MODELLING, SIMULATION AND CONTROL OF THE TRAFFIC AT THE TERMINUS STATIONS OF URBAN UNDERGROUND RAILWAY LINES

G. Campion, V. Vanbreusegem and G. Bastin

Laboratoire d'Automatique de Dynamique et d'Analyse des Systèmes, Université Catholique de Louvain, Louvain-La-Neuve, Belgium

Abstract. An original state space formulation of a linear model of Underground Railway line traffic is proposed, allowing on-line implementation of optimal control. In this paper the model is extended to take into account the particular structure of a terminus station. The properties of this structure are discussed and simulation results show the benefit to be expected from this traffic control policy.

Keywords. Traffic control; modelling; rail traffic.

INTRODUCTION

It is well known that a public transportation system has an intrinsically unstable behaviour. Consider, for instance, a delayed vehicle. Because of this delay the time interval since the last vehicle is greater than the nominal one, and more people than expected have to enter the vehicle, with a resulting increasing delay. It is therefore necessary to implement a control policy in order to restore a disturbed traffic to an acceptable situation. For a high traffic density system with passengers arriving randomly at the stations and getting on the first available vehicle, regardless of the nominal time schedule, it is necesary, from the passengers' viewpoint, to control the traffic in order to minimize the waiting times at the stations, i.e. to keep the time intervals between successive trains as close as possible to their nominal values. On the other hand, as connections with other transportation systems have to be considered, it is also necessary to control the traffic according to the nominal schedule. For better service to the passengers the control has therefore to respect the trade-off between these two objectives. The control action consists of instructions (such as speed during the running time between stations, waiting time at the stations ...) given by the centralized traffic controller, and elaborated on the basis of the available information, consisting mainly of the situation of each train in the system. Physical constraints are of course imposed on the control, e.g. maximum speed, minimum waiting times at the station, minimum distance between successive trains, or other security rules imposed by traffic lights.

A linear model for such a transportation system has been proposed by Sasama and Okhawa (1983) as well as state space representations and a suboptimal control policy. An original state space representtion allowing the practical implementation of optimal control has been proposed by Campion et al. (1985). This model and this state space formulation are shortly presented in section 1, while section 2 discusses the practical implementation for the Brussels Underground Railways line.

This communication deals mainly with the extension of the model to the future configuration of Brussels Underground Transportation System, taking into account the particular structure of the planned end -of- line stations. This structure, with its properties and the nominal operation conditions are discussed in section 3. Section 4 proposes simulation results obtained with the extended model and shows the benefit to be expected from the implemetation of the proposed optimal control policy.

1. LINEAR MODEL, STATE-SPACE FORMULATIONS AND OPTIMAL CONTROL

Let us consider I trains (upper index $i=1,...,I$) and a line with K+1 stations (lower index $k=0,1,..K$) and define

t_k^i as the actual departure time of ith train from k th station,

T_k^i as the corresponding nominal departure time

x_k^i as the deviation, i.e.

$$x_k^i = t_k^i - T_k^i$$

Sasama and Okhawa (1983) propose a linear model for the generation of the x_k^i, leading to a linear relationship between

$$x_{k+1}^i,\ x_{k+1}^{i-1},\ x_k^i \text{ and } u_k^i$$

where u_k^i is the control action applied to ith train, during its transfer from kth station to (k+1) th station. This relation means that the deviation of i th train at (k+1) th station depends linearly on its deviation at k th station, on the deviation of the precedent train at (k+1) th station (x_{k+1}), and on the applied control action u_k^i.

Let us consider the deviations array

$$\begin{matrix} x_0^1 & x_0^2 & \cdots & x_0^I \\ x_1^1 & & & \vdots \\ \vdots & & & \vdots \\ x_K^1 & & \cdots & x_K^I \end{matrix}$$

Two state space formulations have been proposed by Sasama, based on the definition of the state vector. In these formulations the state vector is defined respectively as a row (dimension I) or as a column (dimension K+1) of this deviations array. A third state space formulation is proposed by Campion (1985), with the state vector defined as a diagonal of the deviations array, i.e. as

$$X_j = \begin{bmatrix} x_1^{j-1} \\ \vdots \\ x_K^{j-K} \end{bmatrix}$$

The components of the state vector X_j are characterized by the fact that the sum of the upper and lower indices is equal to j. A first advantage is that the resulting dynamical matrix has a simpler form, but the main advantage results from the fact that the components of the state vector X_j are available is a short time interval. As these components are known nearly simultaneaously the index j plays the role of a time index. This property allows real time implementation of an optimal control policy.

