightness θ (taken in absolute value).
- Because of the CRS assumption, $\eta(\theta^e)$ is also the elasticity of $M(V, U)$ with respect to unemployment.

Under the condition $\gamma = \eta(\theta^e)$, called the "*Hosios condition*" (Hosios, 1990), the search externalities are internalized by the ex-post Nash bargain.
Under the Hosios condition, the total surplus generated by a match is shared in such a way that the externalities are exactly balanced, so that efficiency is restored.

Is the equilibrium efficient?

"Basically, the Hosios condition says that in order to maximize the aggregate gains from trade, less transaction costs, the traders' bargaining shares must reflect their marginal contribution to the value of the aggregate transaction flow. This condition is satisfied in the case of a linearly homogeneous matching technology if and only if agents' shares equal the elasticities of the matching function with respect to the stocks of buyers and sellers in the market." (Mortensen and Wright, 2002)

Is the equilibrium efficient?

- Notice that the Hosios condition expresses that workers should have a positive bargaining power. (Very different from most union models where the underlying reference in the absence of unions is the competitive labor market).
- Notice also that a certain level of unemployment is present when the outcome is efficient. (Very different from union models).
- If $\gamma > \eta$, equilibrium unemployment is above its efficient level.
- Conversely, if $\gamma < \eta$, equilibrium unemployment is inefficiently low.
- Too few (resp. too many) vacancies are created when γ is too high (resp. too low).
- Major difficulty: Measuring workers' bargaining power.
- For a more general condition (avoiding this difficulty and also valid under risk aversion), see Landais, Michaillat and Saez (2018a) and Michaillat and Saez (2021).

Is the equilibrium efficient?

- There is no reason why the Hosios condition should be fulfilled. Hence, in general, search equilibria with ex post Nash bargaining are typically inefficient.
  See later what happens if directed search is assumed instead of random search.
- The *laissez faire* economy (without taxes and unemployment insurance) under the Hosios condition is not the only efficient outcome.
  When the bargaining power does not fulfill the Hosios condition, taxation can restore efficiency because a positive marginal tax rate (resp. a negative one) decreases (resp. increases) the share of the surplus that accrues to the workers (Boone and Bovenberg, 2002).
- What about efficiency when workers are risk averse? See Lehmann and Van der Linden (2007) and Michau (2015).

Out-of-stationary-state dynamics

We skip most of this section (Section 6 in the book)

The study of out-of-stationary-state dynamics allows to diagnose the origin of the perturbations that affect movements in employment. Remedies adopted to reduce under-employment will vary with this diagnosis.

Aggregate shocks	Change in aggregate demand or supply of goods, and would *not* shift the Beveridge curve
Aggregate shocks	= A change in y, r, z, or γ
Reallocation shocks	A restructuring of production units, which would shift the Beveridge curve but perhaps “not much” the equilibrium vacancy rate.
Reallocation shocks	= A change in function $m(.)$ or in q

Out-of-stationary-state dynamics

The initial unemployment rate (at $t = 0$) differs from its steady-state value (designated below by a superscript *). The environment remains time-invariant. What is the dynamic adjustment of this economy?

Free entry of vacancies[9] and Nash bargaining at each moment!

Express value functions out of steady state (like Eq. (14) above).

Decisions of agents are directed toward the future ("forward-looking")
$\Rightarrow$ The number of vacancies (and hence, θ) *and* the wage immediately "jump" to the stationary value.

