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A strict Lyapunov function for boundary control of hyperbolic systems of conservation laws

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***Abstract*— We present a strict Lyapunov function for hyperbolic systems of conservation laws that can be diagonalised with Riemann invariants. The time derivative of this Lyapunov function can be made strictly definite negative by an appropriate choice of the boundary conditions. It is shown that the derived boundary control allows to guarantee the local convergence of the state towards a desired set point. Furthermore, the control can be implemented as a feedback of the state only measured at the boundaries. The control design method is illustrated with an hydraulic application, namely the level and flow regulation in an horizontal open channel.**

***Index Terms*— Partial differential equations, Hyperbolic systems, Boundary control, Lyapunov function, Conservation laws.**

I. INTRODUCTION

In this paper, we are concerned with systems of conservation laws that are described by partial differential equations, with an independent time variable $t \in [0, +\infty)$ and an independent space variable on a finite time interval $x \in [0, L]$. For such systems, the boundary control problem that we consider is the problem of designing control actions at the boundaries (i.e. at $x = 0$ and $x = L$) in order to ensure that the smooth solution of the Cauchy problem converges to a desired steady-state. In previous papers (see [4] et [5]), we have used an entropy of the system as a Lyapunov function having a *semi negative definite* time derivative. In the present paper, our contribution is different : assuming that the system can be diagonalised with the Riemann invariants, we exhibit a strict Lyapunov function which is an extension of the entropy but whose time derivative can be made *strictly negative definite* by an appropriate choice of the boundary controls. We give a theorem which shows that the boundary control allows to prove the local convergence of the system trajectories toward s a desired set point. Furthermore, the control can be implemented as a feedback of the state only measured at the boundaries. The control design method is illustrated with an hydraulic application : the regulation of the level and the flow in an horizontal reach of an open channel. For the sake of simplicity, our presentation is limited to second order systems (i.e. systems of two scalar conservation laws). But, as we shall indicate in the conclusions, the method can be easily extended to a wide class of higher order systems.

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II. BOUNDARY CONTROL OF HYPERBOLIC SYSTEMS OF CONSERVATION LAWS : STATEMENT OF THE PROBLEM

Let Ω an non-empty connected open set in $I\!R^2$. We consider a system of two conservation laws of the general form :

$$\partial_t Y + \partial_x f(Y) = 0 \tag{1}$$

where :

- t and x are the two independent variables : a time variable $t \in [0, +\infty)$ and a space variable $x \in [0, L]$ on a finite interval;
- $Y = (y_1 \;\; y_2)^T; [0, +\infty) \times [0, L] \to \Omega$ is the vector of the two dependent variables;
- $f : \Omega \to I\!R^2$ of class C^1 is the flux density.

We are concerned with the *smooth* solutions of the Cauchy problem for the system (1) over $[0, +\infty) \times [0, L]$ under an initial condition :

$$Y(0, x) = Y_0(x) \qquad x \in [0, L]$$

and two boundary conditions of the form :

$$b_0(Y(t, 0), u_0(t)) = 0 \qquad t \in [0, +\infty)$$
$$b_L(Y(t, L), u_L(t)) = 0 \qquad t \in [0, +\infty)$$

where $u_0 : [0, +\infty) \to I\!R$ and $u_L : [0, +\infty) \to I\!R$ are the control actions.

Steady state : For constant control actions $u_0(t) = \bar{u}_0$ et $u_L(t) = \bar{u}_L$, a steady state solution is a constant solution $Y(t, x) = \bar{Y}$ $\forall t \in [0, +\infty)$, $\forall x \in [0, L]$ which satisfies (1) and the boundary conditions $b_0(\bar{Y}, \bar{u}_0) = 0$ and $b_L(\bar{Y}, \bar{u}_L) = 0$.

The *boundary control problem* is then the problem of finding control actions $u_0(t)$ et $u_L(t)$ such that, for any smooth enough initial condition $Y_0(x)$, the Cauchy problem has a unique smooth solution converging towards a desired steady state $\bar{Y}$ (called " set point ").

