Proceedings of the 8th International
IEEE Conference on Intelligent Transportation Systems
Vienna, Austria, September 13-16, 2005



# A second order model for road traffic networks

Bertrand Haut
Aspirant FNRS
CESAME
haut@auto.ucl.ac.be
Université Catholique de Louvain
Bâtiment Euler
Avenue Georges Lemaître, 4-6
1348, Louvain-la-Neuve, Belgium

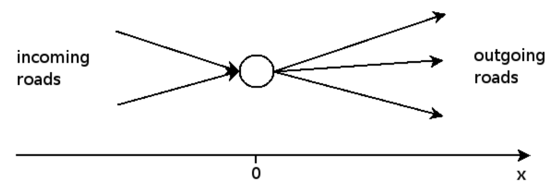
Georges Bastin
CESAME
bastin@auto.ucl.ac.be
Université Catholique de Louvain
Bâtiment Euler
Avenue Georges Lemaître, 4-6
1348, Louvain-la-Neuve, Belgium

*Abstract*— **This article deals with the modelling of a road network from a macroscopic point of view. After an introduction explaining how some macroscopic road network models are established, some basic properties of the Aw & Rascle second order model are recalled. A junction model compatible with the Aw & Rascle model is then established and some examples are presented in order to illustrate the plausibility of the model.**

## I. INTRODUCTION

In the fluid paradigm for road traffic modelling, the traffic is described in terms of two basic macroscopic variables : the density and the speed of the vehicles at position $x$ along the road and time $t$ (denoted $\rho(x,t)$ and $v(x,t)$). A usual way to describe a network traffic model is as follows :

- First, the equations binding the values of $\rho$ and $v$ to the initial conditions on an infinite single road are considered. These equations are usually a set of partial differential equations (PDE). A traditional problem studied for such systems is the Riemann problem which is an initial value problem where the initial condition consists of two constant values :
$$(\rho(x,0), v(x,0)) = \begin{cases} (\rho_l, v_l) & \text{if } x < 0 \\ (\rho_r, v_r) & \text{if } x \geq 0 \end{cases} \qquad (1)$$
The Riemann problem is important, not only because it allows an explicit solution but also because the solution of any initial value problem with arbitrary initial conditions can be constructed from a set of appropriate Riemann problems (see e.g. [3]).
- Then, the junctions at the nodes of the network are introduced. The junctions represent the connections between different roads, for example the merging of two roads in one or the fork of one road in two. An appropriate description of the behaviour of the drivers at the junction must then be provided. One way to do this, is to describe the solution of the Riemann problem at the junction.
If we consider a junction with some incoming and some outgoing roads (see Fig. I), the initial state is
$$(\rho_i(x,0), v_i(x,0)) = (\rho_{i,0}, v_{i,0}) \qquad \forall x,\ \forall i \qquad (2)$$




where subscript $i$ refers to road $i$. The traffic state will first evolve at the junction. Let $(\bar\rho_i, \bar v_i)$ be the new state on road $i$ at the border of the junction just after this evolution :
$$(\rho_i(0,t), v_i(0,t)) = (\bar\rho_i, \bar v_i) \qquad \forall t > 0.$$
We then face a new Riemann problem on each road :
$$(\rho_i(x,0), v_i(x,0)) = \begin{cases} (\bar\rho_i, \bar v_i) & \text{if } x = 0 \\ (\rho_{i,0}, v_{i,0}) & \text{if } x \neq 0. \end{cases} \qquad (3)$$
The choice of the values $(\bar\rho_i, \bar v_i)$ and the resolution of the Riemann problems (3) will provide the solution of the Riemann problem at the junction (1).
Of course, the waves produced on the incoming (resp. outgoing) roads must have a negative (resp. positive) velocity to go away from the junction in order to get a sensible model. To take this constraint into account in the model, we will restrict the set of possible values of $(\bar\rho_i, \bar v_i)$ to a subset of $\mathbb{R}^2$ called the *admissible region* such that all waves produced by the Riemann problem (3) have negative (resp. positive) speed if $i$ corresponds to an outgoing (resp. incoming) road.

The first time and space continuous models that were developed in the literature were based on the LWR model (see [12], [14], [9], [5], [8] and [10]). The LWR model is a first order model which means that there is only one PDE describing the evolution of the traffic state and that the solution to the Riemann problem (1) consists of one *wave* connecting the two initial states.

In this paper we intend to establish a second order traffic **network** model based on the Aw and Rascle second order **single road** model (see [2]). In Section II, we recall the Aw & Rascle model, in Section III the admissible regions

This paper presents research results of the Belgian Programme on Interuniversity Attraction Poles, initiated by the Belgian Federal Science Policy Office. The scientific responsibility rests with its author(s)

are established and some additional conditions are presented in Section IV in order to guarantee a unique and realistic solution to the Riemann problem at the junction. Some conclusions are drawn in Section V.

