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A LYAPUNOV APPROACH TO CONTROL IRRIGATION CANALS MODELED BY SAINT-VENANT EQUATIONS

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Abstract

This paper deals with the regulation of irrigation canals. We consider the distributed and nonlinear nature of a single reach delimited by two regulator gates and which can be described by the Saint-Venant equations. By means of a Lyapunov approach we propose a class of locally exponentially stabilizing controllers.

1 Introduction

The control of most canals all over the world is made under manual operation. This involves an action which is only based on local information about the canal state and depends on the ability, experience and mobility of the operating personnel. The efficiency of the water distribution is poor with respect to the potential performance of the canals [gous]. Although various means exist that may help in improving the traditional management of canals, the introduction of automatic control in the canal operation has been increasingly promoted in recent years when the technical and the socio-economic circumstances make it possible [plus,buya,ruiz]. Two trends can be identified in the literature: a first trend focuses on the development of control algorithms while another one is more oriented towards practical implementation aspects on real canals.

A state of the art can be found in the proceedings of the international workshop RIC'97 (Regulation of Irrigation Canals) which was recently held in Marrakesh [ric].

Concerning modeling and control, finite dimensional models linearized around steady state values are most often used with classical PID or simple heuristic controllers. Such controllers have often poor performances in terms of precision, stability and robustness. This is due to the fact that canals are large, interconnected, nonlinear, delayed and strongly perturbed systems.

To deal with these problems, in a few recent applications, optimal, predictive and adaptive control concepts have been proposed, e.g. in [geor1,geor2,mala1,mala2,mart,rode,sawa]. Recently, in [boun,xu] for example, the distributed nature of these systems is considered, but the authors deal with the linearized PDE system around steady state values, without giving information for the nonlinear system even if the state is "close" to the steady state values.

In this paper we consider a single reach delimited by two regulator gates and modeled by Saint-Venant equations.

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We take into account the distributed and nonlinear nature of the system. By means of a Lyapunov approach we propose stabilizing boundary control laws which give a natural solution to the well known problem dependence of the time-delay in open-channel hydraulic systems.

In Section 2 we give the dynamical PDE describing our system and we state the control problem. In Section 3 we present our Lyapunov control approach leading to asymptotic stabilizing boundary controllers and we state our main exponential stability result given by Theorem 1. Finally, some illustrative simulation results are displayed in Section 4 and we conclude in Section 5.

2 Modeling of the system and statement of the control problem

2.1 Modeling

We consider a one-dimensional portion of irrigation canal delimited by two underflow gates. The reach dynamics are described by PDE Saint-Venant equations (see e.g. [chow]). We restrict our attention to the case of an horizontal reach and viscous friction terms, as well as leaks or withdrawals are neglected, so that the dynamical equations simplify as follows (see Fig. 1):

Continuity equation:

$$\frac{\partial y}{\partial t} + \frac{\partial (Vy)}{\partial x} = 0. \tag{1}$$

Dynamical equation:

$$\frac{\partial V}{\partial t} + g\frac{\partial y}{\partial x} + V\frac{\partial V}{\partial x} = 0, \tag{2}$$

where x is the space coordinate belonging to $[0, L]$, L being the reach's length, t is time, $V(x,t)$ is the water velocity at point x and time t, $y(x,t)$ is the water level (at point x and time t) and g is the gravity constant.

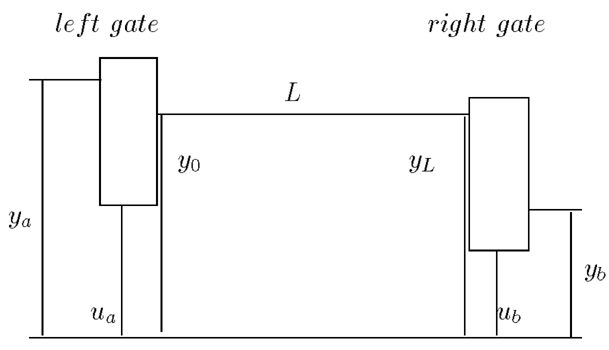

Figure 1: The horizontal reach

The control actions are provided by two underflow gates located at the left end ($x = 0$) and the right end ($x = L$) of the reach (see Fig. 1). A standard discharge relationship of underflow gates is as follows (see e.g. [garc]):

