Copyright © IFAC Modelling and Control of
Biotechnological Processes, Noordwijkerhout, 1985

STABLE ADAPTIVE ALGORITHMS FOR ESTIMATION AND CONTROL OF FERMENTATION PROCESSES

D. Dochain and G. Bastin

Laboratoire d'Automatique, Dynamique, et Analyse des Systèmes, University of Louvain, Batiment Maxwell, 1348 Louvain-la-Neuve, Belgium

ABSTRACT : Fairly simple "ad hoc" adaptive estimation and control algorithms for bioreactors can be shown to be stable. We illustate our approach with four different examples of adaptive observers and adaptive regulators. For three of them a stability or convergence proof is given. The effectiveness of these algorithms is illustrated by simulation results.

1. INTRODUCTION

The aim of this paper is to present stable adaptive algorithms for parameter estimation, state estimation and regulation of fermentation processes.

We consider fermentation plants described by the following 2nd order state-space model :

$$\frac{dX}{dt} = \mu(t)X(t)-D(t)X(t) \qquad (1.a)$$

$$\frac{dS}{dt} = -k_1\mu(t)X(t)+D(t)[S_{in}(t)-S(t)] \qquad (1.b)$$

where X, S, S_{in} represent the biomass concentration, the limiting substrate concentration and the inlet substrate concentration respectively. The parameter k_1 represent the yield coefficient and the parameter $\mu(t)$ the specific growthrate. The dilution rate is D(t) (>0).

It is well known that model (1,a-b) is valid for continous stirred tank (CST) bioreactors (with a constant volume of culture) and for stirred tank fed batch bioreactors as well. In this later case, the volume of culture V(t) is variable and described by :

$$\frac{dV}{dt} = F(t) = D(t)V(t) \qquad (2)$$

with F(t) the input flow rate.

Throughout the paper, the specific growth rate $\mu(t)$ is assumed to be a completely unknown time varying parameter, in line with a number of recent works on estimation and control of fermentation processes (Holmberg and Ranta, 1982; Dekkers, 1983; Holmberg, 1983; Stephanopoulos and San, 1983; Nihtila et al., 1984; Dochain and Bastin, 1984; Stephanopoulos and San, 1984; Bastin and Dochain, 1985).

In case of aerobic fermentation processes, an additional equation is needed to describe the dynamics of the dissolved oxygen (DO) concentration (e.g. Olsson and Hansson, 1976) :

$$\frac{dC}{dt} = k_2W(t)[C_0-C(t)]-D(t)C(t)-k_3\mu(t)X(t)-k_4X(t) \qquad (3)$$

where C represents the DO concentration, C_0 the saturation DO concentration, W(t) the air flow rate and k_2, k_3, k_4 are constants.

Obviously Extended Kalman Filter algorithms can be used for adaptive parameter estimation and state estimation of non stationary and non linear systems like (1,a-b), (2), (3) (e.g. Stephanopoulos and San, 1984; Nihtila et al., 1984) but their stability is not easy to analyze.

In this paper, we show that fairly simple "ad hoc" adaptive estimation (and control as well) algorithms can be shown bo be stable. We illustrate our approach with four different examples of adaptive observers and regulators (one of them being specific to aerobic processes).

In section 2, we describe two adaptive observers :

1) on line estimation of $\mu(t)$ and S(t) from noisy measurements of X(t)

2) aerobic processes : on line estimation of $\mu(t)$, X(t), S(t) from measurements of C(t).

In section 3, we describe two adaptive regulators :

1) Adaptive regulation of unstable anaerobic digestion processes

2) Adaptive regulation of substrate concentration in fed batch processes with a liquid product

The stability and/or convergence of three of these algorithms are analyzed under the following set of (mild and realistic) assumptions :

A1.1 The specific growth rate is positive and bounded (the maximum growth rate μ^* is unknown) :

$$0 < \mu(t) < \mu^* \tag{4}$$

A1.2 The inputs $D(t)$, $S_{in}(t)$, $W(t)$ and the state variables $X(t)$, $S(t)$, $C(t)$ are positive (in accordance with the physical situation)

