ADAPTIVE EXTERNAL LINEARIZATION FEEDBACK CONTROL FOR FLEXIBLE LINK MANIPULATORS: ROBUSTNESS ANALYSIS

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Abstract. The adaptive external linearization control proposed in [2] , [3] was designed for a rigid link manipulator and was proved to be convergent and robust against external input disturbances. Our contributions, in the present communication, is to analyse its robustness against "unmodelled" high frequency modes (which can arise from flexibility, for instance).

Keywords. Adaptive Control; Robots; Stability.

INTRODUCTION

A well known approach to improve the control of robotic manipulators is "the computed torque method" where the control law is designed explicitely, on the basis of a detailed non linear dynamical model, in order to compensate the robot non linearities and to guarantee a desired linear closed loop behaviour. (sec. e.g.[1]).

However, this method requires an accurate knowledge of the physical parameters of the manipulators and instability of the control algorithm can occur in case of uncertainty of the parameters.

A possible way to handle with parameter uncertainty is to implement an adaptive control law as follows:
- to adopt a suitable parametrization of the robot model
- to perform on line estimation of the parameters (to cope with parameter uncertainty)
- to apply the computed torque control law with the parameters substituted by their on line estimates.

We have proposed an "adaptive computed torque" control law of this kind in a previous paper (see [2] and [3]). Another exemple can be found in [4].

The algorithm proposed in [2], [3] was designed for a rigid-link manipulator and was proved to be convergent and robust against external input disturbances.

Our contribution, in the present communication, is to analyse its robustness against "unmodeled" high frequency modes (which can arise from flexibility, for instance).

THE DYNAMICAL MODEL

The general dynamical behaviour of a (n-degrees-of-freedom) manipulator, with n fast "parasitic" stable modes (one for each arm) is described as follows, using the Lagrangian formalism.

$$M(q,p)\ddot{q} + f(q,\dot{q},p) = v \tag{1}$$

$$\mu^2 C_1\ddot{v} + \mu C_2\dot{v} + v = u \tag{2}$$

where q is the n-vector of generalised coordinates
p is the N-vector of pseudo physical parameters (that will be defined hereafter)
$M(q,p)$ is the (nxn) inertia matrix
$f(q,\dot{q},p)$ is the n-vector which accounts for coriolis and centripetal accelerations, torques generated by gravity, springs and (viscous or coulomb) friction
u is the n-vector of torques developped by the actuators
v is the n-vector of the torques which are actually applied to the robot arms.
μ is a small positive scalar
C_1 and C_2 are diagonal nxn matrices describing the dynamics of each stable parasitic mode.

The high-frequency modes are modelled by equation (2) which can be interpreted as a flexible transmission between the developped and the applied torques, whose eigenfrequencies are of the order of $\frac{1}{\mu}$.

The limit as $\mu \to 0$ clearly corresponds to the special case of a completely rigid-link manipulator, described by the following equation :

$$M(q,p)\ddot{q} + f(q,\dot{q},p) = u \tag{3}$$

In [2],[3] we proposed an adaptive control algorithm designed for the rigid-link model (3) which was proved to be globally stable in case of external input disturbances.

Our purpose, in this paper, is to analyse the stability of that algorithm when it is applied to the actual flexible-link manipulator (1)-(2).

THE PARAMETRIZATION

It can be easily shown that the inertia matrix $M(q,p)$ and the torque vector $f(q,\dot{q},p)$ are linear with respect of a suitable set of "pseudo-physical" parameters p_i (i=1,...,N) which are, in turn, non linear functions of the physical robot parameters (like mass, inertia moment, center of mass, friction coefficients, etc...). Several examples of such parametrizations can be found in [2] [3] [4].

This means that M(q,p) and f(q,q̇,p) may be written in the following form :

$$M(q,p) = M_o(q) + \sum_{i=1}^{N} p_i M_i(q) \tag{4a}$$

$$f(q,\dot q,p) = f_o(q,\dot q) + \sum_{i=1}^{N} p_i f_i(q,\dot q) \tag{4b}$$

where $M_i(q)$ and $f_i(q,\dot q)$ are known functions of q and $\dot q$ while the p_i's are unknown but constant coefficients.

We assume however, in accordance with the usual reality, that the physical bounds on the parameters p_i are known by the user :

$$0 \leq p_{imin} \leq p_i \leq p_{imax},$$

$$\text{with } p_{imax} - p_{imin} = \delta_i \tag{5}$$

In addition, we shall restrict ourselves to robots with revolute joints for which the matrix M(q,p) is bounded for any q .