$$\begin{cases} V^2(0,t)y^2(0,t) = u_a(y_a - y(0,t)), \\ V^2(L,t)y^2(L,t) = u_b(y(L,t) - y_b), \end{cases} \tag{3}$$

where u_a and u_b are the physical control variables. They denote respectively the left and right gate openings and y_a and y_b the left and right water levels outside the reach. Equations (3) constitute the boundary conditions at $x = 0$ and $x = L$, associated to the PDEs (1) and (2). Of course for (3) to have meaning, y_a and y_b must satisfy the following inequalities

$$y_a \geq y(0,t)\ ,\ y_b \leq y(L,t)\ ,\ \forall t \geq 0. \tag{4}$$

2.2 Steady-states

For given constant openings $\bar{u}_a$ and $\bar{u}_b$ there exists a steady state solution $(\bar{V}, \bar{y})$ of equations (1), (2) which satisfies, from (3), the following relations:

$$\begin{cases} \bar{y} = \dfrac{\bar{u}_a y_a + \bar{u}_b y_b}{\bar{u}_a + \bar{u}_b}, \\ \bar{V} = \dfrac{\sqrt{\bar{u}_a(y_a - \bar{y})}}{\bar{y}}. \end{cases} \tag{5}$$

2.3 Statement of the control problem

The control objective is to stabilize the water level y and the water velocity V at a given set point $(\bar{V}, \bar{y})$. The control actions are the two gate openings u_a and u_b. The left and right water levels $y(0,t)$ and $y(L,t)$ are supposed to be measured on line at each time instant t. The external constant water levels y_a and y_b are known.

3 Lyapunov control design

Lyapunov design is a classical approach for the control of dynamical systems. The principle is first to look for a Lyapunov function, i.e. a nonnegative radially unbounded function V which is nonincreasing for a suitable choice of the control law and then to use LaSalle's invariance principle to conclude to asymptotic closed-loop stability. For physical systems, a natural candidate for a Lyapunov function is the total energy of the system and the feedback control is used to introduce some kind of artificial viscosity. For our system (1)-(2), it is easily checked that the energy $\int_0^L (y\frac{V^2}{2} + g\frac{y^2}{2})dx$ does not work for this purpose. But one can use, for the Lyapunov approach, any function which is conserved along any solution of system (1)-(2) and which is L-periodic with respect to x. For hyperbolic systems as (1)-(2), there are other quantities than the energy which are conserved (at least for C^1 solutions), namely the entropies (see e.g. [serr, volume 1,

p. 96]). Entropies are functions $(V, y) \to E(V, y)$ such that, for some function $F : (V, y) \to F(V, y)$, called the entropy flux, we have:

$$\frac{\partial F}{\partial y} = V\frac{\partial E}{\partial y} + g\frac{\partial E}{\partial V} \text{ and } \frac{\partial F}{\partial V} = y\frac{\partial E}{\partial y} + V\frac{\partial E}{\partial V}.$$

Indeed, with such functions (E, F), if we let:

$$R = \int_0^L E(V, y)dx, \tag{6}$$

then along the (smooth) solutions of (1)-(2) we have:

$$\dot{R} = -[F(V, y)]_0^L, \tag{7}$$

and therefore $\dot{R}$ depends on the values of (V, y) at $x = 0$ and $x = L$.

For our hyperbolic system (1)-(2), as for general 2×2 hyperbolic systems, there are infinitely many entropies (see e.g. [serr, volume 2, sec. 9.3]).
In order to have a Lyapunov function, it is required that:

$$E(V, y) \geq E(\bar{V}, \bar{y}),$$

with equality if and only if $(V, y) = (\bar{V}, \bar{y})$. After a few computations, it can be seen that the "simplest" entropy is:

$$E(V, y) = y\frac{(V - \bar{V})^2}{2} + g\frac{(y - \bar{y})^2}{2}, \tag{8}$$

the corresponding flux being:

$$F(V, y) = yV\frac{(V - \bar{V})^2}{2} + gVy(y - \bar{y}) - g\bar{V}\frac{y^2}{2}. \tag{9}$$

Therefore, to obtain an asymptotic stabilizing boundary controller we consider the following Lyapunov function candidate given by (6):

$$R = \int_0^L [y\frac{(V - \bar{V})^2}{2} + g\frac{(y - \bar{y})^2}{2}]dx. \tag{10}$$

R is positive and is zero only at the equilibrium point $(\bar{V}, \bar{y})$. Using (7) we know that the time derivative of R is given by:

$$\dot{R} = F(V_0, y_0) - F(V_L, y_L), \tag{11}$$

where F is the flux given by (9), $V_0 = V(0, t)$, $V_L = V(L, t)$, $y_0 = y(0, t)$ and $y_L = y(L, t)$.

