NON LINEAR ADAPTIVE CONTROL ALGORITHMS FOR FERMENTATION PROCESSES

G.BASTIN* and D.DOCHAIN°

*Laboratoire d'Automatique, Dynamique et Analyse des Systèmes
Catholic University of Louvain
Bâtiment Maxwell, Place du Levant,1348 LOUVAIN LA NEUVE (BELGIUM)
[Member of Greco-Sarta (CNRS-France)]

°Ecole Polytechnique de Montreal, Case Postale 6079 (Succ.A)
MONTREAL H3C 3A7, CANADA

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ABSTRACT. Adaptive linearizing control algorithms for fermentation processes are proposed. They are valid in continuous as well as in discrete time.The design is based on non linear state space models which are linearly parameterized. The convergence of the proposed algorithms is analysed in the discrete time case. Two practical examples (anaerobic digestion and ethanolic fermentation) are used to illustrate the approach.

0. INTRODUCTION

A commonly used approach for the adaptive control of non linear systems is to consider them as time varying linear systems and to use black-box linear approximate models to implement the control law. But, since the underlying process is non linear, improved control is to be expected from exploiting the non linear structure for designing the control law. The aim of this paper is precisely to present " non linear adaptive controllers" for non linear fermentation processes. The processes are described by non linear state space models obtained from usual mass balance equations (section 3). The specific reaction rates are assumed to be completely unknown time varying parameters. These models are state feedback linearizable (e.g. [1],[2]) and linearly parameterised. Two different situations are considered: in the first one (section 4), the control input is the dilution rate and a specific process structure allows for a simplified design of the adaptive control algorithm; the second one (section 5) involves more general process models and the control input is the influent substrate concentration. In both cases, the parameters are estimated in real time by an un-normalized recursive least square algorithm which is combined with feedback linearization to design linearizing adaptive controllers whose asymptotic convergence is theoretically analyzed.. Two practical examples of fermentation processes (anaerobic digestion and ethanolic fermentation) are used as a matter of illustration. They are briefly presented in the next section.

I. EXAMPLES

In this section we present two mathematical models of fermentation processes dynamics which will be used throughout the paper in order to illustrate the theory.

Example 1 : The anaerobic digestion process

The anaerobic digestion of solubilized organic substrates is commonly considered as a two-phase process : acidifcation and methanization (e.g. [3],[4]). In the acidification phase the organic substrate is fermented into volatil acids and carbon dioxyde by a group of acidogenic bacteria. In the methanization phase, the volatil acids are converted into methane (CH4) and carbon dioxyde (CO2) by a group of methanigenic bacteria. The dynamics of this process in a continuous stirred tank reactor is as follows :

$$\dot{X}_1 = \mu_1 X_1 - DX_1 \quad (1.a)$$

$$\dot{S}_1 = -k_1\mu_1 X_1 - DS_1 + DS_{in} \quad (1.b)$$

$$\dot{X}_2 = \mu_2 X_2 - DX_2 \quad (1.c)$$

$$\dot{S}_2 = -k_2\mu_2 X_2 - DS_2 + k_3\mu_1 X_1 \quad (1.d)$$

$$Q_1 = k_4\mu_1 X_1 + k_5\mu_2 X_2 \quad (1.e)$$

$$Q_2 = k_6\mu_2 X_2 \quad (1.f)$$

with : X_1 the acidogenic biomass concentration, X_2 the methanigenic biomass concentration, S_1 the organic substrate concentration, S_2 the volatil acids concentration, Q_1 the CO2 gas flow rate, Q_2 the CH4 gas flow rate, μ_1 and μ_2 the specific growth rates, k_1 to k_6 the yield coefficients, D the dilution rate, S_{in} the influent substrate concentration.

Example 2 : The ethanolic fermentation process

A plausible and commonly used model of the growth of yeasts (e.g. saccharomyces cerevisiae) on glucose with ethanol production in a fed-batch stirred tank reactor is as follows (e.g. [5]) :

$$\dot{X} = \mu X - DX \quad (2.a)$$

$$\dot{P} = \nu X - DP \quad (2.b)$$

$$\dot{S} = -k_1\mu X - k_2\nu X - DS + DS_{in} \quad (2.c)$$

$$Q = k_3\mu X + k_4\nu X \quad (2.d)$$

with : X the yeasts concentration, P the ethanol concentration, S the glucose concentration, Q the CO2 gas flow rate, μ the specific growth rate, ν the specific production rate, k_1 to k_4 the yield coefficients, D the dilution rate, S_{in} the influent substrate concentration.

