

ADAPTIVE CONTROL OF NON-LINEAR BACTERIAL GROWTH SYSTEMS

G. Bastin, D. Dochain and M. Installe

*Laboratoire d'Automatique et d'Analyse des Systèmes, Université de Louvain, Bâtiment Maxwell,
B-1348 Louvain-la-Neuve, Belgium*

Abstract. This paper suggests how nonlinear adaptive control of non linear bacterial growth systems could be performed. The process is described by a time varying model (linear in the parameters) obtained using usual material balance equations. It does not require any specific analytic description of the bacterial growth rate. The parameters (which have a clear physical meaning) are identified in real time using a standard RLS algorithm. This parameter estimation algorithm is then combined with a Clarke-Gawthrop controller to obtain an adaptive controller. The special case of biomethanization (involving anaerobic waste water treatment) is analyzed. Three different control problems are considered : depollution control, methane gas production control and wash-out control. For each of these cases an adaptive control algorithm is proposed and its effectiveness is shown by stimulation experiments.

Keywords. Modeling of biochemical process, non linear systems, self tuning adaptive control.

0. INTRODUCTION

A commonly used approach for the adaptive control of non linear systems is to consider them as time varying linear systems and to use black-box linear approximate models to implement the control law. This approach has been used by the authors in previous works on the control of fermentation processes [1,4]. But, since the underlying process is non linear, improved control can be expected by exploiting the non linear structure of the model. Such an idea is pursued in the present paper : we suggest how non linear adaptive control can be performed for the control of non linear bacterial growth systems. A similar idea has been recently used for the adaptive dissolved oxygen control in waste water treatment [12], but under a somewhat different form than in the present paper.

The lay out of the paper is as follows. In section 1, the process is described by a non linear state-space time varying model obtained from usual mass balance equations. This model does not require any specific analytical description of the bacterial growth-rate. In section 2, the (physical) parameters are identified in real time with a standard RLS algorithm. Then we specialize to the analysis of biomethanization plants; the parameter estimation algorithm is combined with Clarke-Gawthrop controllers to obtain adaptive controllers in three different cases : depollution control (section 3), methane gas production control (section 4) and wash-out control (section 5).

1. DESCRIPTION OF THE MODEL

We consider the usual state-space representation of bacterial growth systems by mass balance equations :

$$\dot{X}(t) = (\mu(t) - U(t)) X(t)$$

$$\dot{S}(t) = -k_1 \mu(t) X(t) + U(t)(V(t) - S(t)) \quad (1)$$

$$Y(t) = k_2 \mu(t) X(t)$$

where X is the bacterial concentration
 S and V are the inner and input substrates
 U is the dilution rate
 μ is the growth rate
 k_1 and k_2 are yield coefficients
 Y is the product of the reaction.

In order to facilitate the physical interpretation of the later discussions, we shall specialize to the biomethanization anaerobic waste treatment process [1],[4] where:

$V(t)$ is the input organic load
(i.e. input pollution level).
 $S(t)$ is the output pollution level
 $Y(t)$ is a methane gas flow rate.

but, obviously, the discussion can also apply to other biochemical processes with the same structure.

We could think of adopting an analytical expression for the bacterial growth rate $\mu(t)$ - The most popular expression is certainly the Monod law:

$$\text{Monod: } \mu(t) = \frac{\hat{\mu} S}{K_m + S}$$

but many other expressions have been suggested like :

$$\text{Blackman: } \mu(t) = \frac{\hat{\mu} S}{K_b} \quad S \leq K_b;$$

$$\mu(t) = \hat{\mu} \quad S \leq K_b$$

$$\text{Contois: } \mu(t) = \frac{\hat{\mu} S}{K_c X + S}$$

$$\text{Haldane: } \mu(t) = \frac{\hat{\mu} K_o S}{1 + K_1 S + K_2 S^2}$$

The choice of an appropriate model for $\mu(t)$ is far from being an easy task and is the matter of continuing research (e.g.[6]: Spriet[5] lists nine different models for $\mu(t)$ which have been proposed in the literature, without even mentioning those which involve inhibitions like the Haldane model.

Furthermore, it is well known that important identifiability difficulties occur when estimating the parameters ($\hat{\mu}$ and K_m or K_b or $K_c \dots$) from real life data (e.g.[2],[3],[4]).

Therefore we prefer to "short-circuit" the problem of this choice and to identify the parameter $\mu(t)$ in real time.

Throughout the paper, we shall assume that: the dilution rate $U(t)$ is the control input

the organic load $V(t)$ is an external measurable disturbance input

the output pollution level $S(t)$ is measurable

the output methane gas flowrate $y(t)$ is measurable.