## II. The Aw & Rascle single road model

The Aw and Rascle model (see [2]) for a single road is described by two equations :

$$\begin{aligned} \partial_t \rho + \partial_x(\rho v) &= 0 && (4)\\ (\partial_t + v\partial_x)v + (\partial_t + v\partial_x)p(\rho) &= 0. && (5)\end{aligned}$$

where $p(\rho)$ is an increasing function of the density such as $\frac{d^2}{d\rho^2}(\rho p(\rho)) > 0$. The first equation represents the conservation of the flow while the second equation describes the evolution of the speed of the drivers in function of the surrounding traffic state. Multiplying (4) by $(v + p(\rho))$ and (5) by $\rho$ and adding up these two equations, we obtain

$$\partial_t(\rho(v + p(\rho))) + \partial_x(\rho v(v + p(\rho))) = 0.$$

Therefore the system is composed of two conserved quantities : $\rho$ and $\rho(v + p(\rho))$.

Working with a system like (4)–(5), we have to admit discontinuous solutions. The meaning of the differential equations in presence of discontinuities and the admissibility of these discontinuities are explained in [3]. Because the system is expressed by two equations of conservation, the solutions of a Riemann problem

$$(\rho(x,0), v(x,0)) = \begin{cases} U_l = (\rho_l, v_l) & \text{if } x < 0 \\ U_r = (\rho_r, v_r) & \text{if } x \geq 0 \end{cases}$$

consists of the connection of the left state $U_l$ to an intermediate state $U_c$ by a first wave and the connection of this intermediate state to the right state $U_r$ by a second wave. We have two waves because we have two conservation laws. The two waves are different :

- the first one may be a shock or a rarefaction wave (see Fig. 1). A shock wave is a discontinuity in $\rho$ and/or in $v$ travelling at a constant speed. A rarefaction wave is a self-similar solution, i.e. it depends only on $x/t$.
- the second one must be a contact discontinuity. The contact discontinuity separates two constant states with the same speed but different densities. This contact discontinuity travels at the same speed as the vehicles.

We will not describe here the complete and rigorous description of the Riemann problem (see [2]) but only the two most simple and common cases. In Figure 2, we have represented :

- the curve $(\rho, \rho v_r)$ ;
- the curve $(\rho, \rho K - \rho p(\rho))$ passing by $U_l$;
- the two initial states ($U_l$ and $U_r$) and the intermediate state ($U_c$) which is the intersection of the two previous curves in the $(\rho, \rho v)$ plane;
- the segment connecting $U_l$ to $U_c$.

Two cases a) and b) must be considered.

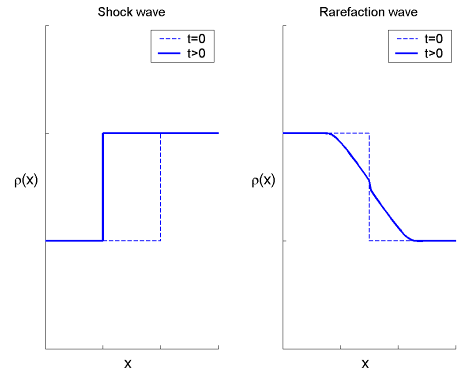



Fig. 1. The possible waves connecting the left state to the intermediate state in the solution of a Riemann problem for the Aw & Rascle model. Only the density is represented here.

*In Fig. 2 a):* the solution consists of a shock wave connecting $U_l$ to $U_c$ followed by a contact discontinuity connecting $U_c$ to $U_r$. The speed of the shock wave is equal to the slope of the line connecting $U_l$ to $U_c$.

*In Fig. 2 b):* the solution consists of a rarefaction wave connecting $U_l$ to $U_c$ followed by a contact discontinuity connecting $U_c$ to $U_r$. The space occupied by the rarefaction wave is

$$\left[ \left.\frac{d(\rho K - \rho p(\rho))}{d\rho}\right|_{\rho=\rho_l} t, \left.\frac{d(\rho K - \rho p(\rho))}{d\rho}\right|_{\rho=\rho_i} t \right].$$

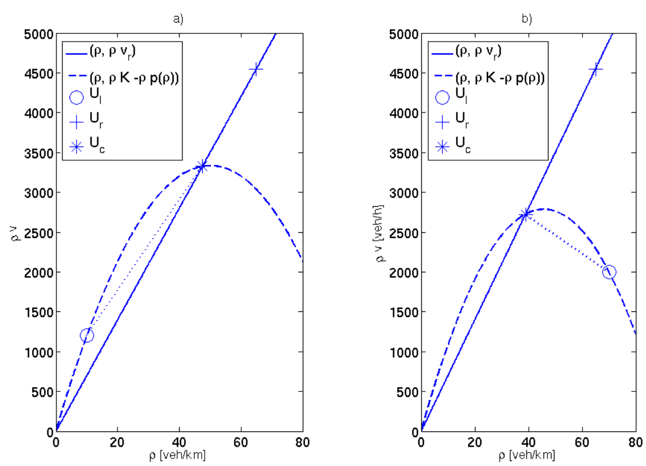
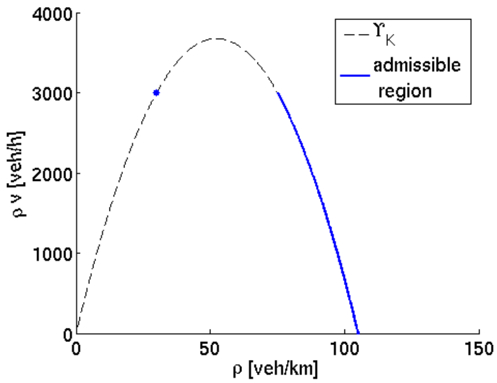



Fig. 2. The initials and intermediate state in the $(\rho, \rho v)$ plane for the Riemann problem.