A wide variety of optimization criteria can be considered depending on the control purposes. The following minimization criterion takes into account the two objectives discussed in the introduction : regularity on the interval between successive trains and regularity with respect to the nominal schedule :

$$J = \sum_{1,K} p_k (x_k^i)^2 + \sum_{1,K} q_k (x_k^i - x_k^{i-1})^2 + \sum_{1,K} (u_k^i)^2$$

The first term penalizes the deviations from the nominal schedule; the second term penalizes the deviations of the time intervals between trains and is therefore related to the average waiting time for the passengers and the congestionn of the trains. The third term is a measure of the control actions which are zero for the nominal schedule. The values of the weighting coefficients p_k and q_k depend on the control purpose and reflect the trade-off between the regulation objectives.

With the linear model and this quadratic performance criterion the optimal control is known to be a linear state-feedback control. The implementation of this optimal control is not possible with the state-space formulations proposed by Sasama (1983), because the components of the state vector are known in a long time interval, so long term prediction should be necssary. For this reason Sasama proposed a simplified sub-optimal control. On the other hand, for the third state-space formulation, on line implementation of the optimal control is possible because the components of the state-vector are known nearly simultaneously. As a particular case, if we consider a one-step optimization criterion, the optimal control u_k^i is of the following form

$$u_k^i = g_{k+1} \, x_k^i + f_{k+1} \, x_{k+1}^{i-1}$$

i.e. depends linearly on the deviation of i-th train at k-th station and of the preceding train at the next station (x_{k+1}^{i-1}) and is therefore particularly straight forward to implement.

2. BRUSSELS UNDERGROUND TRANSPORTATION SYSTEM

The possibility of implementation of the described model as well as the proposed traffic control policy has been investigated for an existing transportation system, with its physical characteristics and constraints, for instance Brussels Underground Railway line (see details in Campion (1985)).

This implementation needs a preliminary analysis of the system, providing :

1. **The topological structure** of the line and its **numerical characteristics.** Brussels underground system consists of two lines (A and B) with a long common section where trains of both lines are operated alternately. The standard running times between stations, as well as the minimum waiting times at a station have to be measured. The characteristic parameters of the dynamical model are obtained from statistical data. The structure of the line, with the stations names and the standard running times, is given in fig. 1

2. **The physical constraints**, such as the maximum speed, minimal distance between trains, or other security rules (for instance, no more than one train at a time in a section between two successive stations). These constraints are imposed by use of traffic lights and automatic stopping procedures.

3. **Practical implementation of the control actions** : Consider the system under operating conditions. The theorical optimal control u_k, to be applied to ith train between k th and (k+1) th station, is calculated by the centralized traffic controller and can be considered as an instruction given to the driver, in order to modify the staying time at kth station and the running time between k and (k+1). The driver has to follow this theoretical instruction, but conforming himself to the other security requirements. The modification of the staying time can be imposed by the traffic lights at the station while the modification of the running time can be realized by a selection between three nominal speeds (slow, normal or high).

A program has been implemented to simulate the complete system, taking into account branching, traffic security requirements, .. This program generates the absolute times t_k, in connection with the nominal schedule. It allows the introduction of control actions, as well as disturbance terms for any train at any station. The proposed traffic policy has been tested on the basis of this simulation program. Detailed simulation results are given in Campion (1985) and show clearly the benefit to be expected from the proposed traffic control policy.