To make $\dot{R}$ negative we may use V_0 and V_L as control variables, since we know from (3) that V_0 and V_L are related to the physical control inputs u_a and u_b via the following relations:

$$u_a = \frac{V_0^2 y_0^2}{y_a - y_0} \text{ and } u_b = \frac{V_L^2 y_L^2}{y_L - y_b}. \tag{12}$$

In fact, we will consider small variations around $\bar{V}$ so that, for Lyapunov analysis we introduce the following control variables:

$$u_0 = V_0 - \bar{V} \text{ and } u_L = V_L - \bar{V}. \tag{13}$$

With these notations, the time derivative of R can be rewritten, using (11):

$$\begin{aligned} \dot{R} = y_0(u_0 + \bar{V})[u_0^2/2 + g(y_0 - \bar{y})] - \bar{V}gy_0^2/2 \\ -y_L(u_L + \bar{V})[u_L^2/2 + g(y_L - \bar{y})] + \bar{V}gy_L^2/2. \end{aligned} \tag{14}$$

We can now propose a class of boundary controllers, making R decrease, as stated in the following proposition.

Proposition 1 *If*

$$g\bar{y} > \bar{V}^2, \tag{15}$$

then the boundary control law u_a and u_b defined by (12), (13) with u_0 and u_L given by:

$$\begin{cases} u_0 = -(1 - \dfrac{\bar{y}}{y_0})(\dfrac{\bar{V}}{2} + \lambda_0), \\ u_L = -(1 - \dfrac{\bar{y}}{y_L})(\dfrac{\bar{V}}{2} - \lambda_L), \end{cases} \tag{16}$$

makes R decrease, i.e. $\dot{R} \leq 0$, in a neighborhood of $(\bar{V}, \bar{y})$, for gains λ_0 and λ_L such that:

$$\begin{cases} \lambda_0 > 0 \in [r_1, r_2] \textit{ with} \\ r_1 = \dfrac{2g\bar{y}\left(1 - \sqrt{1 - \bar{V}^2/(g\bar{y})}\right) - \bar{V}^2}{2\bar{V}}, \\ r_2 = \dfrac{2g\bar{y}\left(1 + \sqrt{1 - \bar{V}^2/(g\bar{y})}\right) - \bar{V}^2}{2\bar{V}} \textit{ and} \\ \lambda_L > 0. \end{cases} \tag{17}$$

Moreover, $\dot{R} = 0$ if and only if $V(0, t) = \bar{V}$, $y(0, t) = \bar{y}$, $V(L, t) = \bar{L}$ and $y(L, t) = \bar{y}$.

Proof : When applying the control (16), equation (14) leads to the following expression:

$$\dot{R} = -g\lambda_0(y_0 - \bar{y})^2 - g\lambda_L(y_L - \bar{y})^2$$
$$+y_0\frac{u_0^3}{2} + \bar{V}y_0\frac{u_0^2}{2} - y_L\frac{u_L^3}{2} - \bar{V}y_L\frac{u_L^2}{2}.$$

In fact, $u_0^3/2$ and $u_L^3/2$ are respectively negligible with respect to $\bar{V}u_0^2/2$ and $\bar{V}u_L^2/2$. Moreover, since $\bar{V}$ and y_L are positive we have: $-\bar{V}y_L\frac{u_L^2}{2} \leq 0$. Of course, the gains λ_0 and λ_L are chosen positive, then, for $\dot{R}$ remaining negative in a neighborhood of $(\bar{V}, \bar{y})$ we have to analyze the sign of the following quantity:

$$\bar{V}y_0\frac{u_0^2}{2} - g\lambda_0(y_0 - \bar{y})^2.$$

Replacing u_0 by its expression given in (16), $\dot{R}$ can be written in a neighborhood of the equilibrium point:

$$\dot{R} = \frac{A}{2\bar{y}}(y_0 - \bar{y})^2 + \frac{B}{2y_L}(y_L - \bar{y})^2, \tag{18}$$

with, if we consider $y_0 \simeq \bar{y}$:

$$A = \bar{V}\left(\frac{\bar{V}^2}{4} + \lambda_0\bar{V} + \lambda_0^2\right) - 2g\lambda_0\bar{y}$$

and

$$B = -\left(\bar{V}(\frac{\bar{V}}{2} - \lambda_L)^2 + 2g\lambda_L y_L\right).$$

