Robustness of Adaptive Observers for Time Varying Systems

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ABSTRACT

We study the properties of two adaptive observers/identifiers, initially proposed for linear time-invariant systems, respectively by Kreisselmeier and Lüders-Narendra, when the parameters are allowed to vary. We show the connections between these two observers, and derive upper bounds for the asymptotic tracking errors.

I. INTRODUCTION

In the 1970's both Kreisselmeier (1977) and Lüders and Narendra (1973) have proposed adaptive observers/identifiers for single-input single-output (SISO) linear time-invariant systems with unknown parameters. They both used special and different canonical state-space forms for the representation of the system. They then derived an estimation algorithm for the estimation of the states and the parameters of the system, and showed that the estimation errors converged to zero exponentially fast provided a regression vector was persistently exciting (P.E.). Here we show that both adaptive observers/identifiers can also be used when the parameters are allowed to be time-varying, and we derive asymptotic bounds on the tracking errors.

Our contributions in this short paper are as follows. First we show the connections between the respective estimators of Kreisselmeier and Lüders-Narendra. These estimators are derived from canonical forms that use a certain number of freely assigned design parameters. We show that the Lüders-Narendra canonical form can be obtained from the Kreisselmeier canonical form by a similarity transformation, provided the two sets of design parameters are related in a specific way. We propose a simplification of the Lüders and Narendra observer, which is then shown to be a reduced-order equivalent of the Kreisselmeier observer.

We extend these identifiers to the case where the full state is measured, in which case the regression vector becomes a regression matrix. We discuss the persistence of excitation conditions, in particular for the new situation where the full state is observed. We show that the sufficient richness condition on the input becomes extremely simple in this case. Finally we discuss the effect of the various design parameters on the asymptotic tracking errors.

II. THE KREISSELMEIER ADAPTIVE ESTIMATOR

We consider linear time varying systems represented in canonical observer form

$$\dot{x}(t) = A(t)x(t) + b(t)u(t) \qquad y(t) = (1 \;\; 0 \; \dots \; 0)\, x(t) = c^T x(t) \tag{2.1}$$

where

$$A(t) = \begin{bmatrix} a(t) & \begin{matrix} I_{n-1} \\ 0 \dots 0 \end{matrix} \end{bmatrix} \tag{2.2}$$

$$a^T(t) = (-a_1(t) \; \dots\dots \; -a_n(t)) \tag{2.3a}$$

$$b^T(t) = (b_1(t) \; \dots \; b_n(t)) \tag{2.3b}$$

We assume that a(t) and b(t) have bounded and continuous derivatives and that (2.1) is BIBO stable. A large family of linear time-varying systems can be transformed to (2.1) by a Lyapunov transformation.

The system (2.1) can be rewritten as

$$\begin{cases} \dot{x}(t) = Fx(t) + \Omega(t)\theta(t) \\ y(t) = c^T x(t) \end{cases} \tag{2.4}$$

where

$$F = \begin{bmatrix} -f_1 & & \\ \vdots & I_{n-1} & \\ -f_n & 0 & 0 \end{bmatrix} \tag{2.5a}$$

$$\underset{(n\times 2n)}{\Omega(t)} = \begin{bmatrix} y & & O & u & & O \\ & \ddots & & & \ddots & \\ O & & y & O & & u \end{bmatrix} \tag{2.5b}$$

and

$$\theta(t) = [f_1-a_1(t)\ldots f_n-a_n(t)b_1(t)\ldots b_n(t)]^T \quad (2.6)$$

$$= \begin{bmatrix} f - a(t) \\ b(t) \end{bmatrix}$$

Here $f = [f_1 \ldots f_n]^T$ is a vector of parameters that can be freely assigned ; they are chosen such that F is a stability matrix. Their influence on the convergence speed and the asymptotic tracking error of the parameter estimator will become apparent soon. The canonical form (2.4) was proposed in Kreisselmeier (1977) for observable SISO linear time-invariant systems, i.e. with $\theta(t) = \theta$ = constant. Suppose now that u(t) and y(t) are measured and that it is required to obtain on-line estimates of x(t) and $\theta(t)$.