3. MODEL EXTENSION

In the actual system configuration the structure of the terminus stations is the following : there are 2 platforms (one for arrival, one for departure) and the track crossing section is located **after** the station. No conflict can occur between a train arriving in the terminus station and stopping at the arrival platform and a train in the crossing section, or leaving the station at the departure platform. On the other hand one of the planned extensions of the system consists in 2 new stations are on line A after Alma : Bornival and, for the new terminus station Stokel (see fig.1). A new structure has been chosen for the terminus station Stockel : there is only one central platform, between the two tracks and the track crossing section is located **before** the station . A train coming from Bornival 2 enter Stockel either

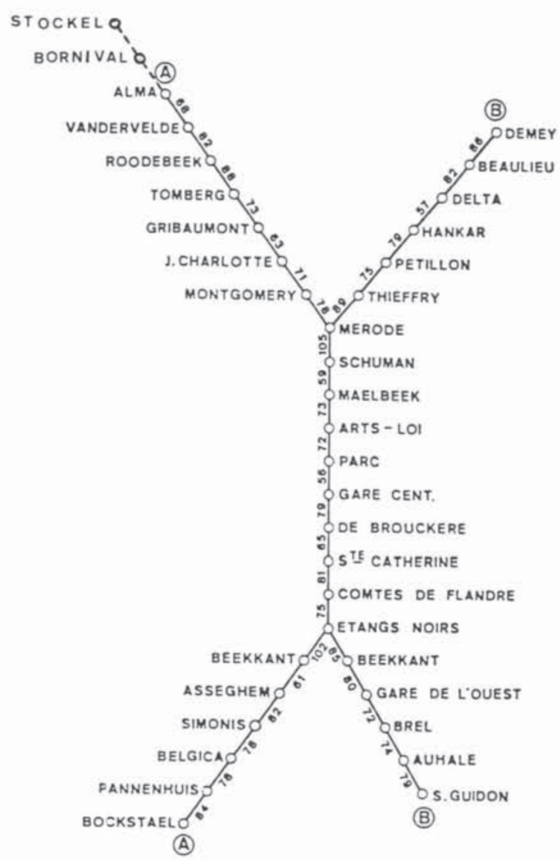

Fig. 1 Brussels Underground Railway lines (Extensions : dotted line)

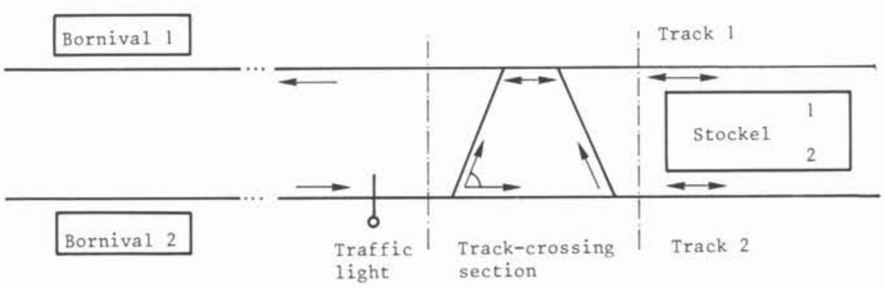

Fig. 2 Terminus Station Structure

directly at Stockel 2, either after track crossing at Stockel 1. On the other hand a train coming to Bornival 1 leaves Stockel, either directly from Stockel 1, either after track crossing from Stockel 2. For security requirements a traffic light is located on track 2 (from Bornival 2 to Stockel), before the crossing section (see fig.2).

This terminus station structure presents two advantages :
1. it avoids a tunnel section used only for maneuvers and reduces therefore the public works costs
2. the distance covered by the trains is reduced (maneuvers before station) with resulting energy saving.
On the other hand there is a risk of conflict in the track crossing section between trains leaving or arriving at Stockel.