The dissolved oxygen dynamics, which is not useful for our subsequent derivations, is not explicitly formulated in this model though it is obviously critical for the process efficiency. Therefore it must be clear that the specific rates μ and ν are assumed to depend on the dissolved oxygen concentration in a suitable (though implicit and unknown) way.

II. A GENERAL MODEL FORMULATION

Most often, the dynamics of biotechnological processes in stirred tank bioreactors operating in batch, fed-batch or continuous mode, can be represented by a general state space model of the following form :

$$\delta\xi = -D\xi + A\rho + U_{in} \qquad (3)$$

$$Q = B\rho \qquad (4)$$

where $\xi \in R^n$ is the state which may include concentrations of biomass, substrates and products in liquid phase ; $Q \in R^q$ is the vector of gazeous products flow rates ; D is the (scalar) dilution rate; $\rho(t) \in R^p$ is the vector of reaction rates (involving both growth and production rates) ; $U_{in} \in R^n$ is a vector of raw material feed rates ; A and B are respectively nxp and qxp matrices of (possibly stoecheometric) yield coefficients.

In equation (3) the meaning of the operator "δ" may be either the continuous time derivative or a first order Euler approximation of the derivative with a unit sampling period, i.e.:

$$\delta\xi = \frac{d\xi}{dt} \quad \text{or} \quad \delta\xi = \xi(t+1) - \xi(t)$$

In the latter case, all the variables in this paper must implicitely be considered at the discrete instant "t".

For the simplicity, we restrict ourselves to processes fed with a single substrate where the input vector U_{in} can be written :

$$U_{in} = HDS_{in} \qquad (5)$$

with S_{in} the (scalar) influent substrate concentration and H a suitable unit column vector.

We illustrate this general model formulation with the examples of section I.

Example 1: Anaerobic Digestion (continued).

$$\xi = \begin{bmatrix} X_1 \\ S_1 \\ X_2 \\ S_2 \end{bmatrix} \quad Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} \quad \rho = \begin{bmatrix} \mu_1 X_1 \\ \mu_2 X_2 \end{bmatrix} \quad H = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 0 \\ -k_1 & 0 \\ 0 & 1 \\ k_3 & -k_2 \end{bmatrix} \quad B = \begin{bmatrix} k_4 & k_5 \\ 0 & k_6 \end{bmatrix}$$

Example 2: Ethanolic fermentation (continued).

$$\xi = \begin{bmatrix} X \\ P \\ S \end{bmatrix} \quad \rho = \begin{bmatrix} \mu X \\ \nu X \end{bmatrix} \quad H = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -k_1 & -k_2 \end{bmatrix} \quad B = \begin{bmatrix} k_3 & k_4 \end{bmatrix}$$

III. STATEMENT OF THE ADAPTIVE CONTROL PROBLEM

Our objective is to control a single output variable which is a measured linear combination of the state variables :

$$Y = C^T\xi \qquad (8)$$

under the following conditions :

C1. One of the process inputs D or S_{in} is the control input ; the other one is known on line either by measurement or by a prior choice of the user.

C2. The gazeous flow rates Q are also measured on line and available for the design of the control law.

C3. The yield coefficients k_i (which appear in the matrices A and B) are constant, strictly positive and unknown.

C4. The specific reaction rates $\rho(t)$ are time varying and unknown.

We shall present solutions to this control problem which are adaptive (to handle with parameter uncertainty) and able to track a linear reference model arbitrarily closely.

We shall investigate two different situations, characterized as follows :

First situation

A1. $p = q$ and the matrix B is full rank

A2. The dilution rate D is the control input

A3. $C^T H \neq 0$

Second situation

B1. $p < q$

B2. The influent substrate concentration S_{in} is the control input

B3. $C^T H = 0$

The first situation will be illustrated with example 1 (anaerobic digestion) and the second one with example 2 (ethanolic fermentation).