Adaptive observer/identifier

Consider the following adaptive observer for (2.4):

$$\dot{\hat{x}}(t)=F\hat{x}(t)+\Omega(t)\hat{\theta}(t)\ \Psi(t)\Gamma\varphi(t)[y(t)-C^T\hat{x}(t)] \quad (2.7a)$$

$$\dot{\hat{\theta}}(t) = \Gamma\varphi(t)[y(t)-C^T\hat{x}(t)] \quad (2.7b)$$

where $\Gamma = \Gamma^T > 0$ is a gain matrix, and $\Psi(t)$ and $\varphi(t)$ are respectively the nx2n matrix state and the 2n vector output of the auxiliary filter :

$$\dot{\Psi}(t) = F\Psi(t) + \Omega(t) \quad (2.8)$$

$$\varphi^T(t) = C^T\Psi(t)$$

Notice that this is a full-order observer, i.e. the whole state x(t) is estimated, even though $C^Tx(t)$ is measured.

Simplified adaptive observer/identifier

It is very easy to check that

$$\frac{d}{dt}(\hat{x} - \Psi\hat{\theta}) = F(\hat{x}-\Psi\hat{\theta}) \quad (2.9)$$

i.e. $\hat{x}(t)$ converges exponentially fast to $\Psi(t)\hat{\theta}(t)$.
This suggests the following simplified estimator :

$$\hat{x}(t) = \Psi(t)\hat{\theta}(t) \quad (2.10a)$$

$$\dot{\hat{\theta}}(t) = \Gamma\varphi(t)[y(t) - \varphi^T(t)\hat{\theta}(t)] \quad (2.10b)$$

together with (2.8). For constant parameter systems, this was precisely the adaptive observer proposed and analyzed by Kreisselmeier (1977). We now study the robustness of this estimator to parameter variations.

Error system

The error system for both (2.7)-(2.8) or (2.10), (2.8) is

$$\begin{bmatrix} \dot{\tilde{x}} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F-\psi\Gamma\varphi C^T & \Omega \\ -\Gamma\varphi C^T & 0 \end{bmatrix} \begin{bmatrix} \tilde{x} \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} 0 \\ \dot{\theta} \end{bmatrix} \quad (2.11)$$

where $\tilde{x} = x-\hat{x}$ and $\tilde{\theta} = \theta-\hat{\theta}$. By defining $e \triangleq \tilde{x}-\Psi\tilde{\theta}$, it can be rewritten as

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F & 0 \\ -\Gamma\varphi C^T & -\Gamma\varphi\varphi^T \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} -\Psi\dot{\theta} \\ \dot{\theta} \end{bmatrix} \quad (2.12)$$

We have the following uniform boundedeness theorem.

Theorem 2.1

Assume that
A1) the system (2.1) is BIBO stable
A2) u(t) is continuous and bounded :

$$\sup_t |u(t)| \leq U < \infty$$

A3) $\sup_t |\dot{\theta}(t)| \leq M < \infty$

A4) the input u(t) is such that the regressor vector $\varphi(t)$ is persistently exciting, i.e. $\exists s_0$, T, α, β > 0 such that

$$\alpha I \leq \frac{1}{T}\int_s^{s+T} \varphi(t)\varphi^T(t)dt \leq \beta I \quad \forall s \geq s_0$$

A5) f_1, ..., f_n are chosen such that F is a stability matrix.

Then
P1) the state $\psi(t)$ of the auxiliary filter (2.8) is uniformly bounded :

$$\lim_{t\to\infty} \sup \|\psi(t)\| \leq K_1U \quad \text{for some } K_1 > 0$$

P2) the estimation errors of the estimators (2.7) or (2.10) are bounded and

$$\lim_{t\to\infty} \sup \left\|\begin{matrix}\tilde{x}(t) \\ \tilde{\theta}(t)\end{matrix}\right\| \leq C(U).M$$

where C(U) is a strictly increasing, positive function of U.

Proof : The proof follows directly from the error system in the form (2.12). See Gevers et al. (1987) for details.