In this paper, we consider the special case where :

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1) the system (1) is strictly hyperbolic, i.e. the Jacobian matrix of the application f :

$$A(Y) = \frac{\partial f}{\partial Y}$$

has two non-zero real distinct eigenvalues for all $Y \in \Omega$; it is furthermore assumed that the two eigenvalues of $A(Y)$ have opposite signs : $\lambda_2(Y) < 0 < \lambda_1(Y) \;\; \forall Y \in \Omega$;

2) each control action is expressed as a feedback of the system state at the boundaries :

$$\begin{aligned} u_0(t) &= \tilde{u}_0(Y(t,0), Y(t,L)) \\ u_L(t) &= \tilde{u}_L(Y(t,0), Y(t,L)). \end{aligned}$$

III. CHARACTERISTIC FORM AND RIEMANN INVARIANTS

Under the above assumptions, the system (1) is rewritten as :

$$\partial_t Y + A(Y)\partial_x Y = 0. \qquad (2)$$

This sytem can be diagonalised with the *Riemann invariants* (see for instance[1, Tome II, Chap. 12]). This means that there exists a change of coordinates $\xi(Y) = (\xi_1(Y) \;\; \xi_2(Y))^T$ whose Jacobian matrix is denoted $D(Y)$:

$$D(Y) = \frac{\partial \xi}{\partial Y}$$

and diagonalises $A(Y)$ in Ω :

$$D(Y)A(Y) = \Lambda(Y)D(Y) \qquad Y \in \Omega$$

with :

$$\Lambda(Y) = \text{diag}(\lambda_1(Y), \lambda_2(Y))$$

In the coordinates ξ, the system (2) can then be rewritten in the following (diagonal) *characteristic form* :

$$\partial_t \xi + \Lambda(\xi)\partial_x \xi = 0 \qquad (3)$$

where $\Lambda(\xi) = \text{diag}(\lambda_1(\xi), \lambda_2(\xi))$ with $\lambda_i(\xi)$ the eigenvalues of $A(Y)$ expressed in the ξ coordinates. Indeed, premultiplying equation (2) by the matrix$D(Y)$ gives :

$$\begin{aligned} & D(Y)\partial_t Y + D(Y)A(Y)\partial_x Y = 0 \\ \Longleftrightarrow \quad & D(Y)\partial_t Y + \Lambda(Y)D(Y)\partial_x Y = 0 \\ \Longleftrightarrow \quad & \partial_t \xi + \Lambda(\xi)\partial_x \xi = 0. \end{aligned}$$

We observe that each quantity :

$$\partial_t \xi_k + \lambda_k \partial_x \xi_k \qquad k = 1,2$$

can then be viewed as the total derivative $d\xi_k/dt$ of the function $\xi_k(t,x)$ at a point (t,x) of the plane, along a curve having a slope :

$$\frac{dx}{dt} = \lambda_k(\xi).$$

This curve is called " charcteristic curve " and the solution $\xi_k(t,x)$ is called " characteristic solution ". Since $d\xi_k/dt = 0$ on the characteristic curve, it follows that $\xi_k(t,x)$ is constant along the characteristic curve. this explains why the characteristic solutions are called *Riemann invariants*.

The change of coordinates $\xi(Y)$ is clearly defined up to a constant. It can therefore be selected in such a way that $\xi(\bar{Y}) = 0$ and the control problem can be restated as the problem of determining the control actions in such a way that the characteristic solutions converge towards the origin. Our contribution in this paper is to propose a control design method based on a strict Lyapunov function that is presented in the next section.

IV. A STRICT LYAPUNOV FUNCTION FOR BOUNDARY CONTROL DESIGN

Let us consider the linear approximation of the characteristic form (3) around the origin :

$$\begin{cases} \partial_t \xi_1 + c_1 \partial_x \xi_1 = 0 \\ \partial_t \xi_2 - c_2 \partial_x \xi_2 = 0 \end{cases} \qquad (4)$$

with $c_1 = \lambda_1(\bar{Y})$ et $c_2 = -\lambda_2(\bar{Y})$ (let us remind that $\lambda_2(\bar{Y}) < 0 < \lambda_1(\bar{Y})$).