More precisely consider the following steady-state nominal operation conditions (corresponding to the rush-hours): the time interval between 2 successive trains is 5 min, for each line (A and B); that means a time interval of only 2'30" on the common section. In addition the nominal staying time at Stockel cannot be less than 5'. Under these conditions every 5 minutes one train arrives in Stockel, and one train leaves this station, with alternance of platforms (1 or 2). The operation nominal sequence is the following :
- trains k and k+1 are staying in Stockel (k at Stockel 1 and (k+1) at Stockel 2)
- k th train leaves Stockel 1 to Bornival 1 (no track crossing)
- (k+2) th train arrives at Stockel 1 (track crossing)
- (k+1) th train leaves Stockel 2 (track crossing)
- (k+3) th train arrives at Stockel 2 (no track crossing)
To avoid crashes the security requirements are the following :
1) (k+2) th train (with destination Stockel 1, with track crossing) does not leave Bornival 2, if the track crossing section is not free, and is stopped at the traffic light until this section is free, if it has been occupied since its departure from Bornival.
2) (k+1) th train (with destination Bornival 1, with track crossing) does not leave Stockel 2 if there is a train between Bornival 2 and Stockel. Conflicts can therefore occur between trains (k+1) (Stockel 2-Bornival 1) and (k+2) (Bornival 2 Stockel 1). The solution consists either in stopping (k+2) th train at Bornival 2 or at the traffic light, either in stopping (k+1) th train at Stockel 2, until the track crossing section becomes free. For satisfying operation conditions an upper bound has therefore to be considered for the nominal staying time at Stockel. As (k+1) th train must leave the track crossing section before (k+2) th train leaves Bornival 2, the upper bound for the nominal staying time at Stockel is 10 minutes (i.e. two times the time interval between 2 trains on line A) — the running time for Bornival 2 to Stockel 1 — the running time from Stockel 2 to the end of the track crossing section.
In our case this upper bound is 7'45". This limitation must be kept in mind for the elaboration of the nominal schedule.

The simulation program described in section 2 has been extended to the augmented system, with the particular structure of the new terminus station. This program generates, as a first result, a nominal time schedule coherent with the dynamics of the system and satisfying all the given requirements (nominal time interval, security requirement, no conflict at Stockel). Simulation results corresponding to a disturbed situation are given in the next section.

4. SIMULATION RESULTS

The general conditions of these simulation are the following
- The time interval between two successive trains on each line is 5', and the nominal staying time at the terminus station Stockel is taken to be 5'30", allowing a satisfying nominal behaviour at Stockel, as seen in section 3. In any case this staying time cannot be less than 5'.
- A delay of 45" is imposed to the 4th train of line A, in the direction Stockel, at Montgomery (first station after the divergence). No other disturbances are imposed to the system.
Trains of line A and B have respectively odd and even numbers, so the index of the disturbed train is 7.

We are interested in the evolution of the delays of train 7 as well as in the propagation of the initial disturbance to the other trains, in the section Montgomery 2 - Stockel, at the terminus Stockel, in the section Stockel - Merode 1, and in the common section Merode 1 - Etangs Noirs 1. We intend to compare two situations.
- *Free System* : no expicit control is applied. The regulation is provided only by the security requirements.
- *Regulated System*. We apply the optimal control proposed in section 1, with $p_k=q_k=1$. This control is bounded by the security requirements but we impose an additional severe constraint. The speed is imposed and cannot be modified. The only control action consists in modifying the waiting time at the stations, with a maximum percentage of 10% with respect to the nominal value.

The results are summarized by several diagrams giving the deviations of the trains at different stations (a positive deviation means a delay). For stations belonging to the section Montgomery — Stockel only odd numbers are of interest, while for stations belonging to the common section odd and even indices can be considered.
Fig 3 and 4 give the deviations at Bornival 2 and Bornival 1, for respectively, the free and the regulated systems.
Fig 5 and 6 give the waiting times at Stockel (free and regulated system). These values have to be compared with the nominal staying time : 330"
Fig 7 and 8 compare the situation for free and regulated system respectively at Merode 1 and Etangs Noirs 1 (first and last station of the common section).