A1.3 The inlet substrate concentration $S_{in}(t)$ is bounded :

$$0 < S_{in}(t) < S_{mx} \tag{5}$$

A1.4 $\mu(t)=0$ when $S(t)=0$ or $C(t)=0$ (6)

A1.5 The initial conditions fulfill the following inequalities :

$$k_1 X(0) + S(0) < S_{mx} \qquad C(0) < C_0 \tag{7}$$

A1.6 The time derivative of $\mu(t)$ is bounded :

$$\left|\frac{d\mu}{dt}\right| < M_1 \tag{7}$$

Then, we have the following preliminary result (Goodwin et al., 1983; Dochain and Bastin, 1984) :

Lemma 1 : Under assumptions A1.1 to A1.5, the state variables $X(t)$, $S(t)$, $C(t)$ are bounded for all t :

$$0 < S(t) < S_{mx}$$

$$0 < X(t) < (1/k_1)S_{mx} \tag{9}$$

$$0 < C(t) < C_0$$

It must be emphasized that, from lemma 1, the states of the process are bounded without imposing any upper bound on the inputs $D(t)$ and $W(t)$.

For the readibility of the paper, the theoretical proofs are relegated in Appendix.

2. ADAPTIVE OBSERVERS

2.1. Example 1 : On line estimation of $\mu(t)$ and $S(t)$ from noisy measurements of $X(t)$.

Statement of the algorithm

Consider a bioractor described by equations (1.a-b).

Assume that :

- the inputs $D(t)$ and $S_{in}(t)$ are known.
- the yield coefficient k_1 is known
- a noisy measurement X_m of the biomass concentration X is available :

$$X_m(t) = X(t) + \epsilon(t) \tag{10}$$

with ϵ the measurement noise.

Then the following adaptive observer can be used
- to estimate on line the specific growth rate $\mu(t)$
- to estimate on line the substrate concentration $S(t)$:

$$\frac{d\hat{X}}{dt} = [\hat{\mu}(t) - D(t) + \lambda_1 (X_m(t) - \hat{X}(t))]X_m(t) \tag{11.a}$$

$$\frac{d\hat{\mu}}{dt} = \lambda_2 X_m(t)[X_m(t) - \hat{X}(t)] \tag{11.b}$$

$$\frac{d\hat{S}}{dt} = -k_1 \hat{\mu}(t) X_m(t) + D(t)[S_{in}(t) - \hat{S}(t)] \tag{11.c}$$

$\lambda_1 > 0$, $\lambda_2 > 0$

Convergence analysis

Defining the estimation errors:

$$ex = X(t) - \hat{X}(t), \quad e\mu = \mu(t) - \hat{\mu}(t)$$
$$es = S(t) - \hat{S}(t) \tag{12}$$

we have the following convergence results:

Assumptions B2.1

a) $X_m(t)$ is strictly positive :

$$X_m(t) > \eta > 0 \tag{13}$$

b) The measurement noise is bounded :

$$|\epsilon(t)| < M_2$$

c) $\lambda_1 = \frac{\alpha}{4} (\lambda_2)^2$

d) the dilution rate is bounded:

$$0 < D(t) < D_m$$

Theorem 2.1.

Under assumptions A1.1 to A1.6 and B2.1

a) $$\lim_{t\to\infty} |ex| < \left(\frac{\beta_1}{\lambda_1} + \beta_2\right)M_2 + \frac{\beta_3}{\lambda_1^2} M_1 \tag{16}$$

b) $$\lim_{t\to\infty} |e\mu| < (\beta_4 \lambda_1 + \beta_5)M_2 + \frac{\beta_6}{\lambda_1} M_1 = G \tag{17}$$

c) $$\lim_{t\to\infty} |es| < \beta_7 G \tag{18}$$

with β_i (i=1,7) positive constants independent of M_1, M_2, λ_1.

Comment

From expressions (16) and (17), it follows clearly that there exists an optimal value of the design parameter λ_1, minimizing the asymptotic upper bounds on $|e\mu|$ and $|es|$:

$$\lambda_1 = \sqrt{\left(\frac{\beta_4}{\beta_6}\right)}$$

Simulation results.