THE ADAPTIVE CONTROL LAW

Suppose that the control purpose is to track a "desired" linear dynamical behaviour described by a reference model which relates a reference input r to the desired generalised coordinates q_d as follows :

$$\ddot q_d + K_v \dot q_d + K_p q_d = r \tag{6}$$

where K_v and K_p are design parameters at the user's disposal, but chosen such that the following matrix A_1 is strictly stable :

$$A_1 = \begin{pmatrix} 0 & I_n \\ -K_p & -K_v \end{pmatrix} \tag{7}$$

Our adaptive control law is then obtained by using a standard "computed torque" control law, based on the rigid-link model (3), but with the parameters p_i substituted by their on line estimates :

$$u(q,\dot q,\hat p) = f(q,\dot q,\hat p) + M(q,\hat p)\,[\ddot q_d + K_p(q_d - q) + K_v(\dot q_d - \dot q)] \tag{8}$$

The on line parameter estimation is carried out as follows :

$$\dot{\bar p}_i = \alpha_i^2\, \phi_i^T\, [P_1(q_d - q) + P_2(\dot q_d - \dot q)]$$

$$\dot{\hat p}_i = o \quad \text{if} \quad \hat p_i = p_{imax} \text{ and } \dot{\bar p} \geq o \tag{9}$$

$$\text{or if} \quad \hat p_i = p_{imin} \text{ and } \dot{\bar p} \leq o$$

$$\dot{\hat p}_i = \dot{\bar p}_i \text{ otherwise}$$

Where the regressor ϕ_i is the solution of the linear system

$$M(q,p)\phi_i = [M_i(q)\ddot q + f_i(q,\dot q)] \tag{10}$$

The coefficients α_i are design parameters and the (nxn) matrices P_1 and P_2 are solution of the following Lyapunov equations :

$$K_p P_1 + P_1 K_p = Q_1 \tag{11a}$$

$$K_v P_2 + P_2 K_v = Q_2 + P_1 \tag{11b}$$

with Q_1, Q_2 arbitrary constant symmetric definite positive matrices.

THEORETICAL PROPERTIES FOR A RIGID-LINK MANIPULATOR

The adaptive control law (8), (9), (10), (11), when it is applied to a rigid-link manipulator, can be shown to be globally stable and to ensure the convergence of the tracking errors to zero, i.e. :

$$\lim_{t\to\infty} (q - q_d) = o \qquad \text{line}_{t\to\infty} (\dot q - \dot q_d) = o \tag{12}$$

Furthermore, the same control law remain globally stable in case of external non measured bounded disturbances. These properties are demonstrated in [3].

ROBUSTNESS ANALYSIS FOR A FLEXIBLE-LINK MANIPULATOR

We now investigate the stability properties of the proposed adaptive control algorithm, when applied to flexible link manipulators, as modelled in (1)-(2), with $\mu > o$

We first introduce some useful notations :

$$x = \begin{bmatrix} q_d - q \\ \dot q_d - \dot q \end{bmatrix} \quad \text{and} \quad z = \begin{bmatrix} v \\ \mu \dot v \end{bmatrix}$$

and the N-vector of the parameter estimation errors $\tilde p$ as

$$\tilde p = \begin{bmatrix} p_1 - \hat p_1 \\ \vdots \\ p_N - \hat p_N \end{bmatrix}$$

With u given by (8), equations (1) and (2) are rewritten as follows :

$$\dot x = A_1 \tilde x + \Phi \tilde p + A_{12} z + B_1 u(q,\dot q,\hat p) \tag{13a}$$

$$\mu \dot z = A_2 z + B_2 u(q,\dot q,\hat p) \tag{13b}$$

where * A_1 is defined in (7)

* $A_{12} = \begin{bmatrix} 0 & 0 \\ -M(q,\hat p)^{-1} & 0 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ M(q,\hat p)^{-1} \end{bmatrix}$

depend on q and $\hat p$

* $A_2 = \begin{bmatrix} 0 & I_n \\ -C_1^{-1} & -C_1^{-1}C_2 \end{bmatrix}$, $B_2 = \begin{bmatrix} 0 \\ C_1^{-1} \end{bmatrix}$

* Φ is a 2nxN matrix of the form

$$\Phi = \begin{bmatrix} \ldots 0 \ldots \\ \ldots\ldots\ldots \\ \phi_1 \quad \phi_N \end{bmatrix}$$