Since $B \leq 0$, for $\dot{R}$ to be a negative semi-definite function of $(y_0 - \bar{y})$ and $(y_L - \bar{y})$, we must have $A \leq 0$ or equivalently:

$$A = \bar{V}\lambda_0^2 + (\bar{V}^2 - 2g\bar{y})\lambda_0 + \frac{\bar{V}^3}{4} \leq 0.$$

This polynomial in λ_0 is negative if its discriminant $\Delta = 4g\bar{y}(g\bar{y} - \bar{V}^2)$ is positive and if λ_0 is in $[r_1, r_2]$, where r_1 and r_2 are the roots of A given by (17). In fact, the condition Δ positive is ensured by (15) and therefore we conclude that $\dot{R} \leq 0$ in a neighborhood of the equilibrium point $(\bar{V}, \bar{y})$.

Moreover, using (18), we see that $\dot{R}$ is zero if and only if $y_0 = \bar{y}$ and $y_L = \bar{y}$, which from (16) implies $u_0 = u_L = 0$ and using (13), this leads to $V_0 = \bar{V}$ and $V_L = \bar{V}$.

This ends the proof. $\diamondsuit$

Now in the following theorem, we show that the solution $(V(x,t), y(x,t))$ locally exponentially converges to the equilibrium point.

Theorem 1 *There exist three strictly positive constants ϵ, M and μ such that, for any $(\tilde{V}, \tilde{y})$ in $C^1([0,L])^2$ satisfying the compatibility conditions:*

$$\left\{\begin{array}{l} \tilde{y}^2(0)(g\tilde{y}'(0) + \tilde{V}(0)\tilde{V}'(0)) + \\ \bar{y}(\bar{V}/2 + \lambda_0)(\tilde{V}(0)\tilde{y}'(0) + \tilde{V}'(0)\tilde{y}(0)) = 0 \\ \textit{and} \\ \tilde{y}^2(L)(g\tilde{y}'(L) + \tilde{V}(L)\tilde{V}'(L)) + \\ \bar{y}(\bar{V}/2 - \lambda_L)(\tilde{V}(L)\tilde{y}'(L) + \tilde{V}'(L)\tilde{y}(L)) = 0, \end{array}\right. \tag{19}$$

($'$ denoting the partial derivative w.r. to x) and such that:

$$\mid \tilde{V} - \bar{V} \mid_{C^1([0,L])} + \mid \tilde{y} - \bar{y} \mid_{C^1([0,L])} < \epsilon,$$

the hyperbolic system (1)-(2), with the boundary conditions (13) and (16) and the initial conditions:

$$V(x,0) = \tilde{V}(x),\ y(x,0) = \tilde{y}(x),\ \forall x \in [0,L],$$

has one and only one solution of class C^1 on $[0,L] \times [0,+\infty)$ and this solution satisfies:

$$\begin{array}{l} \mid V(.,t) - \bar{V} \mid_{C^1([0,L])} + \mid y(.,t) - \bar{y} \mid_{C^1([0,L])} \leq \\ M(\mid \tilde{V} - \bar{V} \mid_{C^1([0,L])} + \mid \tilde{y} - \bar{y} \mid_{C^1([0,L])})e^{-\mu t},\ \forall t \geq 0. \end{array}$$

Proof :

The proof of this theorem is a direct application of [gree, Theorem 2].