These results need some remarks.
1. Consider first the *free system*. Because of the intrinsic instability of the system, the delay of train 7 increases from 45" to 65" at Bornival 2, while, in the other hand, the next train (9) arrives in Bornival 2 25" before time (fig 3). Because of the delay of tain 7, a conflict occurs in Stockel and the preceding train (5) is delayed (staying time of 378" instead of 330" - fig. 5). On the other hand train 7 leaves Stockel after only 300" staying time and reduces therefore its delay (35" at Bornival 1 - fig 3). These disturbances increase from Bornival 1 to Merode 1 (convergence station with line B) : for instance 80" and 65" delays for trains 5 and 7. Because the constraint of alternance between lines A and B on the common section, the disturbances on line A propagate to trains of line B at the convergence station (Merode 1) (see fig 7). At the end of the common section (Etangs Noirs 1) trains 5 to 12 are delayed, with a delay of 260" for train 5 (fig 8).
2. For the *regulated system* the evolu tion is much more satisfying. Because of the control policy the deviation of train 7 does not increase from Montgomery to Bornival 2 (45"-see fig 4) and the next

trains (9-11-13) present only small delays. A conflict occurs at Stockel between trains 5 and 7, and the staying time of train 5 increases up to 360" (30" delay), while train 7 leaves Stockel only 15" behind time, after reduction of its staying time to 300" (fig 6). The propagation of these disturbances is controlled (only 20" delay at Merode 1) and the propagation to trains of line B is rather slight. At the end of the common section (Etangs Noirs 1), the delays of trains 5 to 10 are very small (max delay of 15" for train 5). Deviations of trains 5 to 12 at Merode 1 and Etangs Noirs 1, for both situations, can be summarized as follows :

Train Index	Free System		Controlled System	
	Mer.	E. Noirs	Mer.	E. Noirs
5	80	260	22	15
6	76	225	7	6
7	65	175	10	7
8	48	131	0	5
9	31	87	4	5
10	14	42	0	4
11	0	-3	0	0
12	0	-17	0	0

These results show that, even with a severe limitation on the control, the proposed regulation policy is very efficient. Better results (shorter transient) can be obtained by relaxing somewhat this limitation the control action.

The conclusion of these simulations is that this structure of terminus stations can be selected, but only if an efficient traffic control is implemented. Without control even a small initial disturbance is amplified and propagated leading to inacceptable operation conditions at the terminus.

CONCLUSIONS

1) The proposed linear model and the corresponding state space representation has been shown very efficent to simulate Underground Railway Line behaviour. The simulation programme is very flexible and can easily be adapted in order to take into account particular problems, such as a new structure of terminus station.
2) The control policy based on the third proposed state-space formulation is easily implemented on line and shows to be very efficient. For instance without this control the exploitation of new structure terminus stations should be critical.
3)This methodology can be extended. In fact the proposed control based on a one step optimization criterion is decentralized : the information to be processed to elaborate the control action to be applied at a given station must be collected in only 2 stations : the given station and the next one. No centralized information processing is necessary. A more efficient control can be elaborated on the basis of a multistep optimization criterion. The implementation of such a control needs to centralize the information available in more than two stations. This more sophisticated control policy would increase singificantly the performance of the system, mainly for the critical stations such as terminus stations, or branching stations. These developments are presently under investigation.

REFERENCES

H. Sasama and Y. Okhawa (1983). "Floating traffic control for public transportation systems". Proc. 4th IFAC Conf. on Control and Transportation, Baden-Baden, April 1983.

G. Campion, V. Van Breusegem, P. Pinson, G. Bastin (1985). "Traffic Regulation of an underground railway transportation system by state feedback", to appear in Optimal Control, Applications and Methods.

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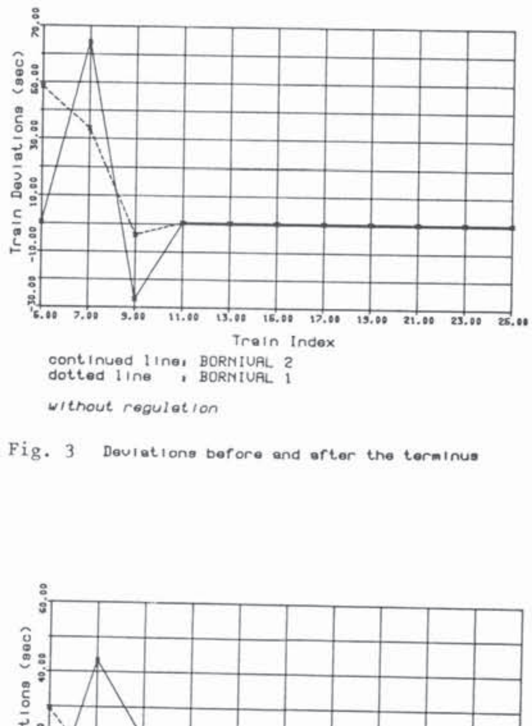