2. ON LINE PARAMETER ESTIMATION

Using a first-order Euler approximation for $X(t)$ and $S(t)$, with a sampling period T , we have:

$$\begin{aligned} X_{t+1} &\approx X_t + T\mu_t X_t - TU_t X_t \\ S_{t+1} &\approx S_t - Tk_1 \mu_t X_t + TU_t (V_t - S_t) \\ Y_t &= k_2 \mu_t X_t \end{aligned} \quad (2)$$

We make the following approximation:

$$Y_{t+1} - Y_t \approx k_2 \mu_t (X_{t+1} - X_t) \quad (3)$$

Then, substituting for X_t and X_{t+1} from (3) into (2), we have:

$$Y_{t+1} = a_t Y_t - T U_t Y_t + \varepsilon_{t+1}^1 \quad (4)$$

$$S_{t+1} = k_t Y_t + S_t + T U_t (V_t - S_t) + \varepsilon_{t+1}^2 \quad (5)$$

with $a_t = 1 + T\mu_t$

$$k_t = -T \frac{k_1}{k_2}$$

A time varying parameter k_t is considered in order to allow for parameter variations "due to unobservable physiological or genetic events" [2].

ε_{t+1}^1 and ε_{t+1}^2 represent errors due to noise, discretization and approximation(3).

Equations(4) and (5) constitute the basic model for the parameter estimation and the adaptive control.

Since the basic model is linear in the parameters a_t and k_t , recursive least-squares estimates can be readily obtained([1]) :

$$\hat{a}_{t+1} = \hat{a}_t + \sigma_t P_t Y_t [Y_{t+1} + TU_t Y_t - \hat{a}_t Y_t] \quad (5)$$

$$\hat{k}_{t+1} = \hat{k}_t + \sigma_t P_t Y_t [S_{t+1} - S_t - TU_t (V_t - S_t) - \hat{k}_t Y_t] \quad (6)$$

$$P_t = P_{t-1} \left[1 - \sigma_t \frac{(\gamma-1)(\gamma+Y_t^2 P_{t-1}) + Y_t^2 P_{t-1}}{\gamma(\gamma+\sigma_t Y_t^2 P_{t-1})} \right]$$

$$0 < \gamma \leq 1 \quad \sigma_t = 0 \text{ or } 1 \quad P_o \gg 0 \quad (7)$$

$$P_t = P_{t-1} \text{ if } \sigma_t = 0.$$

γ is the forgetting factor to allow the tracking of time varying parameters and σ_t (equal to 0 or 1) is a switching coefficient to hold the parameter estimates constant whenever the prediction errors $Y_t - Y_t$ or $S_t - S_t$ become smaller than a prespecified bound (see[1]for the details).

Notice that the estimation of both parameters is decoupled but with a common gain P_t .

3. DEPOLLUTION CONTROL

The aim of the control is to regulate the output pollution level S_t at a prescribed (usually low) level S^* despite the disturbance input V_t , by acting on the dilution rate U_t .

Notice that it is not interesting to reach levels lower than S^* since they would necessarily correspond to unusefully low treatment rates.

A discrete-time Clarke-Gawthrop controller [9], with a dynamic control weight $\lambda(1-z^{-1})$ in the performance index [10], is considered. At each sampling time, the control input U_t is computed by minimizing the criterion :

$$J_1 = (S_{t+1} - S^*)^2 + \lambda^2 (U_t - U_{t-1})^2 \quad (8)$$

where \hat{S}_{t+1} is a prediction of the output pollution level.

λ is a free parameter
From the basic model(S), it is natural to define \hat{S}_{t+1} as follows:

$$\hat{S}_{t+1} = \hat{k}_t Y_t + S_t + TU_t(V_t - S_t)$$

The non linear control law is readily obtained since \hat{S}_{t+1} is linear in U_t :

$$U_t = \frac{1}{T^2(V_t - S_t)^2 + \lambda^2} [T(V_t - S_t)(S_t^* - S_t - \hat{k}_t Y_t) + \lambda^2 U_{t-1}] \quad (9)$$

A block diagram of the closed loop system is presented in fig. 1.

4. METHANE GAS PRODUCTION CONTROL.

The biomethanization process can be viewed as an energy conversion process. An amount of "organic" energy is available in the influent under the form of the input organic load $V(t)$. This energy is converted into methane gas $Y(t)$ by the reactor. Obviously, the output energy $Y(t)$ cannot, in the mean, be larger than the available input energy. When the aim of the plant is not depollution but energy production (as in industrial farms) the control objective is to continuously adapt the output production $Y(t)$ to the available input load $V(t)$. Therefore, the desired gas production $Y^*(t)$ is defined as follows:

$$Y^*(t+1) = \beta V(t) - \beta_0 \quad \beta > 0 \quad \beta_0 > 0$$

The coefficients β and β_0 have to be selected carefully by the user since if, by lack of knowledge, β is chosen too large or β_0 too small (i.e. if we require from the fermentor more methane gas than it can actually provide) then the process can be driven by the controller to a wash-out steady state.[7], i.e. to a state where the bacterial life has completely disappeared and where the reactor is definitely stopped.*

The control objective is thus to bring $Y(t)$ to follow the set-point $Y^*(t) = \beta V(t) - \beta_0$: it is a kind of load tracking.