We have thus shown that the Kreisselmeier observer is robust to parameter variations. This is due to the uniform asymptotic stability of the homogeneous part of (2.11). That error system has been analyzed in some detail in Mareels et al. (1987), where some robustness properties to multiplicative errors have also been demontrated.

Comment 2.1.

A crucial condition for the boundedness of the estimation errors is the PE condition A4. Since the auxiliary filter (2.8) is stable and output reachable, this condition can be translated into a sufficient richness condition on (y u) : see the form (2.5b) of the input Ω . Given that the relation between u and y is given by the linear time-varying system (2.1), this is in turn can be translated into a sufficient richness on u(t) and some bounds on the spped of parameter variations, using the results of Mareels and Gevers (1988).

III. THE LUDERS-NARENDRA ADAPTIVE ESTIMATOR

The Kreisselmeier observer was derived using the fact that any observable SISO system can be written in the canonical form (2.4) - (2.5) where f_1, ..., f_n can be freely chosen. In Lüders and Narendra (1974) it was shown that any observable SISO system can be transformed to the following canonical form where c_2, ..., c_n can be freely chosen :

$$\dot{z}(t) = Gz(t) + \Omega(t)\zeta(t)$$
$$y(t) = c^T z(t) \qquad (3.1)$$

$\zeta(t)$ is a 2n-vector of parameters, $\Omega(t)$ and c^T are as before, and

$$G = \begin{bmatrix} 0 & 1 & \ldots\ldots & 1 \\ \vdots & -c_2 & \ddots & O \\ 0 & O & & -c_n \end{bmatrix} \qquad (3.2)$$

In Lüders and Narendra, $\zeta(t)$ was of course constant. We first establish that the canonical forms (2.4)-(2.6) and (3.1)-(3.2) are related by a similarity transformation provided the following relation holds between the respective design parameters :

$$\det(sI-F) = s^n+f_1 s^{n-1}+\ldots+f_{n-1}s+f_n$$
$$=s(s+c_2)\ldots(s+c_n) = \det(sI-G) \qquad (3.3)$$

This requires in particular that $f_n = 0$. Let then Q_1, Q_2 be, respectively, the observability matrices of the pairs (c^T,F) and (c^T,G) and define

$$z = Tx, \quad T = Q_2^{-1}Q_1 \qquad (3.4)$$

Then it is easy to see that (2.4) is equivalent with (3.1) with

$$\zeta(t) = T\theta(t) \qquad (3.5)$$

Adaptive observer/identifier

For the representation (3.1), with $\zeta(t) = \zeta =$ constant, Lüders and Narendra (1974) proposed the following adaptive observer/identifier :

$$\dot{\hat{z}}(t) = G\hat{z}(t)+\Omega(t)\hat{\zeta}(t)+ \begin{bmatrix} c_1 \\ \Psi_2(t)\Gamma\varphi(t) \end{bmatrix} (y(t)-\hat{z}_1(t)) \qquad (3.6a)$$

$$\dot{\hat{\zeta}}(t) = \Gamma\varphi(t)\ (y(t)-\hat{z}_1(t)) \qquad (3.6b)$$

where c_1 is an arbitrary positive constant, $\Gamma=\Gamma^T>0$ is a gain matrix and Ψ_2 (of dimension (n-1)x2n) and φ are defined via the auxiliary filter :

$$\dot{\Psi}_2(t) = G_2\Psi_2(t) + \Omega_2(t) \qquad (3.7)$$
$$\varphi^T(t) = (1 \ \ldots\ldots \ 1)\ \Psi_2(t) + \Omega_1(t)$$

$$G_2 = -\mathrm{diag}(c_2,\ldots,c_n) \in \mathbb{R}^{(n-1)x(n-1)} \qquad (3.8)$$

$$\Omega(t) = \begin{bmatrix} \Omega_1(t) \\ \Omega_2(t) \end{bmatrix} \text{ with } \Omega_1(t)\in\mathbb{R}^{1x2n}, \Omega_2(t)\in\mathbb{R}^{(n-1)x2n} \qquad (3.9)$$

Note that the original Lüders-Narendra observer looks rather more complicated than (3.6) - (3.9), but it can be rewritten in the present form.