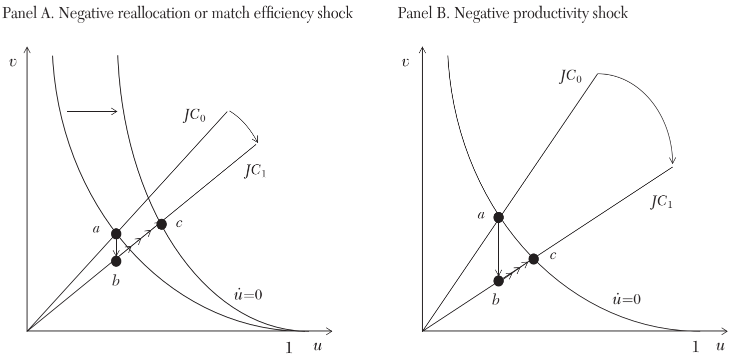
When labor market tightness has reached its stationary value θ^*, the differential equation (11) describing the evolution of the unemployment rate becomes:

$$\dot{u}(t) = q + n - [q + n + \theta^* m(\theta^*)]\, u(t) = (q + n + \theta^* m(\theta^*))(u^* - u(t))$$

[9] Criticized by Coles and Kelishomi (2018) following Diamond (1982).

Dynamic adjustment in the basic model

Source: Elsby, Michaels and Ratner (2015)



"Despite these qualitative successes, however, the standard search model faces challenges in explaining (...) crucial quantitative features of observed Beveridge curve dynamics" (Elsby, Michaels and Ratner, 2015, p. 586)

Out-of-stationary-state dynamics

Extensions: Real-Business-Cycle models with matching frictions

Standard approach since Merz (1995) and Andolfatto (1996):

- Output (of an aggregate good) can either be consumed, invested, or spent to cover the cost of vacant jobs.
- A (representative risk averse) household = a large extended family which contains a continuum of members. Members of the family perfectly insure each other against fluctuations in income due to employment and unemployment.
- The household has standard preferences over consumption and the fraction of their members who are not working ("leisure").
- The household chooses consumption and savings given the law of motion of employment.
- The market for the aggregate good clears.
- Some parameters follow a random process with idiosyncratic and/or aggregate shocks.

Critique of the Matching Model with standard Nash Bargaining

1) Poor cyclical performances of the matching model?
   "Shimer's critique": According to Shimer (2005a), "the textbook search and matching model cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to [productivity] shocks of a plausible magnitude."

   The mechanism at work is well explained in the following quote: "(...) an increase in productivity increases the value of a match. As a consequence, firms post more vacancies which boosts workers' job finding rate, raising their outside option (the value of being unemployed). The net result is that wages rise, eating up much of the gain received by firms associated with the increase in productivity, thereby lowering the response of vacancies." (Gomme and Lkhagvasuren, 2015, p. 107)

Critique

The “Shimer’s critique” led to a major controversy. Main answers[10]:

a) **Introduce wage stickiness.**
In the Nash product (22), the threat point for bargaining is the payoff of each partner if they separate (resp. V_u and Π_v). Plausible calibrations leads then to an elasticity of w with respect to y close to 1.

According to Hall and Milgrom (2008), the realistic threat is to extend the bargaining (not to terminate it) as long as a solution can be found in $[\underline{w}, \overline{w}]$. Under some assumptions, wages are fully rigid.
Applications of the approach of Hall and Milgrom (2008) can be found e.g. in Christiano, Eichenbaum and Trabandt (2016) and Boitier and Lepetit (2018).

[10] Many other propositions exist in the literature.

1) Reply to Shimer's critique (Continued)

   b) **Introduce a fixed cost of matching.**
   Pissarides (2009) argues that wages in *new* matches are actually as cyclical as in the textbook model under Nash bargaining. Moreover, all what matters for job creation is the gap between (expected) productivity and wages in *new* matches.

   The free-entry condition emphasizes that the firm's cost of creating a vacancy is *proportional* to the expected duration of it.

   "Other matching costs, such as training, negotiation, and one-off administrative costs of adding a worker on the payroll, are neglected by the model" (Pissarides, 2009, p. 1363). Pissarides recommends to add such a fixed cost. Call it H. Then the left-hand side of (18) becomes $H + h/m(\theta)$.

   The fixed cost H restrict the marginal cost of hiring from declining in recessions.

Ljungqvist and Sargent (2017) provide an overview of the responses to the Shimer's critique. See also Michaillat (2012).