As in the previous section, a Clarke-Gawthrop controller is used.

U_t is chosen to minimize the criterion:

$$J_2 = (\hat{Y}_{t+1} - Y_{t+1}^*)^2 + \lambda^2 (U_t - U_{t-1})^2 \quad (11)$$

By equation (4), we have:

$$\hat{Y}_{t+1} = \hat{a}_t Y_t - T U_t Y_t$$

(*) Further details on wash-out steady-states together with a steady-state analysis and a stability analysis of the biomethanization process can be found in ref.[7] and [8].

and then :

$$U_t = \frac{1}{T^2 Y_t^2 + \lambda^2} [-T Y_t (\beta V_t - \beta_0 - \hat{a}_t Y_t) + \lambda^2 U_{t-1}] \quad (12)$$

A block diagram of the closed-loop is shown in fig. 2.

5. PREVENTING A WASH-OUT

An imperative requirement, for any control scheme of continuous biomethanization processes, is to prevent a wash-out of the plant. Wash-out steady-states occur when, for a fixed dilution rate U , the input organic load drops below a critical level and becomes insufficient to maintain the bacterial life. In such a case, the only efficient action is to quickly decrease the dilution rate U_t . A wash-out steady state is characterized by the following steady-state values:

$$X = 0 \quad Y = 0 \quad S = V \quad (13)$$

These expressions suggest that prevention from wash-out can be performed by monitoring $(V_t - S_t)$: if $(V_t - S_t)$ becomes smaller than a prespecified bound:

$$\frac{V_t - S_t}{V_t} \leq C \quad 0 < C < 1 \quad (14)$$

then the Clarke-Gawthrop controller is disconnected and replaced by :

$$U_{t+1} = \alpha U_t \quad 0 < \alpha < 1 \quad (15)$$

which ensures a quick decrease of dilution rate U_t .

Obviously, the Clarke-Gawthrop controller is reconnected whenever $V_t - S_t > C V_t$.

6. CONCLUSIONS

Simple adaptive controllers for a class of bacterial growth systems have been proposed. Their effectiveness has been demonstrated by simulation experiments which will be shown during the presentation at the workshop.

An advantage of the non linear control approach of this paper is that the identified parameters correspond clearly to physical parameters (namely growth rate and yield coefficient): therefore they can provide useful information, in real time, on the state of the biomass.

Although the model(1) is well suited for industrial applications like most treatment in sugar industries where the organic load V_t is acetic acid, in many other applications, the model (1) is only the last stage of a complex multi-stage reaction: a typical situation is a five-state twelve-parameter model (e.g.[1,4] describing a sequence of three reactions (solubilization,

acidification, methanization). This is a further reason to explore the possibility of simple control schemes for the different stages of such high-order highly non-linear systems.