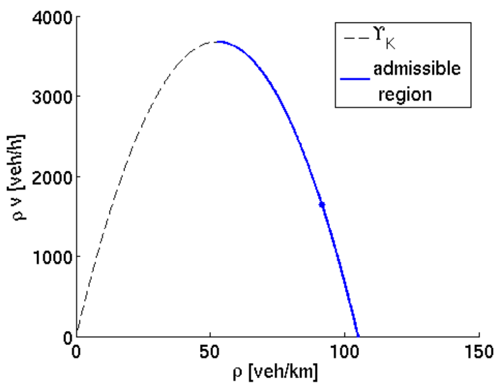
## III. The admissible regions for the junction models

As explained in the introduction, in order to formulate a junction model we need first to explicit for each road the admissible regions for the values of $(\bar{\rho}_i, \bar{v}_i)$. The shape of this admissible region will be different if we consider an incoming or an outgoing road.

### A. Incoming road




(a) $\rho_{i,0} \leq \sigma_{\Upsilon_K}$




(b) $\rho_{i,0} > \sigma_{\Upsilon_K}$

Fig. 3. The admissible regions for an incoming road.

On incoming roads, the only possible waves produced by the Riemann problem

$$(\rho_i(x,0), v_i(x,0)) = \begin{cases} (\bar{\rho}_i, \bar{v}_i) & \text{if } x = 0 \\ (\rho_{i,0}, v_{i,0}) & \text{if } x < 0 \end{cases}$$

must obviously have a negative speed. Because, the speed of the second wave (which is equal to the speed of the drivers) is necessarily positive, the only admissible wave is a wave of the first type. Hence $(\bar{\rho}_i, \bar{\rho}_i\bar{v}_i)$ must be on the curve $\Upsilon_K$ passing through $(\rho_{i,0}, \rho_{i,0}v_{i,0})$ where we define $\Upsilon_K$ as

$$\Upsilon_K = (\rho, \gamma_K(\rho)) = (\rho, \rho K - \rho p(\rho))$$

and

$$\sigma_{\Upsilon_K} = \arg\max_{\rho} \gamma_K(\rho).$$

In terms of solutions described in Section II, we have

- $(\rho_{i,0}, v_{i,0}) = U_l$ ;
- $(\bar{\rho}_i, \bar{v}_i) = U_r = U_c$.

Because $U_r = U_c$, we do not have the second wave which has always a positive speed.

*a) If $\rho_{i,0} \leq \sigma_{\Upsilon_K}$:* the only possibility to have a wave with negative speed, is to have $\bar{\rho}_i > \tau_{\Upsilon_K}(\rho_{i,0})$ (see Fig. 3(a))[1] where, for each $\rho \neq \sigma_{\Upsilon_K}$, $\tau_{\Upsilon_K}(\rho)$ is the unique number $\tau_{\Upsilon_K}(\rho) \neq \rho$ such that

$$\gamma_K(\rho) = \gamma_K(\tau_{\Upsilon_K}(\rho)).$$

In that case, the wave along the incoming road is a shock wave with a negative speed.

*b) If $\rho_{i,0} \geq \sigma_{\Upsilon_K}$ and $\bar{\rho}_i \geq \rho_{i,0}$:* we will have a shock wave with a negative speed. In the other case ($\bar{\rho}_i \leq \rho_{i,0}$), we will have a rarefaction wave. In order that the right limit of this rarefaction wave has a negative speed, we need that $\bar{\rho}_i \geq \sigma_{\Upsilon_K}$ (see Fig. 3(b)).

The admissible region for an incoming road is thus composed of the part of the curve $\Upsilon_K$ represented in Fig. 3 and, of course, $(\rho_{i,0}, v_{i,0})$ for which there isn't any wave.

We can notice a great similarity with the LWR first order models. For these models, a value called the "sending capacity" or the "traffic demand" was introduced by Daganzo (see [6] for details). This value represents the greatest possible outflow of a road segment and is equal to

$$\text{sending capacity} = \begin{cases} Q(\rho) & \text{if } \rho \leq \arg\max_\rho Q(\rho) \\ \max Q(\rho) & \text{if } \rho > \arg\max_\rho Q(\rho) \end{cases}$$

where $Q(\rho)$ represents the flow associated to the density $\rho$. In our second order model, the greatest possible outflow of a road segment is the maximal flow for a point on the admissible region and is equal to

$$\text{sending capacity} = \begin{cases} \gamma_K(\rho) & \text{if } \rho \leq \arg\max_\rho \gamma_K(\rho) \\ \max \gamma_K(\rho) & \text{if } \rho > \arg\max_\rho \gamma_K(\rho). \end{cases}$$

The similarity is obvious with the replacement of $Q(\rho)$ by $\gamma_K(\rho)$. In the second order model, the sending capacity is function of the density but also of the speed (via the value of $K$).

