Green Growth or Low Growth: Modelling the Balanced Transition to a Sustainable Economy

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Abstract

We present a simple mathematical model for the transition to a sustainable economy in order to explore the long-run evolution of an economy that achieves environment protection, full recycling of material resources and limitation of greenhouse gas emissions. The main concern is to investigate how balanced economic paths are modified under public policies for transition to sustainability. We consider a world economy with two subregions that are endowed with greenhouse gas emissions and ecological footprint of OECD and non-OECD countries respectively. Then, for the OECD subregion, three different options are investigated : a green growth option that focuses on accelerating the green technological change, a low growth option that focuses on shifting the structure of the economy towards low carbon and low capital intensive activities and a combined green-low growth option that focuses on the limitation of material resources and the abatement of the ecological footprint.

Keywords: modelling, sustainability, balanced growth, transition.

1. Introduction

In the line proposed by Peter Victor [Victor and Rosenbluth, 2007, Victor, 2012] and Tim Jackson [Jackson, 2009, Appendix 2], we present a simple mathematical model for the transition to a sustainable economy. The modelling approach is in the continuation of the "Limits to Growth" of [Meadows et al., 1972, 2004] which have emphasized the unsustainable character of the current economic trend as well as the necessity of a major change in the economic structure and the consumption behaviour. The "Limits to Growth" projections are confirmed by [Turner, 2008] in his comparison with empirical data.

Some authors have expressed their doubts as to the possibility of correctly analysing the sustainable transition with the toolbox of mainstream economics and ask for the development of an epistemological questioning. Although we totally agree with the relevance of the epistemological issue, we believe that the current debate may be clarified by looking more closely into the potential and the limits of the neo-classical formalism for the understanding of the sustainability transition.

The model is built to assess public policies to attain sustainability. Our objective is to use the model to explore long-run evolution of an economy that achieves environment protection, full recycling of material resources and limitation of atmospheric carbon dioxide (CO_2) which is the major contributor to greenhouse gas emissions.

The model is a conceptual representation of a "decentralized economy" (see e.g. [Wickens, 2008, Chapter 5]) where the decisions of producers, consumers and government are distinguished. In order to address the objectives mentioned above, the model involves the main economic and environmental variables that are essential for analyzing a sustainable economy. In addition to standard

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macroeconomic variables (such as production, consumption, investment, capital and labour), we therefore also consider environmental variables (such as CO_2 emissions, CO_2 intensity, ecological footprint, material intensity and recycling rate). As it is usual in macroeconomic modeling, the model consists essentially of "flow balance equations" that combine aggregate stock variables and flow functions.

We restrict our attention to balanced economic paths. Our main concern is to investigate how balanced paths are modified under public policies for transition to sustainability. The reason for restricting to balanced paths is to have consistent models that are as simple and flexible as possible. We want simple models that can be easily implemented, even by users who are not familiar with the use of optimal control methods in neo-classical economic theory (see e.g. [Bréchet et al., 2011] and [Boucekkine et al., 2012] for the use of optimal control in the analysis of sustainable economic growth). We want models that are flexible to easily include variants and extensions like subregions, economic subsectors or explicit fiscal policies.

We define a fictional pseudo-world economy with two subregions that are endowed with the CO_2 emissions and ecological footprint of OECD and non-OECD countries respectively. Then, for the OECD subregion, we examine two major options towards sustainability: the "Green Growth" option and the "Low Growth" option. In the green growth option, it is believed that the greenhouse gas emissions will be limited by developing public novel technical innovations without changing the final output nor the economic structure. In contrast, the low growth option aims at developing zero or low carbon emission activities without having to rely on major discoveries of new green technologies, which results in structural change and lower growth.