IV. FIRST SITUATION

From equations (3), (4), (5), (8) and under condition A1, we have the following dynamics for the controlled variable Y :

$$\delta Y = -DY + Q^T\theta + C^T HDS_{in} \qquad (9)$$

with $\theta^T = C^T AB^{-1}$ a (1xq) vector of unknown parameters θ_i (i=1,...,q) which are non linear combinations of the yield coefficients k_i.

It must be emphasized that this equation (9), which will be the basis for the derivation of the control algorithm, is completely independent of the reaction rates ρ.

Example 1 : anaerobic digestion (continued)
We are concerned with the control of the anaerobic digestion process when it is used for waste treatment purpose. The control objective is to regulate the output pollution concentration (denoted Y) at a prescribed level Y^*, despite the input pollution (namely S_{in}) fluctuations by acting on the dilution rate D. Clearly, the output pollution concentration is

$$Y = S_1 + S_2 \qquad (10)$$

$$\text{i.e. } Y = C^T\xi \text{ with } C^T = (\,0\,1\,0\,1\,) \qquad (11)$$

Now, from (6) and condition C3, the matrix B is full rank and, according to (6) and (9), the dynamics of Y is written :

$$\delta Y = -DY + \theta_1 Q_1 + \theta_2 Q_2 + DS_{in} \qquad (12)$$

since $C^T H = 1$ and with :

$$\theta_1 = \frac{k_3 - k_1}{k_4} \qquad \theta_2 = \frac{(k_1 - k_3)k_5 - k_2}{k_6} \qquad (13)$$

Model reference linearizing control
We adopt the following first order reference model for the control error $\varepsilon = Y^* - Y$:

$$\delta\varepsilon + \lambda\varepsilon = 0 \qquad (14)$$

where λ is obviously chosen such that (14) is strictly stable.

It is readily shown that the following control law achieves this linear reference model exactly :

$$D = \frac{\lambda(Y^* - Y) + \delta Y^* - Q^T\theta}{C^T DHS_{in}} \qquad (15)$$

We shall now demonstrate that, in case of parameter uncertainty, an adaptive certainty equivalence form of (15) can be designed which asymptotically tracks the reference model (14). We first present the parameter estimation algorithm.

Parameter estimation
Operating on both sides of (9) by the stable low pass filter $(\delta + \omega)^{-1}$, we obtain the "filtered model"

$$Y = \Psi^T\theta + W \qquad (16)$$

$$\text{with : } W = \frac{1}{\delta + \omega}[(\omega - D) + C^T HDS_{in}] \qquad (17.a)$$

$$\Psi = \frac{1}{\delta + \omega}Q \qquad (17.b)$$

ω is a scalar arbitrary design parameter.

For the filtered model (16), which is linearly parameterized, we consider the following unnormalized least squares estimation algorithm (see [6], chapter 7):

$$\delta\hat{\theta} = \gamma R\Psi e \qquad (18.a)$$

$$e = Y - \Psi^T\hat{\theta} - W \qquad (18.b)$$

$$\delta R = -\gamma R\Psi\Psi^T R \qquad (18.c)$$

The following theorem establishes the needed properties of this algorithm (in discrete time) :

Theorem 1: For the parameter estimation algorithm (18) applied to the model (9) :

(i) $\hat{\theta}$ is bounded

(ii) $\lim_{t\to\infty} |e(t)| = 0$

Proof:

(i) Define the parameter error :

$$\tilde{\theta} = \theta - \hat{\theta} \qquad (19)$$

Consider the Lyapunov function:

$$V = \tilde{\theta}^T R^{-1}\tilde{\theta} \qquad (20)$$

Then, using (18.a) and (18.b), it can be shown that :

$$\delta V = -\gamma e^2 \qquad (21)$$

Also, from (18.c), $\delta R < O$ thus R is decreasing and R^{-1} is increasing. Hence:

$$\tilde{\theta}^T(O)R^{-1}(O)\tilde{\theta}(0) \geq \tilde{\theta}^T(t)R^{-1}(t)\tilde{\theta}(t) \geq \tilde{\theta}^T(t)R^{-1}(O)\tilde{\theta}(t) \qquad (22)$$

$$\text{which implies: } |\tilde{\theta}(t)| \leq |\tilde{\theta}(O)| \qquad (23)$$

It then follows that θ is bounded.