The admissible region satisfies some intuitive ideas :

- if there is nobody on the incoming road ($\rho_{i,0} = 0$), the maximal flow allowed to leave the road is zero;
- if there are few vehicles on the incoming road ($\rho_{i,0} \ll \sigma_{\Upsilon_K}$), the flow allowed to leave the road is low (less than $\rho_{i,0}v_{i,0}$) ;
- if there is a lot of vehicles on the incoming road ($\rho_{i,0} \gg \sigma_{\Upsilon_K}$), the flow allowed to leave the road may be high (up to $\gamma_K(\sigma_{\Upsilon_K})$).

### B. Outgoing road

On outgoing roads, the only possible waves produced by the Riemann problem

$$(\rho_i(x,0), v_i(x,0)) = \begin{cases} (\bar{\rho}_i, \bar{v}_i) & \text{if } x = 0 \\ (\rho_{i,0}, v_{i,0}) & \text{if } x > 0 \end{cases}$$

[1] For the illustrations in this paper, the functions $p(\rho)$ used are of the following form :

$$p(\rho) = \frac{v_{ref}}{\gamma}\left(\frac{\rho}{\rho_{\max}}\right)^{\gamma}, \qquad \gamma > 0$$

which is a plausible $p(\rho)$ relation (see [1]).

must obviously have a positive speed. This case is more complex than the previous one because we may have here the presence of the two waves. The second wave has always a positive speed, we can thus connect any intermediate state $U_c$ on the curve $(\rho, \rho v_r)$ to $U_r = (\rho_r, \rho_r v_r)$. The admissible region is thus the $(\rho, \rho v_r)$ curve plus the region of the $(\rho, \rho v)$–plane which can be connected to the curve $(\rho, \rho v_r)$ with any rarefaction or shock wave with positive speed.

In order to characterise the admissible region we will consider all the curves $\Upsilon_K$ for all the possible values of $K \in ]0, \infty[$.

a) $0 \le K \le v_r$ In this case, the curve $\Upsilon_K$ is necessarily located under the curve $(\rho, \rho v_r)$. The left state $(\bar{\rho}_i, \bar{v}_i)$ will be connected to the intermediate state $U_c$ which is the vacuum ($\rho = 0$) by a rarefaction wave. In order that the left limit of this rarefaction wave has a positive speed, we need that

$$\left.\frac{d(\rho K - \rho p(\rho))}{d\rho}\right|_{\rho=\rho_l} \ge 0.$$

It implies that $\bar{\rho}_i$ must be lesser than $\sigma_{\Upsilon_K}$ (see Fig. 4(a) ).

b) $v_r < K$ & $\gamma_K(\sigma_{\Upsilon_K}) \le \sigma_{\Upsilon_K} v_r$ In this case, the curve $\Upsilon_K$ is greater than the curve $(\rho, \rho v_r)$ at the beginning but crosses the line before its maximum. If $(\bar{\rho}_i, \bar{\rho}_i \bar{v}_i)$ is on the part of $\Upsilon_K$ above the curve $(\rho, \rho v_r)$, it will be connected to the intermediate state $U_c$ by a shock wave with positive speed. If $(\bar{\rho}_i, \bar{\rho}_i \bar{v}_i)$ is on the part of $\Upsilon_K$ under the curve $(\rho, \rho v_r)$, it will be connected to $U_c$ by a rarefaction wave. In order that the left limit of this rarefaction wave has a positive speed, we have that $\bar{\rho}_i$ must be lesser than $\sigma_{\Upsilon_K}$ (see Fig. 4(b)).

c) $v_r < K$ & $\gamma_K(\sigma_{\Upsilon_K}) > \sigma_{\Upsilon_K} v_r$ In this case, the maximum of $\Upsilon_K$ is above the curve $(\rho, \rho v_r)$. If $(\bar{\rho}_i, \bar{\rho}_i \bar{v}_i)$ is on the part of $\Upsilon_K$ under the curve $(\rho, \rho v_r)$, it should be connected to $U_c$ by a rarefaction wave with a negative speed, which is impossible.
If $(\bar{\rho}_i, \bar{\rho}_i \bar{v}_i)$ is above the straight line, it will be connected to $U_c$ by a shock wave whose speed will be the slope of the curve connecting $(\bar{\rho}_i, \bar{\rho}_i \bar{v}_i)$ to $U_c$. In order to have a positive slope, we must have that

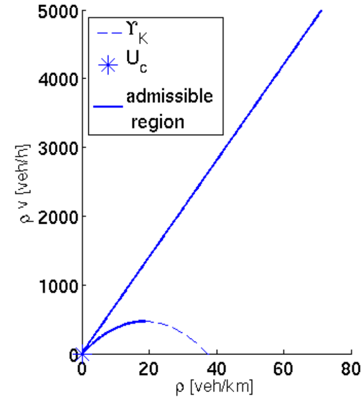
$$\bar{\rho}_i \le \tau_{\Upsilon_K}(\rho_c)$$

where $\rho_c$ is the density of the intermediate state $U_c$ (see Fig. 4(c)).