A simulation experiment has been performed for an anaerobic digestion process with a Monod specific growth rate and with a square wave input $S_{in}(t)$. Fig.1 illustrates the effectiveness of the method.
An application of this algorithm to real life data can be found in Bastin and Dochain (1985).

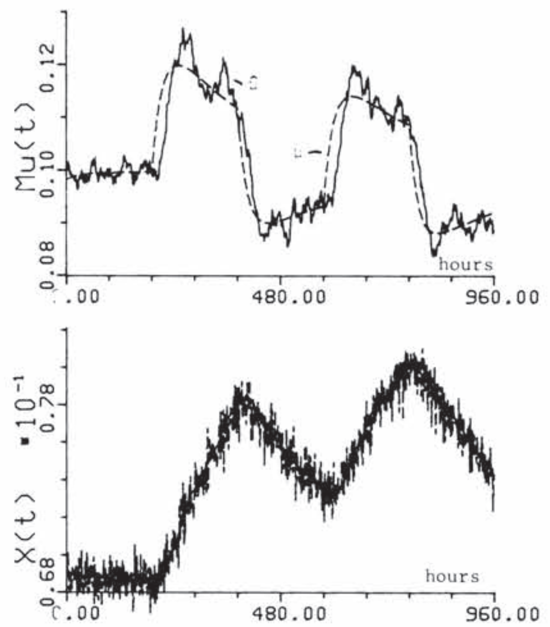

Fig. 1. Estimation of μ with noisy measurements of X(t).

2.2. Example 2 : (Aerobic processes) On line estimation of $\mu(t)$, X(t), S(t) from measurements of C(t).

Statement of the algorithm.

Consider a CST aerobic bioreactor described by equations (1,a-b) and (2).

Assume that

- the inputs $S_{in}(t)$, W(t) and D(t) are known
- the parameters k_i (i=1,4) and the saturation DO C_0 are known
- the DO concentration C(t) is available from measurement.

Then, the following adaptive observer can be used

- to estimate on line the specific growth rate $\mu(t)$
- the state variables X(t) and S(t):

$$\frac{d\hat{C}}{dt} = k_2 W(t)[C_0 - C(t)] - D(t)C(t) - k_3\hat{\varphi}(t) - k_4\hat{X}(t) + \lambda_1[C(t)-\hat{C}(t)] \qquad (19.a)$$

$$\frac{d\hat{X}}{dt} = \hat{\varphi}(t) - D(t)\hat{X}(t) - \lambda_2[C(t)-\hat{C}(t)] \qquad (19.b)$$

$$\frac{d\hat{\varphi}}{dt} = -\lambda_3[C(t)-\hat{C}(t)] \qquad (19.c)$$

$$\frac{d\hat{S}}{dt} = -k_1\hat{\varphi}(t) + D(t)[S_{in}(t)-\hat{S}(t)] \qquad (19.d)$$

$$\hat{\mu}(t) = \hat{\varphi}(t)/\hat{X}(t) \qquad (19.e)$$

$$\lambda_1,\ \lambda_2,\ \lambda_3 > 0$$

Clearly, the parameter $\varphi(t)$ is an on line estimate of the "biological activity" : $\varphi(t) = \mu(t)X(t)$.

Stability analysis.

We define the following estimation errors:

$$e_x = X(t)-\hat{X}(t)\ ,\quad e_\varphi = \varphi(t)-\hat{\varphi}(t)$$
$$e_c = C(t)-\hat{C}(t)\ ,\quad e_s = S(t)-\hat{S}(t) \qquad (20)$$

Assumptions B2.2

a) D(t) is strictly positive :

$$D(t) \geq \eta > 0 \quad \text{for all } t$$

b) $\lambda_1\lambda_2 > \lambda_3$

c) $\left|\frac{d\varphi}{dt}\right| < M_3$

Theorem 2.2

Under assumptions A1.1 to A1.6 and B2.2, there exists a positive finite constant β such that the estimation errors e_x, e_φ, e_c and e_s are all bounded by βM_3.