The state z is formed by a "transient" term and a quasi steady-state term $\bar{z}$ defined as the solution of (13b) with $\mu = 0$, i.e.

$$\bar{z} = -A_2^{-1}B_2u(q,\dot{q},\hat{p}) \qquad (14a)$$

We therefore define the "transient" y as

$$y = z+A_2^{-1}B_2u(q,\dot{q},\hat{p}) \qquad (14b)$$

On the other hand we define the n-vector ξ as

$$\xi = \tilde{x}-\mu A_{12}A_2^{-1}y \qquad (14c)$$

With the transformations (14), the following expressions derive from (13) :

$$\dot{\xi} = A_1\xi + \Phi\tilde{p} + \mu g_1 \qquad (15a)$$

$$\mu\dot{y} = A_2y + \mu g_2 \qquad (15b)$$

where $g_1 = (AA_{12}A_2^{-1}-\dot{A}_{12}A_2^{-1})y - A_{12}(A_2^{-1})^2B_2\dot{u}(q,\dot{q},\hat{p})$ (16a)

$$g_2 = A_2^{-1}B_2\dot{u}(q,\dot{q},\hat{p}) \qquad (16b)$$

Now it can be easily shown that the adaptation law (9) can be written in the following form :

$$\dot{\tilde{p}} = S\Phi^TP\xi - \mu S\Phi^TPA_{12}A_2^{-1}y \qquad (17)$$

where $S = \text{diag}\,[\alpha_i^2]$

P is the solution of the Lyapunov equation :

$$A_1^TP + PA_1 = \begin{bmatrix} -Q_1 & 0 \\ 0 & -Q_2 \end{bmatrix} = -Q \qquad (18)$$

P can also be written :

$$P = \begin{bmatrix} P_0 & P_1 \\ P_1 & P_2 \end{bmatrix} ,$$

with P_1, P_2 defined in (11) and P_0 given by

$$P_0 = P_1K_V + K_VP_1 + P_2K_p + K_pP_2$$

We define a Lyapunov function $V(\xi,y,\tilde{p})$:

$$V(\xi,y,\tilde{p}) = \frac{1}{2}\xi^TP\xi + \frac{1}{2}\mu y^TP_3y + \frac{1}{2}\tilde{p}^TS^{-1}\tilde{p} \qquad (19)$$

where P_3 is the solution of the Lyapunov equation

$$A_2^TP_3 + P_3A_2 = -Q_3$$

with Q_3 arbitrary symmetric definite positive matrix. The time derivatives of V along the trajectories of (15) with the adaptation law (9)-(17) satisfies

$$\dot{V} \leqslant -\frac{1}{2}\xi^TQ\xi - \frac{1}{2}y^TQ_3y + \mu\eta^Th \qquad (20)$$

with $\eta = \begin{bmatrix} \xi \\ y \end{bmatrix}$

and $h = \begin{bmatrix} Pf_1 \\ P_3f_2 + A_2^{-T}P_3\phi S\tilde{p} \end{bmatrix}$

As shown in [3] the strict inequality in (20) corresponds to the case where at least one of the $\dot{\hat{p}}_i$ is set equal to zero, accordingly to (9).

The robustness result is based on the following technical lemma :

<u>Lemma</u> :

Consider a n order differential equation :

$$\dot{x} = f(x,t) \qquad (21)$$

and a definite positive function $V(x)$, with $V(0) = 0$.
Assume there exist two bounded domains in R^n , D_1 and D_2

- containing the origin
- $D_1 \subset D_2$
- $\dot{V} < 0$ on $D_2 \setminus D_1$

Let V_1 be the maximum of V on the boundary of D_1 and V_2 the minimum of V on the boundary of D_2
If $V_1 \leqslant V_2$,

there exist 2 bounded domains B_1 and B_2

$$B_1 = \{\ x \text{ such that } V(x) \leqslant V_1\ \}$$

$$B_2 = \{\ x \text{ such that } V(x) \leqslant V_2\ \}$$

such that all the trajectories of (21) with initial conditions in B_2 enters B_1 in a finite time to, and do not leave B_2 for $t \geqslant t_0$.

<u>Proof</u> :

As $D_1 \subset B_1$ and $B_2 \subset D_2$, $\dot{V}$ is strictly negative on $B_2 \setminus B_1$. The proof follows immediately.