Simplified adaptive observer/identifier

Denote $z^T \triangleq (z_1, z_2^T)$, where $z_1(t)$ and $z_2(t)$ are, respectively, the first and the n-1 remaining components of z(t). Denote also $G_2 \triangleq \mathrm{diag}(-c_2, \ldots, -c_n)$. Then it is easy to see that

$$\frac{d}{dt}(\hat{z}_2 - \psi_2\hat{\zeta}) = G_2(\hat{z}_2 - \psi_2\hat{\zeta}) \qquad (3.10)$$

Compare with (2.9) This leads us to suggest the following simplified adaptive estimator :

$$\dot{\hat{z}}_1(t) = (1\ldots\ldots1)\hat{z}_2(t) + \Omega_1(t)\hat{\zeta}(t) + c_1(y(t) - \hat{z}_1(t)) \qquad (3.11a)$$

$$\hat{z}_2(t) = \psi_2(t)\hat{\zeta}(t) \qquad (3.11b)$$

$$\dot{\hat{\zeta}}(t) = \Gamma\varphi(t)(y(t) - \hat{z}_1(t)) \qquad (3.11c)$$

together with the auxiliary filter (3.7).

Error system

Denote $\tilde{z}(t) \triangleq z(t)-\hat{z}(t)$, $\tilde{\zeta}(t) \triangleq \zeta(t) - \hat{\zeta}(t)$ and

$$e(t) \triangleq \tilde{z}(t) - \begin{bmatrix} 0 \\ \psi_2(t)\hat{\zeta}(t) \end{bmatrix} \qquad (3.12)$$

We then have the following error system for both (3.6) - (3.9) and (3.7) - (3.11) :

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\zeta}} \end{bmatrix} = \left[\begin{array}{c|c} G_1 & \begin{matrix}\varphi^T \\ 0_{(n-1)x2n}\end{matrix} \\ \hline -\Gamma\varphi \quad 0_{2nx(n-1)} & 0_{2nx2n} \end{array}\right] \begin{bmatrix} e \\ \tilde{\zeta} \end{bmatrix} + \begin{bmatrix} 0 \\ -\psi_2 \\ \hline I \end{bmatrix}\dot{\zeta} \qquad (3.13)$$

where

$$G_1 = \begin{bmatrix} -c_1 & 1\ldots\ldots\ldots1 \\ & -c_2 \\ & & \ddots & -c_n \end{bmatrix} \qquad (3.14)$$

The following Theorem shows that the Lüders-Narendra estimator (3.6) - (3.9) and its simplified form (3.11) and (3.7) are robust w.r.t. parameter variations.

Theorem 3.1.

Assume that
B1) the system (3.1) is BIBO stable
B2) u(t) has continuous and bounded derivatives and

$$\sup_t |u(t)| \leq U < \infty$$

B3) $\sup_t |\dot{\zeta}(t)| \leq M < \infty$

B4) A4 holds
B5) $c_1, c_2, \ldots, c_n$ are all positive and different.

Then:

P1′ : $\lim_{t\to\infty} \sup \|\psi_2(t)\| \leq K_1 U$ for some $K_1 > 0$

P2′ : $\lim_{t\to\infty} \sup \left\| \begin{matrix}\tilde{z}(t) \\ \tilde{\zeta}(t)\end{matrix} \right\| \leq C(U)M$

where C(U) is a strictly increasing positive fuction of U.

Proof : See Bastin and Gevers (1988).

Comment 3.1.

Neglecting exponentially decaying terms due to initial conditions, the homogeneous part of the error system (3.13) can be described in shorthand notation as

$$\dot{\tilde{\zeta}}(t) = -\Gamma\varphi(t)\{\frac{1}{s+c_1}(\varphi^T\tilde{\zeta})\}(t) \qquad (3.15)$$

Comment 3.2.

The regressor $\varphi(t)$ has a very simple form

$$\varphi^T = (y \ \frac{1}{s+c_2} y \ \dots \ \frac{1}{s+c_n} y \quad u \quad \frac{1}{s+c_2} u \ \dots \ \frac{1}{s+c_n} u) \tag{3.16}$$

Given that the relation between u and y is linear time-varying, the PE condition A4 can again be translated into a sufficient richness condition on u(t) provided the parameter variations are sufficiently slow or integral small : see Mareels and Gevers (1988) for details.