With a view to the boundary control design, the following candidate Lyapunov function is introduced (see also [8]) :

$$U(t) = U_1(t) + U_2(t)$$

with :

$$\begin{aligned} U_1(t) &= \frac{1}{c_1}\int_0^L \xi_1^2(t,x)e^{-(\mu/c_1)x}dx, \\ U_2(t) &= \frac{1}{c_2}\int_0^L \xi_2^2(t,x)e^{+(\mu/c_2)x}dx. \end{aligned}$$

The time derivative of this function along the trajectories of the linear approximation (4) is :

$$\begin{aligned} \dot{U} = \; & - \; \mu(U_1(t) + U_2(t)) \\ & - \; \left[\xi_1^2(t,x)e^{-(\mu/c_1)x}\right]_0^L \\ & + \; \left[\xi_2^2(t,x)e^{+(\mu x/c_2)x}\right]_0^L \end{aligned}$$

which implies :

$$\begin{aligned} \dot{U} = \; & - \; \mu(U_1(t) + U_2(t)) \\ & - \; \left[e^{-(\mu/c_1)L}\xi_1^2(t,L) - \xi_1^2(t,0)\right] \\ & - \; \left[\xi_2^2(t,0) - e^{(\mu/c_2)L}\xi_2^2(t,L)\right]. \end{aligned}$$

It can be seen that the two last terms depend only on the Riemann invariants at the two boundaries, i.e. at $x = 0$ and at $x = L$. The control laws $u_0(t)$ et $u_L(t)$ can then be defined in order to make these terms negative along the system trajectories.

A simple solution is the to select $u_0(t)$ such that :

$$\xi_1(t,0) = -k\xi_2(t,0) \qquad | k | < 1 \qquad (5)$$

and $u_L(t)$ such that :

$$\xi_2(t,L) = -kA\xi_1(t,L) \qquad | k |< 1 \tag{6}$$

with :

$$A = \sqrt{\exp\left[-(\frac{\mu}{c_1}+\frac{\mu}{c_2})L\right]}.$$

The time derivative of the Lyapunov function is then written :

$$\begin{aligned}\dot{U}(t) = & - \mu(U_1(t)+U_2(t)) \\ & - (1-k^2)\left[e^{-(\mu/c_1)L}\xi_1^2(t,L)+\xi_2^2(t,0)\right]\end{aligned} \tag{7}$$

It can be seen that $\dot{U}(t) \leq -\mu U$ along the trajectories of the linear approximation (4) and that $\dot{U}(t) = 0$ if and only if $\xi_1(t,x) = \xi_2(t,x) = 0$ (i.e. at the system equilibrium).

In the next section, we will show that these boundary controls for the linearized system (4), can also be applied to the non-linear system (3) with the guarantee that the trajectories locally converge to the origin.

V. CONVERGENCE ANALYSIS

Theorem. *For all $\gamma \in [0,\mu)$, there exist two strictly positive constants ε and M such that, for any initial conditions $(\tilde{\xi}_1(x), \tilde{\xi}_2(x))$ in $H^2(0,L)^2$ satisfying the compatibility conditions :*

$$\left\{\begin{array}{l} c_1\partial_x\tilde{\xi}_1(0) = -kc_2\,\partial_x\tilde{\xi}_2(0) \\ c_2\partial_x\tilde{\xi}_2(L) = kAc_1\,\partial_x\tilde{\xi}_1(L)\end{array}\right. \tag{8}$$

and such that :

$$| \tilde{\xi}_1(x) |_{H^2(0,L)} + | \tilde{\xi}_2(x) |_{H^2(0,L)} < \varepsilon,$$

the nonlinear system (3), with the boundary conditions (5) et (6) and the initial conditions :

$$\xi_1(0,x) = \tilde{\xi}_1(x),\ \xi_2(0,x) = \tilde{\xi}_2(x),\ \forall\, x\, \in\, [0,L],$$

has a unique continuous solution in $H^2(0,L)^2$ for $t \in [0,+\infty)$ which satisfies :

$$\begin{aligned}& | \xi_1(t,.) |_{H^2(0,L)} + | \xi_2(t,.) |_{H^2(0,L)} \leq \\ & M(| \tilde{\xi}_1 |_{H^2(0,L)} + | \tilde{\xi}_2 |_{H^2(0,L)})e^{-\gamma t}\,,\ \forall\, t \geq 0.\end{aligned}$$