3. ADAPTIVE REGULATORS.

3.1. Adaptive regulation of unstable anaerobic digestion processes.

Statement of the algorithm.

We consider a CST anaerobic digestion process where the methanization step is rate limiting. The bacterial growth is assumed to be described by (1,a-b) while the methane gas production rate is expressed as follows:

$$Q(t) = k_5\mu(t)X(t) \qquad (27)$$

It is a well known fact that this process ca be unstable, due to inhibitory effects at high substrate concentrations (e.g. Antunes and Installé, 1981). We consider the problem of regulating the reactor when it is used for waste water treatment. The control objective is to regulate the output pollution concentration S(t) at a prescribed level S*, despite the fluctuations of the input pollution level $S_{in}(t)$, by using the dilution rate D(t) as control input.

From equations (1,a-b) and (27), we have:

$$\frac{dS}{dt} = -KQ(t) + D(t)[S_{in}(t)-S(t)] \qquad (28)$$

$$\text{with}\quad K = \frac{k_1}{k_5} \qquad (29)$$

Equation (28) can be viewed as a 1st order dynamic model of the system with a bounded time varying parameter Q(t) (the boundedness of Q(t) results from assumption A1.1 and Lemma 1). This equation is the basis for the derivation of the control algorithm.

Assuming that

- $S_{in}(t)$, S(t), Q(t) are measured on line
- the parameter K is a priori unknown;

Then, the following self-tuning algorithm can be used to regulate S(t) at the level S*:

Let

$$D^{o}(t) = \frac{C_1(t)[S^* - S(t)] + \hat{K}(t)Q(t)}{S_{in}(t) - S(t)} \qquad (29.a)$$

with $C_1(t)$ a strictly positive function ($C_1(t) > 0$) and $\hat{K}(t)$ an adaptive estimate of K.
The control input D(t) and the parameter estimate $\hat{K}(t)$ are calculated as follows:

$$\begin{cases} D(t) = D^{o}(t) & \text{if } 0 < D^{o}(t) < D_m \\ D(t) = 0 & \text{if } D^{o}(t) < 0 \\ D(t) = D_m & \text{if } D^{o}(t) > D_m \end{cases} \qquad (29.b)$$

$$\begin{cases} \hat{K}(0) > 0 \\ \frac{d\hat{K}}{dt} = C_2 Q(t)[S^* - S(t)] \\ \quad \text{except if } \hat{K}(t)=0 \text{ and } S(t)>S^* \\ \quad \text{then } \frac{d\hat{K}}{dt} = 0 \text{ (to ensure } \hat{K}(t) \geq 0 \ \forall t) \end{cases} \qquad (29.c)$$

If we define

$es = S^* - S(t)$ and $ek = K - \hat{K}(t)$

the closed-loop system (28),(29) can be written, when $D(t) = D^{*}(t)$:

$$\frac{d}{dt}\begin{bmatrix} es \\ ek \end{bmatrix} = \begin{bmatrix} -C_1(t) & Q(t) \\ -C_2 Q(t) & 0 \end{bmatrix}\begin{bmatrix} es \\ ek \end{bmatrix} \qquad (30)$$

i.e; as a linear time varying unforced system, with a bounded coefficient matrix.

Convergence analysis.

Assumptions B3.1.

a) The input pollution level is bouded as follows :

$S_{mn} < S(t) < S_{mx}$

b) $S^* < S_{mn}$

c) $D_m > \frac{\mu^* S_{mx}}{S_{mn} - S^*}$

Theorem 3.1.

Under assumptions A1.1 to A1.5 and B3.1,

a) all the closed-loop system variables are bounded;

b) $\lim_{t\to\infty} S(t) = S^*$;

c) $\lim_{t\to\infty} \hat{K}(t) = K$;

d) there exists t_1 such that $D(t) = D^{o}(t)$ for all $t > t_1$;

e) if $C_1(t) = \alpha Q(t)$ ($\alpha > 0$), then the closed loop system is exponentially stable.

Comments.