8. REFERENCES

- [1] BASTIN, G., DOCHAIN D., HAEST M., INSTALLE M., OPDENACKER P. (1982)
Modelling and adaptive control of a continuous anaerobic fermentation process. Proc. IFAC Workshop on Modelling and Control of Biotechnical Processes, Helsinki, Finland, August 17-19, 1982.
- [2] HOLMBERG A. and RANTA J. (1982)
Procedures for Parameter and State Estimation of Microbial Growth Process Models. Automatica, Vol. 13, N°2, pp. 181-193, 1982.
- [3] HOLMBERG A. (1982)
On the accuracy of estimating the parameters of models containing Michaelis-Menten type nonlinearities. Proc. IFIP Working Conference on Modelling and Data Analysis in Biotechnology and Medical Engineering. University of Ghent, Belgium, August 31-September 2, 1982.
- [4] BASTIN G., DOCHAIN D., HAEST M., INSTALLE M., OPDENACKER P. (1982)
Identification and Adaptive Control of a Biomethanization process. Proc. IFIP Working Conference on Modelling and Data Analysis in Biotechnology and Medical Engineering -University of Ghent Belgium, August 31-September 2, 1982.
- [5] SPRIET J.A. (1982)
Modelling of the growth of micro-organisms: a critical appraisal. in "Environmental Systems Analysis and Management". Rinaldi Ed., North-Holland Publ. Cy., 1982.
- [6] ROQUES H., YVE S., SAIPANICH S., CAPDEVILLE B. (1982)
Is Monod's approach adequate for the modelisation of purification processes using biological treatment? Water Resources, Vol. 16, pp.839-847, 1982.
- [7] ANTUNES S. and INSTALLE M. (1982)
The use of phase-plane analysis in the modelling and the control of a biomethanization process. Proc. VIIIth IFAC World Congress, Kyoto, Japan, Vol. XXII, pp.165-170, August 1981.
- [8] VAN DEN HEUVEL J.C. and ZOETMEYER R.J. (1982)
Stability of the Methane Reactor: A simple model including substrate inhibition and cell recycle. Process Biochemistry, May-June 1982, pp 14-19.
- [9] CLARKE D.W. and GAWTHROP P.J. (1979)
Self-Tuning Control. Proc. IEE, 126, N°6, pp.633-640, June 1979.
- [10] BELANGER P.R. (1983)
On type I Systems and the Clarke-Gawthrop Regulator. Automatica, vol. 19, N°1, pp. 91-94, 1983.
- [11] GOODWIN G.C. and SIN K.S. (1983)
Adaptive Filtering, Prediction and Control. Department of Electrical and Computer Engineering - Newcastle- Australia-1980. To be published : Prentice Hall, 1983.
- [12] KO. K.Y., Mc INNIS B.C. and GOODWIN G.C. (1982)
Adaptive control and Identification of the Dissolved Oxygen Process. Automatica, vol. 18. N°6, pp. 727-730, 1982.
- [13] GOODWIN G.C., LONG R.S. and Mc INNIS B.C. Adaptive Control of bilinear Systems. Technical Report EE8017, August 1980, University of Newcastle, Australia.
- [14] BASTIN G.
Adaptive Control of Bacterial growth Systems. Laboratoires d'Automatique, Université de Louvain, Belgium, April 1983.

APPENDIX

CONVERGENCE OF THE DEPOLLUTION CONTROL ALGORITHM.

- In case of - minimum variance control (i.e. $\lambda=0$)
- constant input pollution level ($V(t)=\bar{V}$)
 - RLS estimation with forgetting factor $\gamma=1$

The convergence of the depollution control algorithm can be proved. We give only the main steps of the proof. A detailed demonstration can be found in a workpaper [4].

STEP 1. BIBO stability of the process.

Assume that 1) $0 \leq U(t) \leq U_{\max}$ $t \geq 0$

- 2) The growth-rate μ is a function of S with:
- $\mu(S)=0$ if $S=0$
 - $0 \leq \mu(S) \leq \hat{\mu}$ for all $S \geq 0$
 - $0 < \frac{d\mu}{dS} < C_2 < \infty$ for all $S \geq 0$
 - $0 < C_3 < \frac{d\mu}{dS} < \infty$ for $S=0$.

(Notice that the Blackman, Monod and Haldane models presented in section 1 all fulfill these conditions).

Then if $0 \leq S(0) \leq \bar{V}$ and $0 \leq Y(0) \leq Y_{\max}$
 $S(t)$ and $Y(t), t \geq 0$, are bounded as follows:

$$0 \leq S(t) \leq \bar{V} \quad 0 \leq Y(t) \leq Y_{\max} \quad \text{with}$$

$$Y_{\max} = \frac{k_2}{k_1} \left[U_{\max} \bar{V} + \frac{\hat{u}^2}{C_2} \right]$$

STEP 2. Convergence of the parameter estimation algorithm

Assume 1) the parameter estimation algorithm for k_t (section 2) with $\gamma=1$.

2) ε_{t+1}^2 in (S) represents the discretization error and : $|\varepsilon_{t+1}^2| \leq \Delta; t \geq 0$

3) $\sigma_t = 1$ if $\frac{[S_{t+1} - \hat{S}_{t+1}]^2}{1 + \sigma_t Y_{\max}^2 P_{t-1}} > \Delta^2$, $\sigma_t = 0$ otherwise

Then $\limsup_{t \rightarrow \infty} |S_t - \hat{S}_t| \leq (1 + Y_{\max}^2 P_o) \Delta$

STEP 3. Convergence of the control algorithm

Assume 1) the depollution control algorithm of section 3 with $\lambda = 0$

2) $V - S(t) > 0 \quad t \geq 0$

3) The control law:
$$U_o(t) = \frac{S^* - S_t - k_t Y_t}{T(V_t - S_t)}$$

$U_t = U_o(t)$ if $0 \leq U_o(t) \leq U_{\max}$

$U_t = 0$ if $U_o(t) < 0$

$U_t = U_{\max}$ if $U_o(t) > U_{\max}$

Then: $\limsup |S_t - S^*| \leq (1 + Y_{\max}^2 P_o) \Delta$.

The proof of Step 2 is similar to that used by Goodwin and al. for the control of bilinear systems [13]. The proof of Steps 1 and 3 is established by exploiting the particular structure of the system.