Let us rewrite (1)-(2) using Riemann invariants (see e.g. [serr]). Let:

$$\left\{\begin{array}{l} \alpha = V + 2\sqrt{gy} - \bar{V} - 2\sqrt{g\bar{y}} \\ \beta = V - 2\sqrt{gy} - \bar{V} + 2\sqrt{g\bar{y}}. \end{array}\right.$$

Then (1)-(2) is equivalent to:

$$\left\{\begin{array}{l} \dfrac{\partial\alpha}{\partial t} + C_1(\alpha,\beta)\dfrac{\partial\alpha}{\partial x} = 0 \\ \dfrac{\partial\beta}{\partial t} + C_2(\alpha,\beta)\dfrac{\partial\beta}{\partial x} = 0, \end{array}\right.$$

with:

$$\left\{\begin{array}{l} C_1(\alpha,\beta) = \dfrac{3}{4}\alpha + \dfrac{1}{4}\beta + \bar{V} + \sqrt{g\bar{y}}, \\ C_2(\alpha,\beta) = \dfrac{1}{4}\alpha + \dfrac{3}{4}\beta + \bar{V} - \sqrt{g\bar{y}}. \end{array}\right.$$

The boundary conditions (13) and (16) are, in the (α,β)-variables:

$$F_1(\alpha(0,t), \beta(0,t)) = 0\ ,\ F_2(\alpha(L,t), \beta(L,t)) = 0$$

with:

$$F_1(u,v) = \frac{u+v}{2} + \left(1 - \frac{16\bar{y}g}{(u - v + 4\sqrt{g\bar{y}})^2}\right)(\bar{V}/2 + \lambda_0),$$

$$F_2(u,v) = \frac{u+v}{2} + \left(1 - \frac{16\bar{y}g}{(u - v + 4\sqrt{g\bar{y}})^2}\right)(\bar{V}/2 - \lambda_L).$$

One has:

$$\left\{\begin{array}{l} C_1(0,0) = \bar{V} + \sqrt{g\bar{y}} > 0 \\ C_2(0,0) = \bar{V} - \sqrt{g\bar{y}} < 0 \\ F_1(0,0) = F_2(0,0) = 0. \end{array}\right. \tag{20}$$

Let us now compute the positive quantity A_1A_2 where:

$$A_1 = \mid \frac{\frac{\partial F_1}{\partial v}(0,0)}{\frac{\partial F_1}{\partial u}(0,0)} \mid \text{ and } A_2 = \mid \frac{\frac{\partial F_2}{\partial u}(0,0)}{\frac{\partial F_2}{\partial v}(0,0)} \mid.$$

We obtain:

$$A_1 = \mid \frac{2\sqrt{g\bar{y}} - \bar{V} - 2\lambda_0}{2\sqrt{g\bar{y}} + \bar{V} + 2\lambda_0} \mid$$

and

$$A_2 = \mid \frac{2\sqrt{g\bar{y}} + \bar{V} - 2\lambda_L}{2\sqrt{g\bar{y}} - \bar{V} + 2\lambda_L} \mid. \tag{21}$$

By (15) and (21), one has for all $\lambda_L > 0$:

$$A_2 < \frac{2\sqrt{g\bar{y}} + \bar{V}}{2\sqrt{g\bar{y}} - \bar{V}}.$$

Hence $A_1A_2 < 1$, which (see (20)) is the last condition required to apply [gree,Theorem 2], holds if:

$$A_1 < \frac{2\sqrt{g\bar{y}} - \bar{V}}{2\sqrt{g\bar{y}} + \bar{V}}. \tag{22}$$

But, for $\lambda_0 > 0$, (22) is equivalent to:

$$\lambda_0 < \frac{4g\bar{y} - \bar{V}^2}{2\bar{V}},$$

and therefore holds for $\lambda_0 \in]0, r_2[$. This ends the proof. $\diamondsuit$

Remark 1 *The compatibility conditions (19) are obtained by time differentiation of the boundary conditons (16) and using (1)-(2).*

4 Simulation results

We have considered a reach of length $L = 20\,m$ with outside levels:

$$y_a = 1\,m \text{ and } y_b = 0\,m.$$

We have chosen for the state equilibrium:

$$\bar{y} = 0.72\,m \text{ and } \bar{V} = 0.24\,m/s,$$

which of course satisfies condition (15). Initial conditions are $y(x,0) = 0.50\,m$ and $V(x,0) = 0\,m/s$ for all x in $[0, L]$. The control gains have been chosen as follows:

$$\lambda_0 = \lambda_L = 0.5,$$

one can easily check that conditions (17) are satisfied.

We have numerically integrated PDEs (1) and (2), using a semi-implicit Preissmann scheme with a spatial step $\Delta x = 1\,m$ and a weighting coefficient $\theta = 0.75$.