1) This theorem is based on the assumption that the methane flow rate Q(t) is strictly positive for all t. Within the context of our previous assumptions, it is not possible to **prove** that $Q(t) > 0$ since Q(t) depends on $\mu(t)$ and since we do not specify $\mu(t)$. However for simple models of $\mu(t)$, it is fairly easy to show that Q(t) is effectively > 0 for all t.

2) It is worth noting that the regulator achieves a zero steady-state error even with a varying input disturbance $S_{in}(t)$, since the algorithm includes a feedforward action (see the simulations in the next section).

Simulation results.

A simulation experiment has been performed for an unstable biomethanization plant with a Haldane specific growth rate (fig.2). A pulse of toxic substance is introduced after 1 day. Fig 2. shows clearly the stabilization of the plant by the adaptive controller (while in open loop it is led to wash-out).

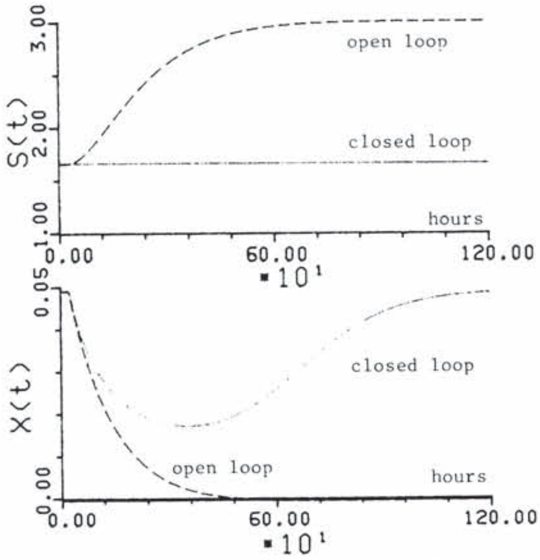

Fig. 2. Adaptive control of an unstable anaerobic digestion process.

3.2. Adaptive regulation of substrate concentration in a fed batch reactor with a liquid product.

Statement of the algorithm.

We consider a fed batch bioreactor described by equations (1,a-b) and (2), with a reaction product, in liquid phase, whose production dynamics is described by the following equation:

$$\frac{dP}{dt} = \gamma(S(t))X(t) - D(t)P(t) \qquad (31)$$

where P(t) is the concentration of the reaction product. The specific production rate $\gamma(S(t))$ is assumed to be a non linear function of S(t), inhibited at high concentrations: a typical example, taken from Takamatsu et al. (1974), is shown in fig.3.

Our conjecture is then that adaptive

regulation of S(t) at values corresponding to a maximum production rate should contribute to a maximization of the yield of the reactor (without any modelling of $\mu(t)$ or $\nu(S(t))$ being necessary, unlike earlier optimal control studies, e.g. Ohno et al., 1975)

In accordance with assumption A1.4, we define the specific growth rate as follows:

$$\mu(t) = \rho(t)S(t) \qquad (32)$$

Then, equation (1.b) is rewritten:

$$\frac{dS}{dt} = -\varphi(t)S(t) + D(t)[S_{in}(t)-S(t)] \qquad (33)$$

$$\text{with } \varphi(t) = k_1\rho(t)X(t) \qquad (34)$$

This equation is the basis for the derivation of the regulation algorithm.

Assuming that

- the flow rate is constant : $F(t) = F_0$ (until the tank is full, of course);
- the inlet substrate concentration $S_{in}(t)$ is the control input;
- the concentration S(t) is measured on line.

Then the following adaptive algorithm can be used to regulate S(t) at a reference level S* :

$$S_{in}(t) = \frac{1}{D_0}\{C_1[S^* - S(t)] + [D_0 + \hat{\varphi}(t)]S(t)\} \qquad C_1 > 0$$

where $\hat{\varphi}(t)$ is an on line estimate of $\varphi(t)$ updated as follows :

$$\frac{d\hat{\varphi}}{dt} = C_2S(t)[S^* - S(t)]$$

except if $\hat{\varphi}(t)=0$ and $S(t)>S^*$,

then $\frac{d\hat{\varphi}}{dt} = 0$ (to ensure $\hat{\varphi}(t) \geq 0 \ \forall t$).