The inequality $\dot{V}(t) \leq -\mu V$ ensures the convergence in $L^2(0,L)$ norm of the solutions of the linear system (4). As mentioned in the statement of the above Theorem, in order to extend the analysis to the case of the nonlinear system (3), it is needed to prove a convergence in $H^2(0,L)$ norm (see for instance [2, Chap. 16, Sec. 1]).

The detailed proof of this Theorem is too long for this short paper. In this communication, we limit ourselves to a sketch of the proof. The fool proof can be found in the extended version [3]. The system(3) is rewritten as follows :

$$\left\{\begin{array}{l} \partial_t\xi_1 + c_1(\xi_1,\xi_2)\partial_x\xi_1 = 0 \\ \partial_t\xi_2 - c_2(\xi_1,\xi_2)\partial_x\xi_2 = 0\end{array}\right. \tag{9}$$

We then consider the dynamics of the firs-order and second-order spatial derivatives of ξ_1 et ξ_2, denoted as follows :

$$\begin{aligned}&\zeta_1(t,x) = \partial_x\xi_1(t,x) \ \text{ et } \ \zeta_2(t,x) = \partial_x\xi_2(t,x),\\ &\eta_1(t,x) = \partial_x\zeta_1(t,x) \ \text{ et } \ \eta_2(t,x) = \partial_x\zeta_2(t,x),\end{aligned}$$

By a time differentiation and using the equation (9), it is readily shown that these quantities satisfy the following dynamics :

$$\left\{\begin{array}{l} \partial_t\zeta_1 + c_1(\xi_1,\xi_2)\partial_x\zeta_1 + \zeta_1\phi_1 = 0 \\ \partial_t\zeta_2 - c_2(\xi_1,\xi_2)\partial_x\zeta_2 - \zeta_2\phi_2 = 0\end{array}\right. \tag{10}$$

$$\left\{\begin{array}{l} \partial_t\eta_1 + c_1(\xi_1,\xi_2)\partial_x\eta_1 + \eta_1\phi_1 + \zeta_1\partial_x\phi_1 = 0 \\ \partial_t\eta_2 - c_2(\xi_1,\xi_2)\partial_x\eta_2 - \eta_2\phi_2 - \zeta_2\partial_x\phi_2 = 0\end{array}\right. \tag{11}$$

with

$$\begin{aligned}\phi_1 &= \zeta_1\frac{\partial c_1}{\partial\xi_1} + \zeta_2\frac{\partial c_1}{\partial\xi_2}\\ \phi_2 &= \zeta_1\frac{\partial c_2}{\partial\xi_1} + \zeta_2\frac{\partial c_2}{\partial\xi_2}\end{aligned}$$

The key point is that the linear approximations (around zero) of systems (10) and (11) have the following form :

$$\left\{\begin{array}{l} \partial_t\zeta_1 + c_1\partial_x\zeta_1 = 0 \\ \partial_t\zeta_2 - c_2\partial_x\zeta_2 = 0\end{array}\right.$$

$$\left\{\begin{array}{l} \partial_t\eta_1 + c_1\partial_x\eta_1 = 0 \\ \partial_t\eta_2 - c_2\partial_x\eta_2 = 0\end{array}\right.$$