(ii) On the other hand, we have from (21):

$$\gamma\sum_{t=0}^{\infty} e^2(t) = \sum_{t=0}^{\infty}(-\delta V(t)) \leq V(0) \qquad (24)$$

$$\text{and thus } \lim_{t\to\infty} |e(t)| = 0 \qquad (25)$$

Adaptive Control.
In the case where the parameter θ is not known but is estimated as above, we propose the following version of the control law (15):

$$D = \frac{\lambda(Y^* - Y) + \delta Y - Q^T\hat{\theta} - [(1-\omega)\Psi + Q]^T\delta\hat{\theta}}{C^T HS_{in} - Y} \qquad (26)$$

Clearly, this controller coincides exactly with the controller (15) in the ideal situation where $\hat{\theta} = \theta =$ constant.

We have the following convergence result (in the discrete time case).

Theorem 2 : For the control law (26), combined with the parameter estimator (18), applied to the model (9):

$$\lim_{t\to\infty} |\varepsilon(t)| = 0$$

Proof:
From (16),(17,(18) we have:

$$(\delta+\omega)e = (\delta+\omega)[\Psi^T\theta - \Psi^T\hat{\theta}] \tag{27}$$
$$= \delta Y - Q^T\hat{\theta} - [(1-\omega)\Psi + Q]\delta\hat{\theta} - D(C^T H S_{in} - Y)$$

Substituting the value of D given by (26) into (27), we obtain:

$$\varepsilon = -\frac{\delta+\omega}{\delta+\lambda}e \tag{28}$$

and theorem 2 holds as aconsequence of theorem 1.

Comment.
The convergence property of theorem 2 is a local result which is valid only outside the singularity occuring in (26) when:

$$Y = C^T H S_{in} \tag{29}$$

Moreover, it is obvious that, for practical implementation, the control action D must be saturated:

$$O \leq D \leq D_{max} \tag{30}$$

This saturation automatically prevents the closed loop from the singularity (29) but the stability analysis is much more involved and not investigated here. However this issue is discussed in [7] (chapter 4) for a similar application.

V. SECOND SITUATION

From (3), (4), (8) and under conditions B1 and B3, the dynamics of the controlled variable is written :

$$\delta Y = -DY + C^T A \rho \tag{31}$$

From this equation we see that the second situation is more complex than the first one for two main reasons :

a) The equation (31) depends on the time varying unknown quantity ρ which cannot be eliminated simply by inverting B as in the first situation.

b) The control input Sin does not appear explicitely in (31) because $C^T H = 0$ by assumption B3 : the simple linearizing design of section IV is therefore not possible here.

In this section we shall show how to take advantage of the special structure of the process model (3), (4) and of the assumptions B1 to B3, to avoid these difficulties and to design a linearizing adaptive controller similar to that of section V.

To simplify and limit the mathematical technicalities within the space allowed to this communication, we shall restrict ourselves to the special case of the ethanolic fermentation (example 2). A more comprehensive treatment can be found in [8].

The objective is to control the ethanol concentration P which is measured on line, by using the glucose concentration Sin as control input. Assumptions B1 to B3 are fulfilled as follows:

$q = 1$ and $p = 2$ such that $q < p$

$Y = C^T\xi = P$ with $C^T = (\,0\,1\,0\,) \Rightarrow C^T H = 0$

We shall proceed in two steps. We first derive an alternative state space model for the process which allows to avoid the difficulties mentioned above. We will then show that there exists, for this alternative model, a linearizing control law which can be made adaptive by using a suitable parameter estimator.

An alternative state space model
In order to put the model (2.a-d) in a form which is convenient for deriving the control law, we write the specific production rate v(t) as follows :

$$v(t) = \alpha(t)\, S(t) \tag{32}$$

with $\alpha(t)$ a bounded positive unknown time varying parameter. There is no loss of generality in writing (32) : this expression, purely technical, just formalizes the evidence that $v(t) = 0$ when $S(t) = 0$, i.e. that there is no ethanol production without glucose.