Simulation results.

On fig.4, we compare closed loop and open loop performance of a fed batch process with a Monod specific growth rate and the specific production rate of fig.3. In both cases the reactor is fed with the same amount of substrate. In open loop, a constant input $S_{in}(t)$ is used, while in closed loop the input $S_{in}(t)$ is computed by the control algorithm. Fig.4 show that the closed loop operation increases the final product concentration by nearly 40%.

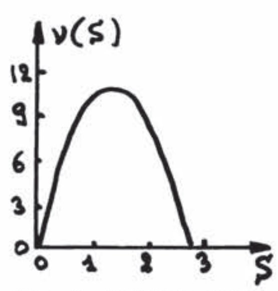

Fig. 3. Specific production rate.

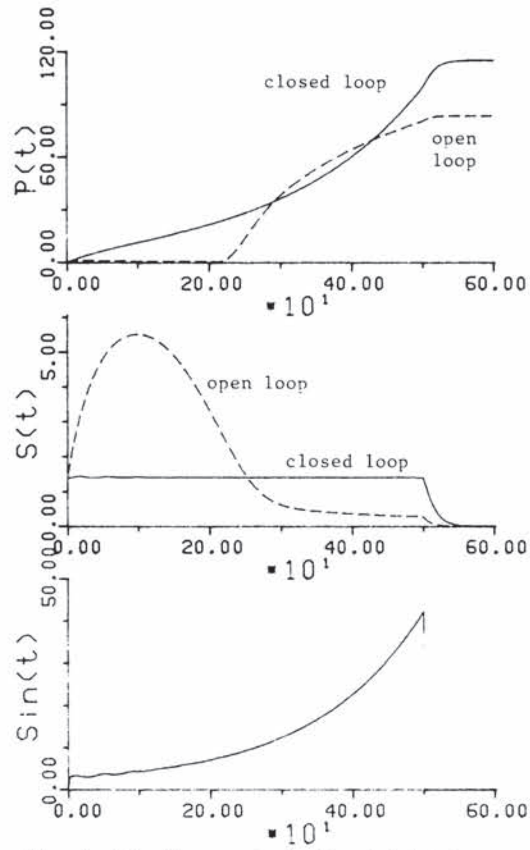

Fig. 4. Adaptive control of a fed-batch process.

APPENDIX.

PROOF OF THEOREM 2.1.

step 1: Defining $e^t = (ex, e\mu)^t$ the following error system" derives from (11.a-b) :

$$\frac{de}{dt} = X_m(t)Ae(t) + B(t)u(t) \qquad (A1)$$

$$\text{with } A = \begin{bmatrix} -\lambda_1 & 1 \\ -\lambda_2 & 0 \end{bmatrix} \qquad (A2)$$

$$B(t) = \begin{bmatrix} D(t) - \mu(t) - \lambda_1 X_m(t) & 0 \\ -\lambda_2 X_m(t) & 1 \end{bmatrix} \qquad (A3)$$

$$u(t) = \begin{bmatrix} \epsilon(t) \\ \frac{d\mu}{dt} \end{bmatrix} \qquad (A4)$$

step 2: The matrix A is stable (since λ_1, λ_2 are >0) and we denote its eigenvalues by $-p_1$, $-p_2$ (<0).

step 3: The solution of system (A1) ca be written :

$$e(t) = \exp[X_m(t)At] + \int_0^t PR^{-1}(t)R(s)P^{-1}B(s)u(s)ds \qquad (A5)$$

with P = matrix of eigenvectors of A and

$$R(t) = \text{diag}\ (\ \exp[p_1\zeta(t)]\ ,\ \exp[p_2\zeta(t)]\ )$$

with $\zeta(t) = \int_0^t X_m(\tau)d\tau$

step 4: $\exp[X_m(t)At]$ tends exponentially to zero, by assumption B2.1 and step 2.