Critiques

2) “Lagos (2000) emphasizes that if the matching function is a reduced-form relationship, one should be concerned about whether it is invariant to policy changes. Addressing this issue requires an explicit model of heterogeneity that gives rise to an empirically successful reduced-form matching function” (Shimer, 2007, p. 1077).

Large outward shifts in the locus of the Beveridge curve are mainly explained by “a deterioration in the matching process” (a “black box”). What explains the latter?

Efforts to address this critique: in particular by Stevens (2007), Mortensen (2009), Ebrahimy and Shimer (2010), and Barnichon and Figura (2015).

Critiques

3) "Random search is dumb search":
Each agent searches in "all directions" at random, nobody uses
   - Wages that firms commit to pay
   - Or a "match maker" / "market maker" (private or public employment agencies, specialized online platforms such as CareerBuilder.com, craigslist.org, mturk.com, upwork.com, burning-glass.com...)

   to "organize a less time-consuming search process".

   Opposite view: Job-seekers **direct their search** on the basis of posted wages (Moen, 1997, and many followers).

Extensions

- Endogenous effort to search for a job (e_u)/for applicants (e_f): $H = M(e_f \cdot V, e_u \cdot U)$ (see e.g. Chap. 5 of Pissarides, 2000; Lehmann and Van der Linden, 2007; Mukoyama, Patterson and Şahin, 2018; For evidence of endogenous recruitment intensity, see Davis, Faberman and Haltiwanger, 2013).
- Stochastic job matching: Ex ante workers and firms are identical but the productivity of a job-worker pair is random (chap. 6 of Pissarides, 2000; with a learning process of job quality in Pries and Rogerson, 2005);
- On-the-job-search Important extension as job-to-job flows are observed to be important.

  This extension raises however several tricky issues in the presence of bargaining. See the seminal paper of Pissarides (1994) (or chap. 4 of Pissarides, 2000). While Shimer (2006) points out that the equilibrium wage distribution is no longer unique in wage bargaining models with on-the-job search, Cahuc, Postel-Vinay and Robin (2006) show that the introduction of renegotiation circumvents the multiple equilibria outcome.

Extensions

- Endogenous size of the labor force and/or of the number of hours worked (chap. 7 of Pissarides, 2000, Garibaldi and Wasmer, 2005, Pries and Rogerson, 2009, Sec. 7 of Elsby, Michaels and Ratner, 2015, Krusell, Mukoyama, Rogerson and Şahin, 2017, or Kudoh and Sasaki, 2011).
- Labor market policies and taxation (see chap. 9 of Pissarides, 2000, Holmlund and Lindén, 1993, Fredriksson and Holmlund, 2001, Lehmann, Parmentier and Van der Linden, 2011).
- Add explicitly a spatial dimension (e.g. Wasmer and Zenou, 2006, Zenou, 2009 in the urban economics literature; Marimon and Zilibotti, 1999, and Decreuse, 2008, where agents are *ex ante* heterogeneous in a broader sense).
- New insights on discrimination (e.g. Rosen, 2003, and Masters, 2009, about statistical discrimination).
- Introducing a life-cycle dimension (heterogeneity in age, a finite age of retirement) (e.g. Chéron, Hairault and Langot, 2011, Menzio, Telyukova and Visschers, 2016).

Extensions

- Risk averse workers, incomplete markets and precautionary savings (Krusell, Mukoyama and Şahin, 2010, and Ravn and Sterk, 2021, extend Merz, 1995, and Andolfatto, 1996).
- Imperfect credit markets: “If credit markets are imperfect, an entrepreneur with an idea but without any capital will encounter some impediments when he turns to credit markets to find the funds required to post a vacancy” (Wasmer and Weil, 2004). See also Boeri, Garibaldi and Moen (2018) and the references therein.
- Replacing free entry of vacancies by a more realistic assumption (Diamond, 1982, Coles and Kelishomi, 2018).
- The indeterminacy in matching models can lead to aggregate demand effects (Farmer, 2011, Michaillat and Saez, 2015).
- Getting rid of *continuous* wage negotiation (Gertler and Trigari, 2009).