IV. THE CASE OF MATRIX REGRESSORS

In some applications, it is desired to construct an adaptive parameter estimator for $\theta(t)$ where $\theta(t)$ is related to a fully observed state vector x(t) by the following model

$$\dot{x}(t) = Fx(t) + \Omega(x,u,t)\theta(t) \tag{4.1}$$

where F is any constant known nxn matrix, $\Omega(x,u,t)$ is a nxp matrix of known functions of the measured state x(t) and an external signal u(t). In analogy with the adaptive estimtor (2.10), (2.8), we then propose the following adaptive observer/identifier :

$$\dot{\hat{\theta}}(t) = \Gamma\psi^T(t)(x - \psi(t)\hat{\theta}(t)) \tag{4.2}$$

with the auxiliary filter :

$$\dot{\psi}(t) = F\psi(t) + \Omega(x,u,t) \tag{4.3}$$

Denote : $\tilde{x} \triangleq x-\hat{x}$, $\tilde{\theta} \triangleq \theta - \hat{\theta}$ and $e = \tilde{x} - \psi\tilde{\theta}$. We then have the following error system :

$$\begin{bmatrix} \dot{e} \\ \dot{\tilde{\theta}} \end{bmatrix} = \begin{bmatrix} F & 0 \\ -\Gamma\psi^T & -\Gamma\psi^T\psi \end{bmatrix} \begin{bmatrix} e \\ \tilde{\theta} \end{bmatrix} + \begin{bmatrix} -\psi \\ I \end{bmatrix} \dot{\theta} \tag{4.4}$$

The tracking properties of the estimator (4.2) - (4.3) are given by the following theorem.

Theorem 4.1.

Assume that
C1) F is a stability matrix
C2) $\sup_t \|\Omega(x,u,t)\| \leq U < \infty$
C3) $\sup_t |\dot{\theta}(t)| \leq M < \infty$
c4) $\Omega(x,u,t)$ is such that $\exists s_0, T, \alpha, \beta > 0$ for which

$$\alpha I \leq \frac{1}{T} \int_s^{s+T} \psi^T(t)\psi(t)dt \leq \beta I \quad \forall s \geq s_0$$

Then the estimation error $\|\tilde{\theta}(t)\|$ is bounded and

$$\limsup_{t\to\infty} \|\tilde{\theta}(t)\| \leq C(U)M$$

where C(U) is a strictly increasing positive function of U.

Proof : Conditions C1 and C2 imply the boundedness of ψ. The proof then follows directly from the structure of (4.4).

We now examine the persistence of excitation condition C4 more closely and translate it into a sufficient richness condition on $\Omega(t)$.

Theorem 4.2

The matrix regressor $\psi(t)$ is PE in the sense of C4 if $\Omega(x,u,t)$ is such that there exist s_1, T_1, α_1 and $\beta_1 > 0$ for which

$$\alpha_1 I \leq \frac{1}{T_1} \int_s^{s+T_1} Z^T Z \, dt \leq \beta_1 I \quad \forall s \geq s_1 \tag{4.5}$$

where

$$\dot{Z} = \gamma Z + \Omega \quad \text{for any } \gamma > 0 \tag{4.6}$$

Proof :Condition C4 can equivalently be stated as:

$\forall c \in \mathbb{R}^p$ with $\|c\| = 1$,

$$\alpha \leq \frac{1}{T} \int_s^{s+T} c^T\psi^T\psi c \, dt \leq \beta \quad \forall s \geq s_0 \tag{4.7}$$

Define $\psi c = x$ and $\Omega c = u$. Then by (4.3)

$$\dot{x} = Fx + u \text{ with } x,u \in \mathbb{R}^n \tag{4.8}$$

Similarly define

$$y = \gamma y + u \quad \text{with } y,\ u \in \mathbb{R}^n \tag{4.9}$$

Conditions (4.5) - (4.6) imply that

$$\alpha_1 \leq \frac{1}{T_1} \int_s^{s+T_1} \|y(t)\|^2 dt \leq \beta_1 \tag{4.10}$$

The required conditon on $\|x(t)\|$, and hence on $\psi^T\psi$, follows by using the swapping lemma of Mareels and Gevers (1988).