The paper is organized as follows. The baseline system is presented in Section 2. It is a simple single-sector economy. The system is supposed to follow a balanced trajectory along which the capital-output ratio is constant. In Section 3, we set up a benchmark numerical model which is initialized with orders of magnitude corresponding to the state of OECD economy during the period 1998-2008 and which is consistent with the empirical data. Section 4 is devoted to modelling of CO_2 intensity and to the quantitative estimation of the relative decoupling between GDP growth and CO_2 emissions. For the simulations, the model equations are solved with Matlab-Simulink. A first "business as usual" simulation experiment is presented in Section 5. In this simulation, the economy continues to follow its current trend and makes the planet reaching unsupportable CO_2 atmospheric concentrations at the end of the century. Section 6 deals with the green growth public policy. The baseline system is extended with a sector producing green technical knowledge. The investment in this sector is assumed to increase proportionally to the excess of CO_2 emissions. The simulation result shows how the investment policy in public green technologies stabilizes the atmospheric CO_2 concentration with a public cost in the range 2-8 % of GDP. In Section 7, we examine how the transition to sustainability may be achieved with a low-growth public policy that consists in fostering the development of activities with low or zero carbon intensity and resulting in lower productivity growth and structural change. For this purpose, we consider an economy with two sectors: a conventional sector endowed with the economic features of the baseline system and an ecological sector of activities having zero carbon intensity and constant labour productivity. In the presented simulation results, the emphasis is on the progressive reallocation of capital and labour between the two sectors in order to reach sustainability. Finally, in Section 8, the model is extended by considering the issues of material resource limitations and environmental protection when the transition to a sustainable economy requires to achieve ecological footprint reduction and full recycling of a-biotic materials in addition to CO_2 abatement.

2. The baseline system

We consider an economy with the fundamental national accounting identity

$$Y(t) = w(t)L(t) + r(t)K(t) = C(t) + I(t) + X(t) - M(t) \tag{1}$$

where t is time, Y is the aggregate output production flow (= GDP), K is the aggregate capital, L is the labour, C, X and M are consumption, export and import flows respectively. For simplicity, we do not distinguish between the flows stemming from the private and public sectors. In equation (1), $w(t)$ is the wage rate and $r(t)$ is the capital rental rate.

The dynamics of the aggregate capital K are represented by the standard balance equation

$$\frac{dK(t)}{dt} = -\delta K(t) + I(t) \tag{2}$$

where I is the aggregate investment allocated to production of the final goods. The constant parameter $\delta \in (0, 1)$ is the capital depreciation rate.

We assume that the labour L is varying over time according to the dynamics

$$\frac{dL}{dt} = \mu(t)L \tag{3}$$

where the specific evolution rate $\mu(t)$ is a time-varying exogenous variable.

Note that, in this paper, we do not use an aggregate "production function" (like e.g. the Cobb-Douglas function) to describe the economy. Therefore "no assumption is made that returns to scale are constant or that factors are paid their marginal products" [Temple, 2006], and neither of those assumptions are required in the paper.

We introduce the following notations for the labour share in GDP, for the specific growth rates of capital, output, wages and capital rental rate :

$$\alpha(t) = \frac{w(t)L(t)}{Y(t)}, \tag{4}$$

$$g_K(t) = \frac{1}{K(t)}\frac{dK(t)}{dt}, \qquad g_Y(t) = \frac{1}{Y(t)}\frac{dY(t)}{dt}, \tag{5}$$

$$g_w(t) = \frac{1}{w(t)}\frac{dw(t)}{dt}, \qquad g_r(t) = \frac{1}{r(t)}\frac{dr(t)}{dt}. \tag{6}$$

Then, from (1)-(2)-(4)-(5)-(6) we have:

$$g_y(t) = \alpha(t)\left(\mu(t) + g_w(t)\right) + (1 - \alpha(t))\left(g_k(t) + g_r(t)\right). \tag{7}$$

A balanced path is defined as the special case where the output-capital ratio $Y(t)/K(t)$ is constant and where imports equal exports $X(t) = M(t)\ \forall t$. Along a balanced path, the capital and output growth rates are equal:

$$\frac{Y(t)}{K(t)} = c = \text{ constant} \implies \frac{d}{dt}\frac{Y(t)}{K(t)} = 0 \implies g_K(t) = g_Y(t) = g(t)\ \ \forall t, \tag{8}$$