Extensions

Endogenous job destruction

The job destruction rate q can be made endogenous

(see chap. 2 of Pissarides, 2000, or CCZ p. 862, and Chéron, Hairault and Langot, 2013, with finite horizon).

The productivity of filled vacancies is hit by random idiosyncratic shocks (often a Poisson process).
Such a shock triggers an *automatic* renegotiation of the wage.[11]

$\Rightarrow$ There is a “reservation” level of productivity below which it is preferable to separate and look for another partner.

$\Rightarrow$ The model can be expressed in terms of a “job creation” curve and a “job destruction” curve in a $(\theta,$ reservation productivity$)$ space.

[11] Critique: It would be more sensible to assume renegotiation by mutual consent, i.e. neither party can force the other to renegotiate the wage. (See Postel-Vinay and Turon, 2010). Only a credible threat of ending the match can trigger renegotiation.

Directed Search
Also called “Competitive Search”

If random search is (in general) not leading to an efficient allocation, Aren't there ways of reducing this inefficiency?

One can think at the presence of *a third party* playing the role of a "market maker" (public employment services, and whatnot). *Without a third party,* firms could also provide information to job-seekers that allows them to **direct** their search.
Let us turn to this now...

References

This subsection is based on CCZ, pp. 603-605 (which is inspired by Moen, 1997).

Main ideas

- **Decentralized labor markets:**
  More specifically, the labor market can be seen as a (very) large number of *distinct* sub-markets, labor pools or “islands”.
  Trade occurs on each island.
  (This idea goes back to the seminal paper by Lucas and Prescott, 1974).
- Firms announce wages before the match takes place, commit to pay the announced wage and they are not willing to renegotiate wages when matched to a worker.
  $\rightarrow$ “**wage posting**”
  So, employers have the power to fix the wage (but they compete to attract applicants).
- Job-seekers have perfect information about wage offers (and about the chances to be recruited), they trust and exploit this information.

The model is developed in continuous time.
Infinitely-lived agents that are risk neutral and forward looking.
The analysis is here conducted in a steady state.

To keep things as simple as possible, some specific assumptions are made:

- As in the case of random matching full specialization in either job-search or production; so no on-the-job search.
- The sub-markets are homogeneous:
  On each island, the same exogenous and fixed values of y, h, and q, and the same matching process.
- A vacant job can be created in any sub-market i (free entry).

Specific assumptions (continued):

- Employers in a sub-market i announce a wage w_i (a single wage per sub-market[12]) and **commit** to pay that wage forever.
- The unemployed observe offers and *direct* their search towards their sub-market of preference (choice of one and only one location - can be generalized).
- They are moreover **perfectly mobile** between sub-markets.
- As search is directed, the source of matching frictions are a coordination failure problem:
  In decentralized markets, it would be very hard to coordinate search decisions. Hence some vacancies receive 0 applications, some others > 1.
  Each sub-market is endowed with a matching function according to which V_i "local" vacant jobs meet U_i applicants present on sub-market i.

[12] This is generalized by Moen (1997).

Are these assumptions making sense?

> *"There are several approaches to assessing directed search. Some use observational data (Faberman and Menzio 2015; Marinescu and Wolthoff 2015; Banfi and Villena- Roldan 2016), while others use data from field experiments (Dal Bo et al. 2013; Belot et al. 2016), to see if higher wages attract more or better job applicants. This seems to be the case if one takes care that jobs are sufficiently comparable." (Wright, Kircher, Julien and Guerrieri, 2021, p. 57)*

The caveat in the last sentence appears to be important: To obtain the above property, Marinescu and Wolthoff (2020) need to condition the analysis on a given job title (e.g. "senior accountant").