This is exactly the same form as the linear approximation (4) of the original system (9). Then, in order to prove that the solutions of the global system (9)-(10)-(11) converge to zero, it is quite natural to consider an extended Lyapunov function of the form :

$$U(t) + V(t) + W(t)$$

where $V(t)$ and $W(t)$ have the format of $U(t)$:

$$\begin{aligned}V(t) &= a_1V_1(t) + a_2V_2(t)\\ V_1(t) &= \frac{1}{c_1}\int_0^L \zeta_1^2(t,x)e^{-(\mu/c_1)x}dx,\\ V_2(t) &= \frac{1}{c_2}\int_0^L \zeta_2^2(t,x)e^{+(\mu/c_2)x}dx\end{aligned}$$

and

$$\begin{aligned}W(t) &= b_1W_1(t) + b_2W_2(t)\\ W_1(t) &= \frac{1}{c_1}\int_0^L \eta_1^2(t,x)e^{-(\mu/c_1)x}dx,\\ W_2(t) &= \frac{1}{c_2}\int_0^L \eta_2^2(t,x)e^{+(\mu/c_2)x}dx.\end{aligned}$$

With the boundary conditions (5)-(6) on ξ for the system (9) and the corresponding boundary conditions on ζ and η (deduced by time differentiation) for the systems (10) et (11), it can then be shown that there are positive constants

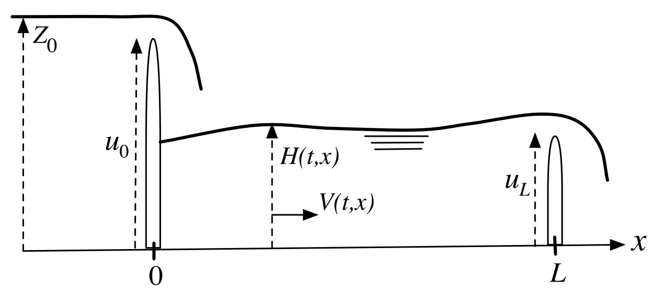

Fig. 1. A reach of an open channel delimited by two adjustable overflow spillways

a_i, b_i such that the time derivative of the Lyapunov function satisfies an inequality :

$$\begin{aligned}\dot{U}(t)+\dot{V}(t)+\dot{W}(t) &\leq -\mu(U(t)+V(t)+W(t))\\ &+ \text{ higher order terms.}\end{aligned}$$

The proof of the theorem is then completed with a classical Lyapunov stability analysis.

VI. APPLICATION TO LEVEL AND FLOW CONTROL IN AN HORIZONTAL REACH OF AN OPEN CHANNEL

In the field of hydraulics, the flow in open-channels is generally represented by the so-called Saint Venant equations which are a typical example of a system of conservation laws.

We consider the special case of a reach of an open channel delimited by two overflow spillways as represented in Figure 1.
We assume that :

1) the channel is horizontal,
2) the channel is prismatic with a constant rectangular section and a unit width,
3) the friction effects are neglected.

The flow dynamics are described by a system of two laws of conservation (Saint-Venant or shallow water equations), namely a law of mass conservation :

$$\partial_t H + \partial_x (HV) = 0,$$

and a law of momentum conservation :

$$\partial_t V + \partial_x (\frac{1}{2}V^2 + gH) = 0.$$

where $H(t,x)$ represents the water level and $V(t,x)$ the water velocity in the reach while g denotes the gravitation constant. The system is written under the form (2) as follows :

$$\partial_t \begin{pmatrix} H \\ V \end{pmatrix} + A(H,V)\partial_x \begin{pmatrix} H \\ V \end{pmatrix} = 0$$

with the matrix $A(H,V)$ defined as :

$$A(H,V) = \begin{pmatrix} V & H \\ g & V \end{pmatrix}.$$

The control actions are the positions u_0 et u_L of the two spillways located at the extremities of the pool and related to the state variables H et V by the following expressions :

$$\begin{aligned}&b_0(H(t,0),V(t,0),u_0(t)) = \\ &H(t,0)V(t,0) - C(Z_0 - u_0(t))^{3/2} = 0 \qquad (12)\end{aligned}$$

$$\begin{aligned}&b_L(H(t,L),V(t,L),u_L(t)) = \\ &H(t,L)V(t,L) - C(H(t,L)_L - u_L(t))^{3/2} = 0 \qquad (13)\end{aligned}$$

where Z_0 denotes the water level above the pool (see Fig. 1) and C is the characteristic constant of the spillways.