We introduce the following auxiliary state variables :

$$Z_1 = k_1 X + k_2 P + S \tag{33}$$

$$Z_2 = k_3 X + k_4 P \tag{34}$$

It is then easily shown that the model (2.a-d) is equivalent from an input-output viewpoint to :

$$\delta Z_1 = -DZ_1 + DS_{in} \tag{35}$$

$$\delta Z_2 = -DZ_2 + Q \tag{36}$$

$$\delta Y = -DY + \varphi^T(Z_1, Z_2, Y)\,\theta \tag{37}$$

with $\varphi^T(Z_1,Z_2,Y) = (Z_1Y, Z_2Y, Z_1Z_2, Z_2^2, Y^2)$

$\theta^T = (\theta_1, \theta_2, \theta_3, \theta_4, \theta_5)$

$$\theta_1 = -\alpha\frac{k_4}{k_3} \qquad \theta_2 = \alpha\left[\frac{k_1k_4}{k_3}\left(1+\frac{1}{k_3}\right) - k_2\right]$$

$$\theta_3 = \frac{\alpha}{k_3} \qquad \theta_4 = -\alpha\frac{k_1}{k_3} \qquad \theta_5 = \alpha\frac{k_4}{k_3}\left(\frac{k_1k_4}{k_3} - k_2\right)$$

The important point is that this equivalent state-space model (35)-(37) has the following properties :

i) The state variables are either measurable (Y) or calculable in line (Z_1, Z_2) independently of the unknown parameters

ii) The model is linear with respect to the parameters θ_i.

Adaptive Control.
The state space model (35)-(37) can be shown to be state feedback linearizable, in such a way that the closed loop matches the following reference model exactly:

$$\delta^2\varepsilon + \lambda_1\delta\varepsilon + \lambda_2\varepsilon = 0 \tag{38}$$

where $\varepsilon = Y^* - Y$ is the control error, Y^* is the set point and λ_1 λ_2 are design parameters.

The linearizing control law is as follows:

$$S^L_{in}(\theta) = \{ [DZ_1(\theta_1 Y + \theta_3 Z_2) - (Q - DZ_2)(\theta_2 Y + \theta_3 Z_1 + 2\theta_4 Z_2) - (\theta_1 Z_1 + \theta_2 Z_2 + 2\theta_5 Y)(\varphi^T\theta - DY)] + \delta^2 Y^* + (\lambda_1\delta + \lambda_2)(Y^* - Y)\}\{(\theta_1 Y + \theta_3 Z_2)D\}^{-1} \qquad (39)$$

where Z_1 and Z_2 are computed on line, in parallel on the process, by integrating equations (35)(36) which are independent of θ.

Inthe case where the parameter θ is not known, it can be estimated on line with the parameter estimator (18,a-b) provided Ψ and W are defined as follows:

$$\Psi = \frac{1}{\delta + \omega}\varphi \qquad (40.a)$$

$$W = \frac{1}{\delta + \omega}(\omega - D)Y \qquad (40.b)$$

Finally, an adaptive version of the control law (38) is implemented as follows:

$$S^A_{in}(\hat{\theta}) = S^L_{in}(\hat{\theta}) - \{ [(1-\omega)\Psi + \varphi]^T \delta\hat{\theta}\}\{(\hat{\theta}_1 Y + \hat{\theta}_3 Z_2)D\}^{-1} \qquad (41)$$

In the ideal situation where the parameter θ is constant, the convergence of this adaptive control scheme is established by the following theorem.

Theorem 3: If θ = constant, for the control law (35),(36),(39),(40),(18):

$$\lim_{t\to\infty} |\varepsilon(t)| = 0 \qquad (42)$$

This theorem can be demonstrated in the same way as theorems 1 and 2 of section IV.

VI. CONCLUSIONS.

In this paper, it has been schown how to design adaptive linearizing controllers for fermentation processes which are valid in continuous as well as in discrete time. The design is based on non linear state space models which are linearly parameterized. The parameters are time varying combinations of the specific reaction rates and of the yield coefficients. The convergence of the proposed adaptive control algorithms has also been analysed in the discrete time case. The continuous time analysis is slightly more involved but would follow the same lines (according to [6]).

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