V. DISCCUSION OF THE ASYMPTOTIC BOUNDS.

We briefly discuss the influence of the design parameters $f_1, \dots, f_n$ and Γ on the asymptotic tracking error bounds for the estimator (2.10), (2.8). For simplicity we shall assume that $\Gamma = \gamma I$. From the block-triangular structure of (2.12), we can write, using A3, P1 and $\|e^{Ft}\| \leq K_2 e^{-at}$,

$$\limsup_{t\to\infty} \|e(t)\| \leq \frac{K_1 K_2}{a} MU \tag{5.1}$$

The second part of (2.12) can be rewritten as

$$\dot{\tilde{\theta}}(t) = -\gamma\varphi(t)\varphi^T(t)\tilde{\theta}(t) + v(t) \tag{5.2}$$

$$\limsup_{t\to\infty} \|v(t)\| \leq (1 + \frac{\gamma}{a} K_1^2 K_2 \|C\|^2 U^2)M \tag{5.3}$$

Using asymptotic upper bounds derived by Sondhi and Mitra (1976) for equations of the form (5.2) we obtain

$$\limsup_{t\to\infty} \|\tilde{\theta}(t)\| \leq (1 + \frac{\gamma}{a} K_1^2 K_2 \|C\|^2 U^2)M \frac{T}{1-e^{-bT}} \tag{5.4}$$

where

$$b = -\frac{1}{2T} \ln(1-\rho) \tag{5.5}$$

$$\rho = \frac{2\alpha\gamma T}{1+\gamma\beta T + \frac{1}{2}\gamma^2\beta^2 T^2} \tag{5.6}$$

Here α, β and T are as defined in the PE condition A4, while γ is the parameter adaptation gain.

Discussion

1) The expression (5.4) shows that the upper bound on the tracking error is proportional to U^2 and M, and inversely proportional to a, with -a being the largest eigenvalue of F. This is to be expected. However, the dependence on the total eigenstructure of F is less clear, since the eigenstructure of F also effects $K_2 K_1^2$. We shall return to this below.
2) The dependence on the adaptation gain γ is rather complicated. Sondhi and Mitra (1976) show that, for fixed parameters α, β and T of the PE condition, there is an optimum value of γ^* for the factor

$$\frac{T}{1-e^{-bT}}.$$

Notice that, since γ also appears in the other factor of (5.4), the optimum adaptation gain will be smaller than γ^*.
3) The use of (5.4) for the choice of the design parameters of F is made complicated by the very complex dependence of K_2 (and hence K_1, which is proportional to K_2) on the eigenvalues of F. For a second order system

$$\begin{bmatrix} \dot{e}_1 \\ \dot{e}_2 \end{bmatrix} = \begin{bmatrix} -f_1 & 1 \\ -f_2 & 0 \end{bmatrix} \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} + \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}$$

with $|u_1(t)| \leqslant U_1$, $|u_2(t)| \leqslant U_2$, and $s^2 + f_1 s + f_2 = (s - \lambda_1)(s - \lambda_2)$, $\lambda_2 < \lambda_1 < 0$, the following bounds can be computed (see Dochain (1986)) :

$$\lim_{t\to\infty} \sup|e_1(t)| \leqslant \frac{2U_1}{\lambda_1-\lambda_2}\left[\left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_1}{\lambda_1-\lambda_2}} - \left(\frac{\lambda_1}{\lambda_2}\right)^{\frac{\lambda_2}{\lambda_1-\lambda_2}}\right] + \frac{U_2}{\lambda_1\lambda_2}$$

$$\lim_{t\to\infty} \sup|e_2(t)| \leqslant U_1 - \frac{\lambda_1+\lambda_2}{\lambda_1\lambda_2} U_2$$

The authors know of no similar expressions for higher order systems. The optimization of these bounds w.r.t. $\lambda_1 = a$ and λ_2, even in this simple case, is not straightforward.

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