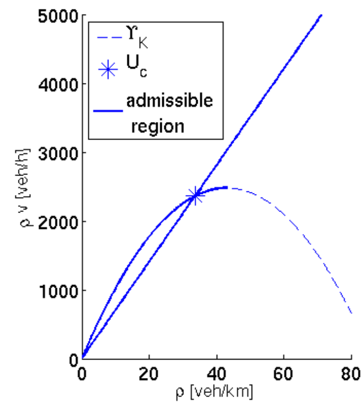
If we combine the admissible regions (AR) for all the values of $K$, we obtain the AR for an outgoing road represented in Figure 5. This AR satisfies some intuitive ideas :

- if there is nearly nobody on the outgoing road ($\rho_{i,0} \approx 0$, $v_{i,0} \gg 0$), the AR is quite large ;
- if there are many vehicles on the outgoing road ($\rho_{i,0} \gg 0$, $v_{i,0} \approx 0$), the AR is smaller.

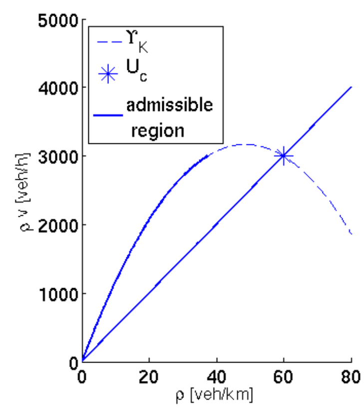
In fact, the AR only depends on the speed of the vehicles on the road. If $v_1 \le v_2$, the AR associated with $v_1$ will be included in the AR associated with $v_2$.




(a) $0 \le K \le v_r$




(b) $v_r < K$ & $\gamma_K(\sigma_{\Upsilon_K}) \le \sigma_{\Upsilon_K} v_r$




(c) $v_r < K$ & $\gamma_K(\sigma_{\Upsilon_K}) > \sigma_{\Upsilon_K} v_r$

Fig. 4. Some parts of the admissible region for an outgoing road.

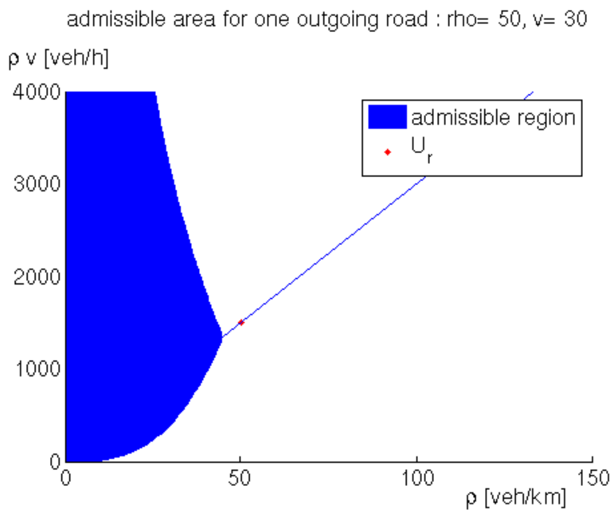



Fig. 5. The admissible region for an outgoing road.

## IV. Additional conditions

In order to have a unique solution, additional conditions are needed :

1) A first condition is indisputable : the conservation of flow. The sum of the entering flows must be equal to the sum of the leaving flows at the junction.
2) The second equation of the Aw and Rascle model (5) describes the behaviour of the drivers. It says that the Lagrangian derivative of the speed is equal to the Lagrangian derivative of $-p(\rho)$. It means that a driver will adapt his speed if the quantity $p(\rho)$ is modified. The most natural extension of this behaviour to the junction is

$$v_a - v_b = p(\rho_a) - p(\rho_b)$$

where the subscripts $a$ and $b$ means "after" and "before" the junction. In other words, the quantity $v + p(\rho)$, which describes the behaviour of the drivers, is "conserved" through the junction by the drivers. Here, the meaning of conservation is not the same as in "conservation of the flow" : the total flow of the quantity $v + p(\rho)$ is not necessary the same before and after the junction but each driver tends to conserve his quantity $v + p(\rho)$ which describes his behaviour. If we have only one incoming road, all the drivers have the same behaviour, and thus :

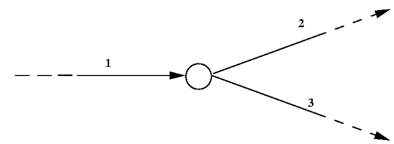
$$v_a + p(\rho_a) = v_b + p(\rho_b).$$

If we consider the case where there are several incoming roads, we may assume that the behaviour of the drivers after the junction will be a mean of the driver behaviours from the incoming roads. In other words :

$$v_a + p(\rho_a) = \sum_i \alpha_i (v_i + p(\rho_i))$$

where $\alpha_i$ is the proportion of the drivers coming from the incoming road $i$.
3) The two previous additional conditions are not sufficient to have a unique solution to the Riemann problem at the junction. In order to get a unique solution, like for the first order model (see [9], [5], [11] and [10]), we may assume that the drivers act such that the flow entering the outgoing roads is maximised with respect to the previous restrictions.