Implications of workers' perfect mobility

- Identical sub-markets $\Rightarrow$ identical matching functions. If there are U_i unemployed selecting sub-market i and V_i vacancies, the exit rate to a job on island i is $\theta_i m(\theta_i)$ and the vacancy filling rate is $m(\theta_i)$, $\theta_i = \frac{V_i}{U_i}$.
- Because of the (strong) assumption of perfect mobility, *The expected utility of an unemployed should in equilibrium be the same whatever the sub-market i she chooses to visit*. Denote it

$$V_u.$$

- The standard Bellman equations have to be written as follows:

$$rV_{ei}(w_i) = w_i + q(V_u - V_{ei}(w_i)) \qquad \forall i \text{ "active" i.e. } \mid w_i \geq rV_u$$

$$rV_u = z + \theta_i m(\theta_i)(V_{ei}(w_i) - V_u) \qquad \forall i \text{ "active" i.e. } \mid w_i \geq rV_u$$

- The first equality implies that in an active market

$$V_{ei}(w_i) - V_u = \frac{w_i - rV_U}{r+q}$$

Substituting this equality in the second relationship yields

$$\theta_i m(\theta_i) = (r+q)\frac{rV_u - z}{w_i - rV_u}, \tag{35}$$

which implicitly defines a one-to-one relationship θ_i as a function of w_i (*conditional on a given value of* V_u), say

$$\theta_i = \Theta(w_i) \text{ or more explicitly } \theta_i = \Theta(w_i; V_u)$$

Conditional on a common value in unemployment, rVu, along (35),

$$\frac{d\theta_i}{dw_i} = \frac{-\theta_i}{(w_i - rV_u)(1-\eta_i)} < 0 \tag{36}$$

where $\eta_i = \eta(\theta_i) \equiv -\frac{\theta_i m'(\theta_i)}{m(\theta_i)} \in (0,1)$.

With a Cobb-Douglas matching function, $H_i = A\ V_i^{1-\eta}\ U_i^{\eta}$, $\eta(\theta_i) \equiv \eta$, and (35) is a convex relationship in a (w_i, θ_i) space.

- According to (35), i.e.

$$\theta_i m(\theta_i) = (r+q)\frac{rV_u - z}{w_i - rV_u}$$

  guaranteeing V_u to all job seekers can be achieved in two ways on a given sub-market i.

  Loosely speaking:

  - Either, a high wage w_i is announced in market i. As such, this attracts many applicants. Then, at a (provisionally) given number of vacant positions V_i, the chances to recruit rapidly a worker will be high.
  - Or, a low wage is posted in market i, attracting few job seekers. Then however, the chances to recruit rapidly a worker are reduced (still at a provisionally given number of vacancies).

- Along (35), as $w_i \overset{>}{\to} rV_u$, $\theta_i \to +\infty$.

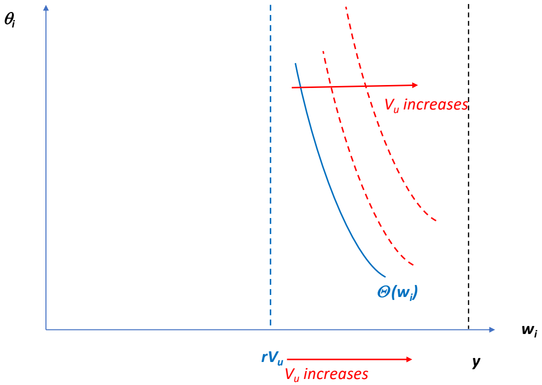


Figure: The iso-relationhip (35) conditional on rV_u.

Wage posting

For a given number U_i of job seekers, employers located in sub-market i choose a wage offer so as to maximize the expected gain of a vacant position Π_{vi} subject to constraint (35).

◊ First, how to write Π_{vi}?

The expected discounted gains of a vacant (resp. a filled) position, Π_{vi} (resp. Π_{ei}) in sub-market i verify:

$$\begin{aligned} r\Pi_{vi} &= -h + m(\theta_i)\left(\Pi_{ei} - \Pi_{vi}\right) \\ r\Pi_{ei} &= y - w_i + q\left(\Pi_{vi} - \Pi_{ei}\right) \Rightarrow \Pi_{ei} = \frac{y - w_i + q\Pi_{vi}}{r+q} \end{aligned}$$

Plugging the last expression into the first Bellman equation and rearranging least to:

$$r\Pi_{vi} = \frac{-h(r+q) + m(\theta_i)(y - w_i)}{r + q + m(\theta_i)} \tag{37}$$

Along an "iso-expected gain" $\Pi_{vi} = \overline{\Pi}$ (i.e. a constant), there is here also a trade-off between creating a lot of vacancies and offering high (take-it-or-leave-it) wage offers. For, from (37)

$$\frac{d\theta_i}{dw_i} < 0.$$

With a Cobb-Douglas matching function, (37) is concave.