For constant spillway positions $\bar{u}_0$ and $\bar{u}_L$, there is a unique steady state solution which satisfies the following relations :

$$\begin{aligned}\bar{H} &= Z_0 - \bar{u}_0 + \bar{u}_L \\ \bar{V} &= \frac{C(Z_0 - \bar{u}_0)^{3/2}}{Z_0 - \bar{u}_0 + \bar{u}_L}.\end{aligned}$$

The control objective is to regulate the level H and the velocity V (or the flow rate $Q = HV$) at the set points $\bar{H}$ and $\bar{V}$ (or $\bar{Q} = \bar{H}\bar{V}$), by acting on the spillway positions u_0 et u_L.

The eigenvalues of the Jacobian matrix $A(H,V)$:

$$\begin{aligned}\lambda_1(H,V) &= V + \sqrt{gH} \\ \lambda_2(H,V) &= V - \sqrt{gH}\end{aligned}$$

are generally called " characteristic velocities " . The flow is said to be " fluvial " (or subcritical) when the characteristic velocities have opposite signs :

$$\lambda_2(H,V) < 0 < \lambda_1(H,V).$$

The Riemann invariants can be defined as follows :

$$\begin{aligned}\xi_1 &= V - \bar{V} + 2(\sqrt{gH} - \sqrt{g\bar{H}}) \\ \xi_2 &= V - \bar{V} - 2(\sqrt{gH} - \sqrt{g\bar{H}}).\end{aligned}$$

By using the relations (5) and (6) for the control definition, combined with the spillway characteristics (12) - (13), the following boundary control laws are obtained :

$$\begin{aligned}u_0 &= Z_0 - \sqrt[3]{\left(\frac{H_0^2}{C}\right)\left(\bar{V} - 2\sqrt{g}\frac{1-k}{1+k}(\sqrt{H_0} - \sqrt{\bar{H}}\right)^2} \\ u_L &= H_L - \sqrt[3]{\left(\frac{H_L^2}{C}\right)\left(\bar{V} + 2\sqrt{g}\frac{1-k_L}{1+k_L}(\sqrt{H_L} - \sqrt{\bar{H}}\right)^2}\end{aligned}$$

where $H_0 = H(t,0)$, $H_L = H(t,L)$ et $k_L = ke^{-(\mu/c_1)L}$.

It can be seen that both controls have the form of a state feedback at the two boundaries. In addition, it can be emphasized that the implementation of the controls is particularly simple since only measurements of the levels

$H(t,0)$ et $H(t,L)$ at the two spillways are required. This means that the feedback implementation does not require neither level measurements inside the pool nor any velocity or flow rate measurements.

VII. CONCLUSIONS

In this paper, we have presented a strict Lyapunov function which can be used for the boundary control design for second order systems of conservation laws and to analyse the convergence of the closed-loop system towards the equilibrium. We have the following additionnal comments :

1) In the special case where $\mu = 0$, our Lyapunov function is just an entropy function of the system under characteristic form, linearised in the space of the Riemann coordinates. In references [4]-[5], the interested reader will find an alternative approach of the boundary control design where the entropy is used as such (i.e. without linearisation) as a Lyapunov function inside the space of the ssystem physical coordinates. It must however be emphasized that the entropy is not a strict Lyapunov function because its time derivative is not negative definite but only semi negative definite (as we can see by setting $\mu = 0$ in equality (7)).
2) The boundary conditions (5) and (6) which are used for the control definition are analog to the conditions derived in [6]. In the latter reference however the convergence analysis is different : it does not make use of a Lyapunov function but is obtained from a general theorem on the stability of the classical solutions of quasilinear hyperbolic systems.
3) In order to solve the control problem, we have selected the particular simple boundary conditions (5) and (6). But obviously many other forms are admissible provided they make $\dot{U}$ negative. For instance it can be interesting to use controls at a boundary which depend on the state at the other boundary, hence introducing some usefull feedforward action in the control.
4) For the sake of simplicity, our presentation was restricted to second order systems of conservation laws. From our analysis, it is however very clear that the approach can be directly extended to any system of conservation laws which can be diagonalised with Riemann invariants. It is in particular the case for networks where the flux on each arc is modelled by a system of two conservation laws (see e.g. [7]).

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