### A. The diverge junction




It is reasonable to assume that, the drivers having fixed destination intention, the proportions of the total flow entering into road 2 and 3 are fixed ($\alpha_2$ and $\alpha_3$).

With the additional conditions presented in section IV, the optimisation problem at the junction can be expressed as

$$\max_{\bar{\rho}_i, \bar{v}_i} \bar{\rho}_1 \bar{v}_1$$

subject to

$$\begin{cases} \bar{\rho}_1 \bar{v}_1 = & \frac{\bar{\rho}_2 \bar{v}_2}{\alpha_2} = \frac{\bar{\rho}_3 \bar{v}_3}{\alpha_3} \\ \bar{v}_1 + p(\bar{\rho}_1) = & \bar{v}_2 + p(\bar{\rho}_2) = \bar{v}_3 + p(\bar{\rho}_3) \\ (\bar{\rho}_i, \bar{v}_i) \in & AR_i \end{cases}$$

where $AR_i$ is the admissible region associated to road $i$ with initial state $(\rho_{i,0}, v_{i,0})$. The solution of this optimisation problem produces the new values of $(\rho_i, v_i)$ at the junctions which allow to solve the Riemann problems on each road. It can be shown that this solution of the optimisation problem has some good properties :

- the solution exists and is unique ;
- if all the drivers take the same road ($\alpha_2 = 0, \alpha_3 = 1$), the solution is the same as the classical Aw & Rascle model for an infinite single road ;
- the solutions are physically acceptable. For example if there is not too much traffic on the outgoing roads, all the flow on the incoming road must pass (see Fig. 6). If the traffic on the outgoing roads is important, only a part of the flow on the incoming road may pass (see Fig. 7). In the Figures, the red cross correspond to the solution, the blue line to the curve $\Upsilon_K$ passing through the solution and the hatched area to the admissible region of the outgoing roads ;
- the only situation where no flow can leave the incoming road is when the speed on one of the outgoing roads is equal to zero (the AR is reduced to curve $(\rho, 0)$ which is associated to a zero flow).
  It implies that, if only one of the outgoing roads is jammed up, the outflow of the other road is equal to zero. This is a rather hard constraint on the outflows. The reason is that the Aw & Rascle model, used to describe the evolution on one road, doesn't make any distinction between the different lanes. In a single lane road, if a driver stops because he is unable to turn left, he also blocks all the drivers wanting to turn right. To

remove this hard constraint, it would be needed to adapt the Aw & Rascle model to take the different lanes into account.

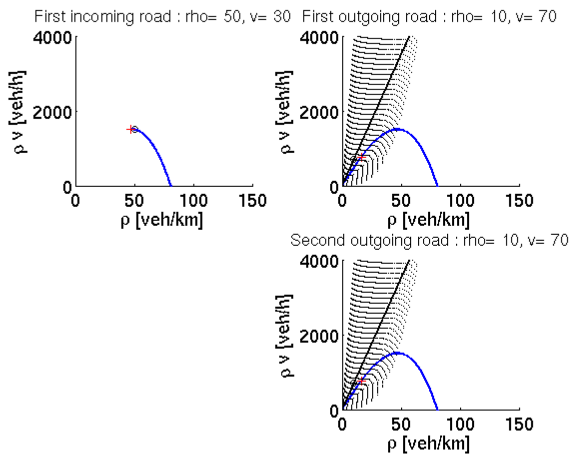



Fig. 6. The solution to the following Riemann problem : $(\rho_{1,0}, v_{1,0}) = (50, 30), (\rho_{2,0}, v_{2,0}) = (10, 70), (\rho_{3,0}, v_{3,0}) = (10, 70), \alpha_2 = \alpha_3 = 0.5$.

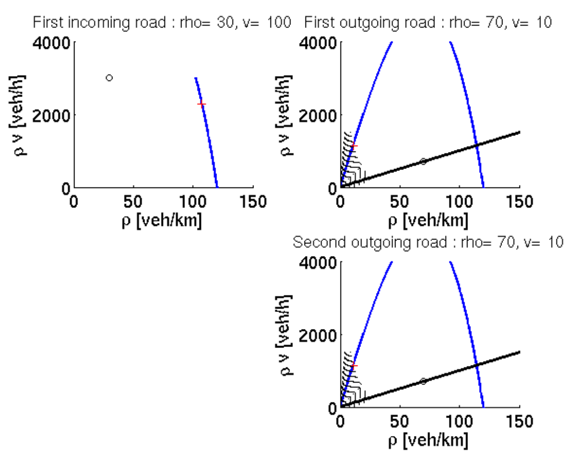
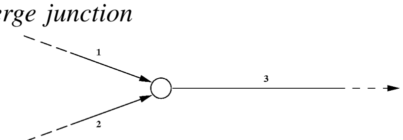



Fig. 7. The solution to the following Riemann problem : $(\rho_{1,0}, v_{1,0}) = (30, 100), (\rho_{2,0}, v_{2,0}) = (70, 10), (\rho_{3,0}, v_{3,0}) = (70, 10), \alpha_2 = \alpha_3 = 0.5$.