As $w_i \to y$, notice that $r\Pi_{vi}$ becomes negative.

The following graph summarizes the theoretical framework:

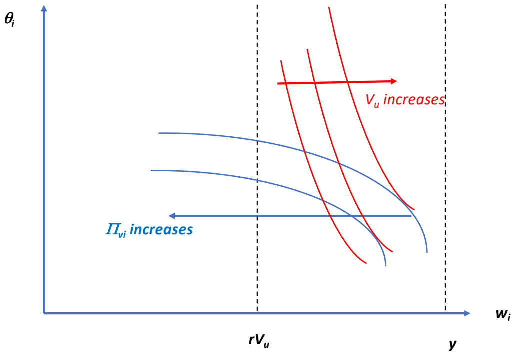


Figure: Charts of iso-relationhips defined by resp. (35) in red, and (37) in blue.

$\diamondsuit$ Now, we return to the employers' maximization on each island i. They choose the announced wage w_i to maximize the value of opening a vacancy (37) subject to *the assumed knowledge of* (35), i.e. to

$$\theta_i = \Theta(w_i).$$

for a given lifetime value in unemployment V_u.
The slope of this relationship is given by (36).

Hence, the maximization problem on sub-market i can be written as:

$$\max_{w_i} \frac{-h(r+q) + m(\Theta(w_i))(y - w_i)}{r + q + m(\Theta(w_i))} \tag{38}$$

This maximization expresses that, given a certain level of V_u, the pair (w_i, θ_i) is a point where the highest "iso-expected gain" will be tangent to (35).

Closing the model to pin down V_u

- Under free entry, in each active market i, one has

$$\Pi_{vi} = 0 \Leftrightarrow \frac{h}{m(\theta_i)} = \frac{y - w_i}{r + q} \tag{39}$$

  The expression on the right-hand side comes from setting the numerator of (37) to zero.
- Given the homogeneity of the sub-markets, it is easily seen that the equilibrium wage and tightness levels are independent of the index i of the sub-markets. *So, ignore this index from now on.*
- Getting rid of the i index, (35) can be rewritten as:

$$rV_u = z + \theta m(\theta) \frac{w - z}{r + q + \theta m(\theta)} \tag{40}$$

- *In sum,* w, θ and V_u solve (38), (39) and (40).
- Knowing w, θ and V_u, the levels of V and U are derived from the definition of θ, $L + U = N$ and $q(N - U) = \theta m(\theta) U$.

Illustration of the solution under free entry

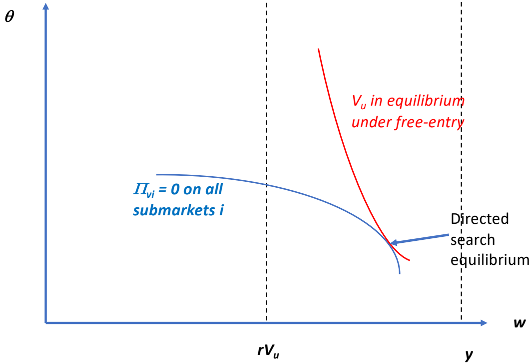


Figure: The equilibrium under free-entry of vacancies (40), in red, and (39) in blue.

Efficiency of the decentralized economy?