step 5: By calculating the last term of (A5), one obtains :

$$\lim_{t\to\infty} ex = \lim_{t\to\infty} \int_0^t \Big(\Big[\frac{1-\omega}{\omega} z_1(s) - \frac{1+\omega}{\omega} z_2(s)\Big][D(s)-\mu(s) - \lambda_1 X_m(s)]\epsilon(s) + \frac{2}{\lambda_1\omega}[z_2(s)-z_1(s)][-\lambda_2 X_m(s)\epsilon(s) + \frac{d\mu}{ds}]\Big) ds \qquad (A8)$$

$$\lim_{t\to\infty} e\mu = \lim_{t\to\infty} \int_0^t \Big(\frac{\alpha\lambda_1}{2\omega}[z_1(s)-z_2(s)][D(s)-\mu(s)-\lambda_1 X_m(s)] + \Big[\frac{1+\omega}{\omega} z_1(s) - \frac{1-\omega}{\omega} z_2(s)\Big][-\lambda_2 X_m(s)\epsilon(s) + \frac{d\mu}{ds}]\Big) ds \qquad (A9)$$

with $\omega = \sqrt{1-\alpha}$

$$z_i(t) = \exp(-p_i\zeta(t)) \quad (i=1,2)$$

step 6: then by computing the asymptotic bounds of (A8) and (A9) one obtains:

$D_m < \mu^* + \lambda_1 M_0$	$D_m > \mu^* + \lambda_1 M_0$
$\beta_1 = 4\mu^*/\eta\omega$	$\beta_1 = 4D_m/\eta\omega$
$\beta_2 = 6M_0/\eta\omega$	$\beta_2 = 2M_0/\eta\omega$
$\beta_3 = 8/\eta\alpha\omega$	$\beta_3 = 8/\eta\alpha\omega$
$\beta_4 = (4-\alpha)M_0/\eta\omega$	$\beta_4 = (2-\alpha)M_0/\eta\omega$
$\beta_5 = 2\mu^*/\eta\omega$	$\beta_5 = 2M_0/\eta\omega$
$\beta_6 = 4(2-\alpha)/\eta\alpha\omega$	$\beta_6 = 4(2-\alpha)/\eta\alpha\omega$

with $M_0 = (1/k_1)S_{mx} + M_2$

Then the asymptotic upper bound on es follows immediately with $\beta_7 = D_m/k_1\eta$.

▽▽▽

PROOF OF THEOREM 2.2.

step 1: Defining $e^t = (ec, ex, e\rho)^t$ the following "error system" derives from equations (19.a-c):

$$\frac{de}{dt} = A(t)e(t) + Bu(t)$$

with $A(t) = \begin{bmatrix} -\lambda_1 & -k_4 & -k_3 \\ \lambda_2 & -D(t) & 1 \\ \lambda_3 & 0 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$u(t) = d\rho/dt$$

step 2: Let the following quadratic function :

$$V = k_4\lambda_3 ex^2 + \lambda_2\lambda_3 ec^2 + (\lambda_2 k_3 + \lambda_1)e\rho^2 + 2\lambda_3 ece\rho$$

This function is positive definite, since by assumption B2.2 we have :

$$\lambda_3 < \sqrt{\lambda_3^2 + \lambda_2^2\lambda_3 k_3} < \sqrt{\lambda_1\lambda_2\lambda_3 + \lambda_2^2\lambda_3 k_3} = \sqrt{\lambda_2\lambda_3(\lambda_2 k_3 + \lambda_1)}$$

Now $\frac{dV}{dt} = -k_4\lambda_3 D(t)ex^2 - \lambda_3(\lambda_1\lambda_2 - \lambda_3)ec^2 - k_3\lambda_3 e\rho^2$

which is negative definite, by assumption B2.2.

Hence the free system $\frac{de}{dt} = A(t)e(t)$ is exponentially stable.

step 3: Then it is a standard result of stability theory (Willems, 1970, theorem 3.1) that there exists a positive constant β such that:

$$\lim_{t\to\infty} \|e\| < \beta M_3$$

and the boundedness of es follows readily.