### *B. The merge junction*




For the merge junction, we need to introduce a coefficient describing how the available space on the outgoing road is spread out between the incoming roads in case of congestion. A simple way to introduce these coefficient is to make them depend on the flows wishing to enter the outgoing road (see [10]) :

$$\alpha_i = \frac{f_i^*}{f_1^* + f_2^*}$$

where $f_i^*$ is the maximal flow able to leave the road $i$ (the "sending capacity"). With the additional conditions introduced above, the optimisation problem at the junction can be expressed as

$$\max_{\bar{\rho}_i, \bar{v}_i} \bar{\rho}_1 \bar{v}_1 + \bar{\rho}_2 \bar{v}_2$$

subject to

$$\begin{cases} \frac{\bar{\rho}_1 \bar{v}_1}{\alpha_1} = & \frac{\bar{\rho}_2 \bar{v}_2}{\alpha_2} = \bar{\rho}_3 \bar{v}_3 \\ \bar{v}_3 + p(\bar{\rho}_3) = & \alpha_1(\bar{v}_1 + p(\bar{\rho}_1)) + \alpha_2(\bar{v}_2 + p(\bar{\rho}_2)) \\ (\bar{\rho}_i, \bar{v}_i) \in & AR_i \end{cases}$$

It can be shown that the solution of this problem has some good properties :

- the solution exists and is unique ;
- if one of the incoming roads is empty ($\rho = 0$), the solution is the same as the classical Aw & Rascle model for an infinite single road ;
- the solutions are physically acceptable. For example if there is not too much traffic on the roads, all the flow on the incoming roads must pass (see Fig. 8). If the traffic on the different roads is important, only a part of the flow on the incoming roads may pass (see Fig. 9).
- the only situation where no flow can leave the incoming road is when there is no traffic on the incoming road or when the speed on the outgoing road is equal to zero (the AR is reduced to curve $(\rho, 0)$ which is associated to a zero flow).
- the model is sometimes able to naturaly represent the *capacity drop phenomenon*. The *capacity drop phenomenon* is a critical phenomenon which represents the fact that the outflow of a traffic jam is significantly lower than the maximum achievable flow at the same location. We can easily understand this phenomenon at a junction where two roads merge in one : if there are too many vehicles trying to access the same road, there is a sort of mutual embarrassment between the drivers which results in an outgoing flow lower than the optimal possible flow. This phenomenon has been experimentally observed (see [4] and [7]). The flow decrease, which may range up to 15 %, has a considerable influence when considering traffic control ([13]). To have of a model describing this phenomenon is thus a critical feature in the establishment of a traffic state regulation strategy.
  To illustrate this phenomenon, we can carry out a series of simulations whose solutions are presented in Table I. To simplify, we consider the case where the state on road one and road two are the same. We start from an equilibrium (row 1) with a total passing flow of 3000 veh/h. If we apply a perturbation (row 2) consisting of an increasing of the incoming flow (3300 veh/h) we obtain the new equilibrium represented in row 3. To this new equilibrium corresponds a total passing flow of 2165 veh/h. This illustrates the capacity drop phenomenon : the incoming flows are higher but the outgoing flow drop down. Moreover, if we try to return to the initial state by a new perturbation on the incoming

roads (row 4), we don't exactly obtain again the same total passing flow but a lower one.

| | state | $\rho_1$ | $v_1$ | $f_1$ | $\rho_3$ | $v_3$ | $f_3$ |
|---|---|---|---|---|---|---|---|
| 1 | equilibrium | 20 | 75 | 1500 | 51.4 | 58.36 | 3000 |
| 2 | perturbation | 30 | 55 | 1650 | 51.4 | 58.36 | 3000 |
| 3 | equilibrium | 80.7 | 13.42 | 1082.5 | 52 | 41.6 | 2165 |
| 4 | perturbation | 20 | 75 | 1500 | 52 | 41.6 | 2165 |
| 5 | equilibrium | 91.5 | 15.9 | 1458.5 | 47.8 | 61 | 2917 |

TABLE I

THE CAPACITY DROP PHENOMENON ILLUSTRATED IN A PARTICULAR CASE.