The FOC of the maximization problem (38) is a complex expression. However, under free-entry, $\Pi_{vi} = 0 \; \forall$ active i, this FOC becomes much simpler, namely:

$$w = rV_u + \eta(y - rV_u) \tag{41}$$

Under **random matching**, a similar expression namely (24) or

$$w = rV_u + \gamma(y - rV_u)$$

was the outcome of Nash bargaining. Remember that under random matching the workers' bargaining power γ has no reason to verify the Hosios condition.

Expression (41) is similar. However γ is replaced by η, the elasticity of $m(\theta)$ with respect to θ (in absolute value). This leads to the conclusion on the next slide...

Efficiency of the decentralized economy?

- From (41), (39) and (40), it can be checked that the level of tightness (in all active sub-markets) verifies

$$\frac{(y-z)(1-\eta)}{r+q+\eta\theta m(\theta)}=\frac{h}{m(\theta)} \tag{42}$$

which is characterizing an **efficient allocation** (see (33) derived when $r \to 0$).

- Why intuitively is the decentralized economy efficient?

> *"Models in the celebrated Diamond-Mortensen-Pissarides tradition of random search (...) tend to treat price formation as an afterthought that has to be sorted out once agents meet (...) but that cannot be used to guide individuals' search decisions..."* `https://voxeu.org/article/guide-directed-search`

Efficiency of directed or competitive search equilibrium

Intuition

> ... *"Competitive search bundles two characteristics: the terms of trade are posted by agents in advance of meetings, and these terms direct search and hence help determine who meets whom. Relative to random search, this is not only a different philosophical approach to the study of markets, but the combination of posting and directed search also alters substantive findings. In particular, posted prices give agents incentives to seek out particular counterparties. This often leads to market efficiency (it is sometimes said the models internalise search externalities)."* `https://voxeu.org/article/guide-directed-search`

For extensions and to understand why "often" in the last quote, you can refer to the overview of the literature by Wright, Kircher, Julien and Guerrieri (2021).

Note on the literature

Links to other branches of the literature

◊ The expression "job matching" is used in other contexts than the "Matching models":
Jovanovic (1979) developed a theory of separation (i.e. change of employer): when a match is formed, the quality of the match is uncertain and revealed though experience.[13]
This theory explains why the separation rate decreases with job-tenure and becomes close to constant after some point.

◊ Search-Matching frictions appear also in papers about:

- Goods and housing markets (starting with Stigler, 1961),
- Financial and money markets (e.g. Rocheteau and Wright, 2005),
- The "Marriage market" (e.g. Burdett and Coles, 1997),
- Schooling application and admission (e.g. Gale and Shapley, 1962).

[13] See Pries and Rogerson (2005) and Menzio, Telyukova and Visschers (2016) for the same idea in the framework developed in this chapter.

◊ The "assignment literature"
Studies how heterogeneous agents (say, job-seekers and job vacancies with different intrinsic productivity) find "appropriate" trade partners when there are complementarities in production.

- "Positive Assortative Matching" (PAM)
  = "better-qualified job-seekers match with better jobs"
  This property comes out in a Walrasian static matching economy (Becker, 1973). Unemployment and vacant jobs cannot coexist.
- In the presence of frictions on the labor market, unemployment and vacant jobs can coexist for all types (productivity levels).
  Despite a certain degree of PAM, there can be some **mismatch**: "some high-productivity firms are forced to hire low-productivity workers whereas some low-productivity firms are able to hire higher productivity workers." (Shimer, 2005b, p. 999).
- On how this literature is related to the evidence in Abowd and Kramarz (1999), see e.g. Lopes de Melo (2018).

Various final information

For an introduction and an overview of the assignment literature, see
`https://www.youtube.com/watch?v=dJVnjtCWs8M`

Batyra and De Vroey (2011) situate the
Diamond-Mortensen-Pissarides search and matching framework in the historical development of economic thought since Marshall.

Danthine and De Vroey (2017) discuss how the matching model was integrated in Macroeconomics.

Diamond-Mortensen-Pissarides search-matching approach has been honored by the Nobel price in 2010.
Albrecht (2011) provides a brief overview of this literature.

The website of the international network of researchers in the field is :
`http://sam.univ-lemans.fr/`

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