Our robustness result can now be expressed as :

<u>Theorem</u> :

Under the assumptions

H_1 : $|h|$ is bounded uniformly in $\tilde{p}$ and t by

$$|h| \leqslant K_1 + K_2\,|\eta|^3$$

H_2 : $\sum_{i=1}^{N} \frac{\delta_i^2}{\alpha_i^2} < \frac{1}{2}\frac{\lambda_2\lambda_3}{K_2}$

where λ_2 is the smallest eigen value of P_3

λ_3 is the smallest eigen value of Q and Q_3

There exist two bounded domains B_1 and B_2 $(B_1 \subset B_2)$, and $\mu_0 > 0$, such that for $\mu \leq \mu_0$

the trajectories of (15) with initial conditions in B_2 enter B_1 in a finite time t_0 and do not leave B_1 for $t \geq t_0$.

Proof :

a) From H_1, $\dot{V} \leq 0$ for $\mu \leq \frac{1}{2} \frac{\lambda_3 |\eta|}{K_1 + K_2 |\eta|^3} = \rho(|\eta|)$

the function $\rho(|\eta|)$ presents a maximum equal to

$$\sigma = \frac{1}{2} \frac{\lambda_3 (\frac{K_1}{2K_2})^{1/2}}{K_1 + K_2 (\frac{K_1}{2K_2})^{3/2}} \quad \text{for } |\eta| = (\frac{K_1}{2K_2})^{1/2}$$

For every $\mu \leq \sigma$ there exist therefore 2 values of η, η_1 and η_2, characterized by

$$\mu = \frac{1}{2} \frac{\lambda_3 \eta_i(\mu)}{K_1 + K_2 \eta_i^3(\mu)} \qquad i = 1,2$$

and such that

$$\eta_1(\mu) \leq (\frac{K_1}{2K_2})^{1/2} \leq \eta_2(\mu)$$

Furthermore,

$$\lim_{\mu \to 0} \eta_1(\mu) = \lim_{\mu \to 0} \eta_2(\mu) = 0 \quad \text{and}$$

$$\lim_{\mu \to 0} \mu \eta_2^2(\mu) = \frac{1}{2} \frac{\lambda_3}{K_2}$$

b) Under assumption H_2 there exists $\mu_0 > 0$ such that for $\mu \leq \mu_0$

$$\frac{1}{2} \Sigma \frac{\delta_i^2}{\alpha_i^2} < \frac{1}{2} \mu \lambda_2 \eta_2^2(\mu) - \frac{1}{2} \lambda_1 \eta_1^2(\mu) < \frac{1}{4} \frac{\lambda_2 \lambda_3}{K_2}$$

where λ_1 is the largest eigen value of P.

Defining 4 bounded domains

$$D_1 = \{(\xi, y, \tilde{p}) \mid |\eta| \leq \eta_1(\mu), |\tilde{p}_i| \leq \delta_i\}$$

$$D_2 = \{(\xi, y, \tilde{p}) \mid |\eta| \leq \eta_2(\mu), |\tilde{p}_i| \leq \delta_i\}$$

$$B_1 = \{(\xi, y, \tilde{p}) \mid V(\xi, y, \tilde{p}) \leq \frac{1}{2} \lambda_1 \eta_1^2(\mu) + \frac{1}{2} \Sigma \frac{\delta_i^2}{\alpha_i^2} \text{ and } |\tilde{p}_i| \leq \delta_i\}$$

$$B_2 = \{(\xi, y, \tilde{p}) \mid V(\xi, y, \tilde{p}) \leq \frac{1}{2} \lambda_3 \eta_2^2(\mu) \text{ and } |\tilde{p}_i| \leq \delta_i\}$$

the result follows immediately by application of the lemma, the boundedness in (ξ, y) implies the local aymptotic boundedness in $\tilde{x}$ and z.

Remarks

1. From the form of f and $u(q, \dot{q}, \hat{p})$ (presence of quadratic terms in $\dot{q}$ introduced by the centripetal and Coriolis accelerations) it can be verified that H_1 is satisfied for revolute joint manipulators if $\dot{r}$ (or equivalently $\ddot{q}_d$) is uniformly bounded, which is a rather realistic requirement.

2. For given δ_i, the α_i^2 can always be chosen to satisfy H_2. Great values of α_i^2 guarantee therefore the local asymptotic boundedness but can be lead to unacceptable transient behaviour.

CONCLUSION

This communication presents a robustness analysis of the proposed adaptive algorithm ([2],[3]) in presence of unmodelled high-frequency parasitic modes. It has been shown that local asymptotic boundedness of the error system is ensured under restrictions on the parasitic modes and the assumption of boundedness of the time derivative of the reference input.

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