Fig. 3 Deviations before and after the terminus

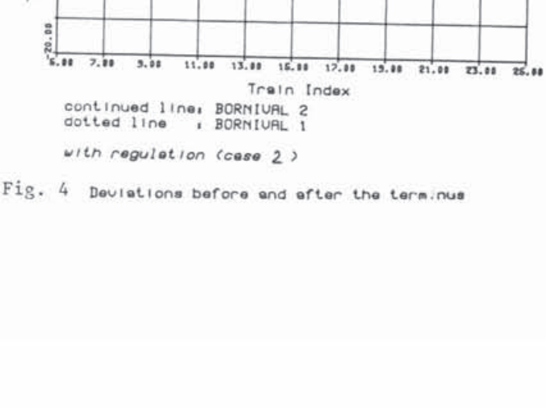
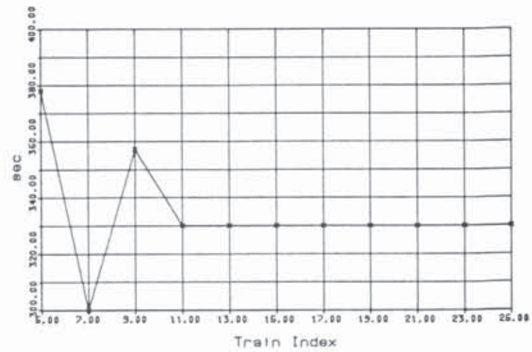
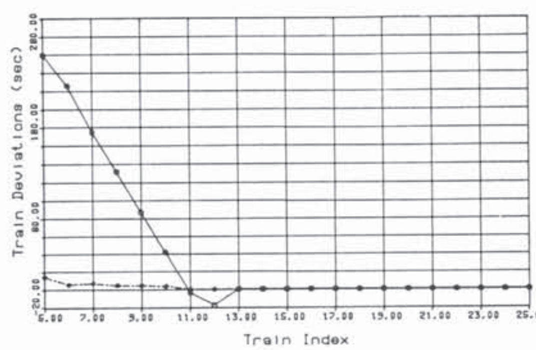


Fig. 4 Deviations before and after the terminus



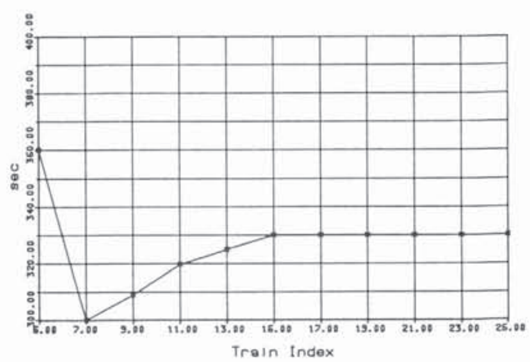
without regulation

Fig. 5 Staying Time at Terminus



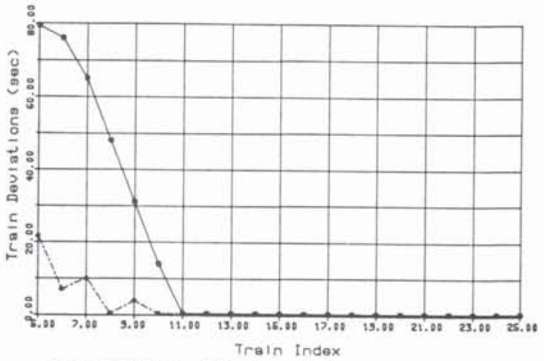
continued line: without regulation
dotted line : with regulation (case 2)

Fig. 8 Deviations with and without regulation at ETANGS NOIRS 1 station



with regulation (case 2)

Fig. 6 Staying Time at Terminus



continued line: without regulation
dotted line : with regulation (case 2)

Fig. 7 Deviations with and without regulation at MERODE 1 station