▽▽▽

PROOF OF THEOREM 3.1.

a) trivial from lemma 1 and by the definition of the control algorithm.

b) see Dochain and Bastin, 1984b.

c) and d) Since S(t) is continuous and derivable, we have from b):

$$\lim_{t\to\infty} \frac{dS}{dt} = 0 \text{ and } \lim_{t\to\infty} D(t) = \lim_{t\to\infty} \frac{KQ(t)}{S_{in}(t)-S^*}$$

Now $0 < \frac{K\epsilon}{S_{mx}-S^*} < \frac{KQ(t)}{S_{in}(t)-S^*} < \frac{KQ_{mx}}{S_{mn}-S^*} < D_m$

Hence $\lim_{t\to\infty} D(t) = \lim_{t\to\infty} D^0(t) = \lim_{t\to\infty} \frac{\hat{K}(t)Q(t)}{S_{in}(t)-S^*}$

and c) and d) follows readily.

e) for $t > t_1$, the closed loop system is equivalent to th system (30). We prove that this system is exponentially stable when $C_1(t) = \alpha Q(t)$ and $\alpha > C_2 > 1$: let the following positive definite function

$$V(es, ek) = \frac{\alpha}{2(C_2-1)}(es^2 + ek^2) - esek$$

then

$$\frac{dV}{dt} = -Q(t)\Big[\Big(\frac{\alpha^2}{C_2-1} - C_2\Big)es^2 + ek^2\Big]$$

is clearly, a negative definite function and the system (30) is exponentially stable for $t > t_1$. And, from a), the closed loop system is exponentially stable for all $t > 0$.

REFERENCES.

Antunes S. and M.Installe (1981). The use of phase plane analysis in the modelling of a biomethanization process. Proc. 8th IFAC World Cong., Kyoto, 22, pp 165-170.

Bastin G. and D.Dochain (1985). Adaptive estimation of microbial growth rates. 7th IFAC Symp. on Identification, York,U.K, 3-7 July

Dekkers R.M. (1983). State estimation of fedbatch baker's yeast fermentation. in Mod. and Cont. of Bioteh. Proc. (ed. Halme). Pergamon,pp 201-212.

Dochain D. and G.Bastin (1984). Adaptive identification and control algorithms for non linear bacterial growth systems. Automatica,20(5), pp621-634.

Dochain D. and G.Bastin (1984b). Stability and convergence analysis of a continuous time adaptive control algorithm for fermentation processes. in Anal. and Opt. of Syst. Springer Verlag, pp468-474.

Goodwin G.C., B.McInnis and R.S.Long (1987). Adaptive control algorithms for waste water treatment and pH neutralization. Opt. Cont. Appl. and Meth., 3(4), pp443-460.

Holmberg A. (1983). A micro-processor based estimation and control system for the activated sludge process. in Mod. and Cont. of Biotech. Proc. (ed. Halme), Pergamon, pp 111-120.

Holmberg A. and J.Ranta (1982). Procedures for parameter and state estimation of microbial growth process models. Automatica,13,181-193.

Nihtila M., P Harmo, M.Perttula (1984). Real time growth estimation in batch fermentation. Proc. 9th IFAC World Cong., July 2-6, 225-230.

Ohno H., E.Nakanishi, T.Takamatsu (1976) Optimal control of a semi batch fermentation. Biotech. and Bioeng., 18, pp847-864.

Olsson G and O.Hansson (1976). Modelling and identification of an activated sludge process. Proc. 4th IFAC Symp. on Ident., Tbilissi.

Stephanopoulos G. and K-Y San (1983) On line estimation of time varying parameters: application to biochemical reactors. in Mod. and Cont. of Biotech. Proc. (ed. Halme), Pergamon, pp195-199.

Stephanopoulos and K-Y San (1984). Studies on on-line bioreactor identification. Biotech. and Bioeng., 26, pp 1176-1188.

Takamatsu T. et al. (1975). Theory and practice of optimal control in continuous fermentation process. Automatica, 11, pp 141-148.

Willems J.L. (1970). Stability theory of dynamical systems. Nelson.