In Figure 2 the left and right water levels are displayed, when applying first our feedback control law (12), (13) with u_0 and u_L given by (16) and then (in dashed lines), when applying the open-loop constant controls $\bar{u}_a$ and $\bar{u}_b$ given by (12) when considering the values at the equilibrium point, i.e. $y_0 = y_L = \bar{y}$ and $V_0 = V_L = \bar{V}$.

In Figure 3 profiles of the reach are displayed at time $t = 5$s, also with our control law and then when applying the constant controls $\bar{u}_a$ and $\bar{u}_b$. One can observe that our closed-loop control strategy improves the transient behaviors, since oscillations are significantly reduced.

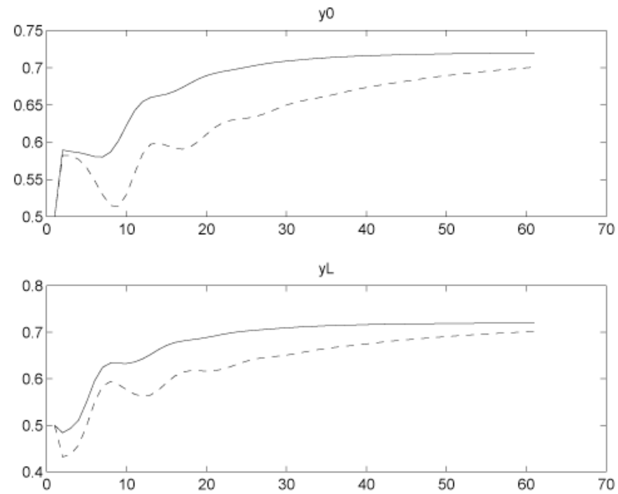

Figure 1: Left and right water levels

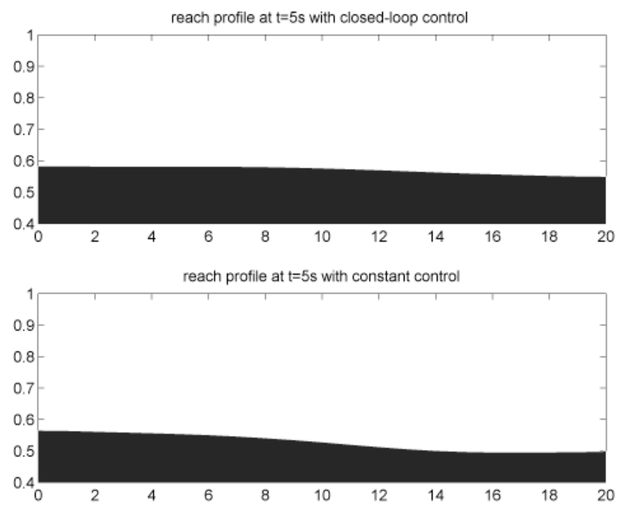

Figure 2: Reach profiles at t=5s with closed-loop and constant controls

5 Conclusion

Our main contribution in this paper has been to propose a Lyapunov control design strategy for a canal described by the Saint Venant equations. However, it must be emphasized that the proof of Theorem 1 does *not* rely on the Lyapunov approach. But it may be expected that with the Lyapunov approach, a more global result than Theorem 1 could be achieved. The main difficulty to get a global result is that, for large initial deviations from the equilibrium, wave shocks will appear (whatever are the controls). Therefore, it is needed to deal with entropic solutions, with, for physical reasons, equation (2) replaced by:

$$\frac{\partial(yV)}{\partial t} + \frac{\partial}{\partial x}(yV^2 + gy^2/2) = 0. \tag{23}$$

Note that, for solutions of class C^1, (1)-(2) is equivalent to (1)-(23) but not for solutions which are less regular.

On the whole real line, ($x \in (-\infty, +\infty)$), the existence of entropic solutions for (1)-(23) and large initial deviations is proved in [lion]. But, for $x \in [0, L]$ and with boundary conditions at $x = 0$ and $x = L$, as (13)-(16) with a suitable "trace" meaning, the existence of entropic solutions for large initial deviations is still open.

Finally, we have illustrated our control strategy by some numerical simulation results. Future works will consist in evaluating the robustness of our class of feedback laws with respect to some perturbations, such as small (but unknown) slopes or lateral leaks or withdrawals.

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