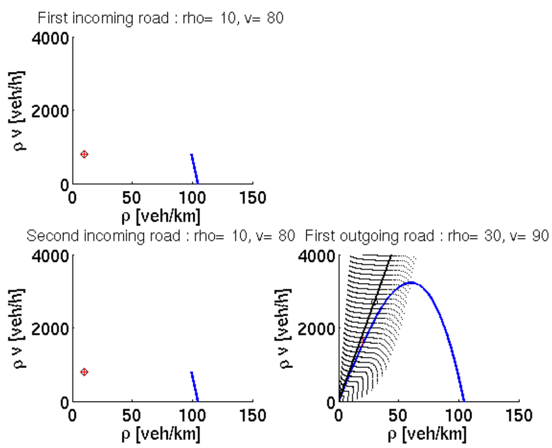



Fig. 8. The solution to the following Riemann problem : $(\rho_{1,0}, v_{1,0}) = (10, 80), (\rho_{2,0}, v_{2,0}) = (10, 80), (\rho_{3,0}, v_{3,0}) = (30, 90)$.

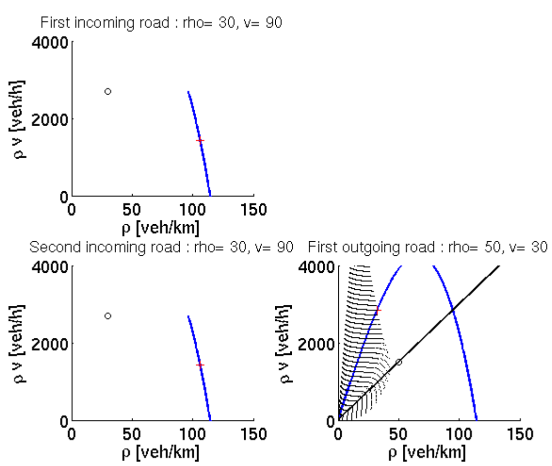



Fig. 9. The solution to the following Riemann problem : $(\rho_{1,0}, v_{1,0}) = (30, 90), (\rho_{2,0}, v_{2,0}) = (30, 90), (\rho_{3,0}, v_{3,0}) = (50, 30)$.

## V. CONCLUSIONS

Adding only one new assumption ("conservation" of the quantity $v + p(\rho)$ representing the behaviour of the drivers) to some commonly admitted assumptions (conservation of the flow, sharing of the available space on the outgoing road based on coefficients function of the sending capacities, drivers acting such as maximising the passing flow), we obtain a coherent and realistic model for the junction able to represent the capacity drop phenomenon. This junction model, which doesn't add any new parameter in addition to those introduced by the Aw & Rascle single road model (the function $p(\rho)$), combined with this single road model, provides a complete description of the traffic evolution on a road network.

The extension of the junction model to n-incoming-1-outgoing and 1-incoming-n-outgoing roads is direct. It does not need to introduce additional assumptions. If we consider multiple incoming and multiple outgoing roads, the extension is more difficult. Additional criteria must be added in order to describe how the different flows interact.

## REFERENCES


[1] A. Aw, A. Klar, T. Materne, and M. Rascle. Derivation of continuum traffic flow models from microscopic follow-the-leader models. *SIAM J. APPL. MATH.*, 63:259–278, 2002.

[2] A. Aw and M. Rascle. Ressurection of "second order" models of traffic flow. *SIAM J. APPL. MATH.*, 60:916–938, 2000.

[3] Alberto Bressan. *Hyperbolic Systems of Conservation Laws – The One-dimensional Cauchy Problem*. Oxford University Press, 2000.

[4] M. J. Cassidy and R. L. Bertini. Some traffic features at freeway bottlenecks. *Transportation Research Part B*, B33:25–42, 1999.

[5] G. M. Coclite, M. Garavello, and B. Piccoli. Traffic flow on a road network. *to appear in SIAM J. Math. Anal.*, 2004.

[6] C.F. Daganzo. The cell transmission model 2: network simulation. *Transportation Research B*, 29B(2):79–93, 1995.

[7] F. L. Hall and K. Agyemang-Duah. Freeway capacity drop and the definition of capacity. *Transportation Record*, (1320):99–109, 1991.

[8] M. Herty and A. Klar. Modeling, simulation, and optimization of traffic flow networks. *SIAM J. Sci. Comput.*, 25(3):1066–1087, 2003.

[9] H. Holden and N. H. Risebro. A mathematical model of traffic flow on a network of unidirectional roads. *Siam J. Math. Anal.*, 26(4):999–1017, 1995.

[10] W.L. Jin and H.M. Zhang. On the ditribution schemes for determining flows through a merge. *Transportation Research PartB*, (37):521–540, 2003.

[11] JP Lebacque. The Godunov scheme and what it means for first order traffic flow models. In *Transportation and traffic flow theory, Proceedings of the 13th ISTTT,* Pergamon, 1996.

[12] M. J. Lighthill and J. B. Whitham. On kinematic waves. i: Flow movement in long rivers. ii: A theory of traffic flow on long crowded roads. *Proc. Royal Soc. London Ser. A*, (229):281–345, 1955.

[13] Markos Papageorgiou. Freeway ramp metering : An overview. *IEEE transactions on intelligent transportation systems*, 3(4):271–281, december 2002.

[14] P. I. Richards. Shock waves on the highway. *Oper. Res*, (4):42–51, 1956.