Furthermore

$$r(t)\frac{K(t)}{Y(t)} = 1 - \alpha(t) \implies g_r = -\Big(\frac{c}{r(t)}\Big)\frac{d\alpha(t)}{dt}. \tag{9}$$

Moreover, from the definition of the labour share in the GDP, we have

$$\alpha(t) = \frac{w(t)L(t)}{Y(t)} = \frac{w(t)}{A(t)}$$

where $A(t)$ denotes the labour productivity. By differentiating this expression and using (9) we get an alternative formula for g_r from the following implication:

$$\frac{d\alpha(t)}{dt} = \alpha(g_w - \gamma) \implies g_r(t) = \Big(\frac{c\alpha(t)}{r(t)}\Big)(\gamma(t) - g_w(t)) \tag{10}$$

where γ denotes the growth rate of the labour productivity. Finally, by substituting g_w from (10) into (7), and using (8), we obtain

$$g(t) = \gamma(t) + \mu(t).$$

From now on, for mathematical simplicity, the time argument "t" will be most often omitted in the equations.

3. Identification of a benchmark numerical model

The setting of a numerical simulation model requires to select parameter values and to specify initial conditions. The year is taken as the time unit. The capital depreciation rate is set to $\delta = 0.08$.

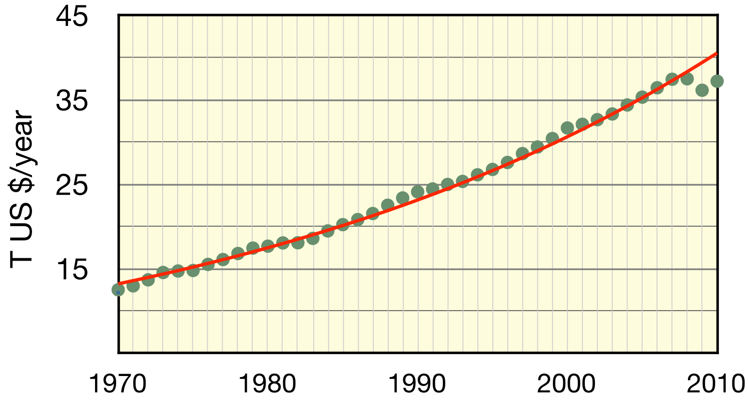

Fig.1: *GDP in OECD from 1970 to 2010 (constant prices 2005). The green dots are empirical data from* `stat.oecd.org`. *The red curve represents a superimposed LS estimate of the balanced path.*

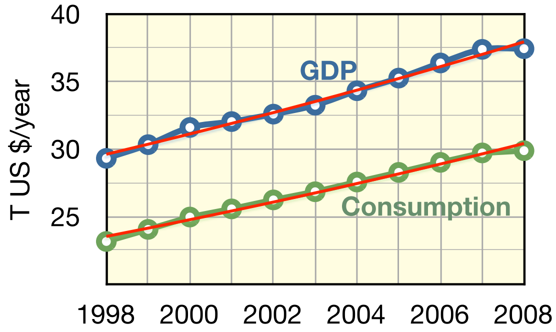

Fig.2: *GDP and Consumption in OECD from 1998 to 2010 (constant prices 2005). The dots are annual empirical data from* `stat.oecd.org`. *The red curves represent superimposed LS estimates of the balanced path.*

In order to get simulation results having a realistic flavour, we set up a benchmark model which is initialized with orders of magnitude corresponding to the state of OECD economy during the period 1998 - 2008. The evolution of GDP, consumption, employment, imports, exports and labour share in GDP are shown in Fig.1-2-3-4-5. Employment is taken as the measurement of labour L.

These empirical data show small economic fluctuations around an exponential path (represented by the red curves fitted on the data) which is assumed to be a balanced path. From the empirical data of Fig.1 and Fig.3, the following least-squares estimates are computed:

$$\frac{1}{Y}\frac{dY}{dt} = g \simeq 0.028, \qquad \frac{1}{L}\frac{dL}{dt} = \mu \simeq 0.011. \tag{11}$$

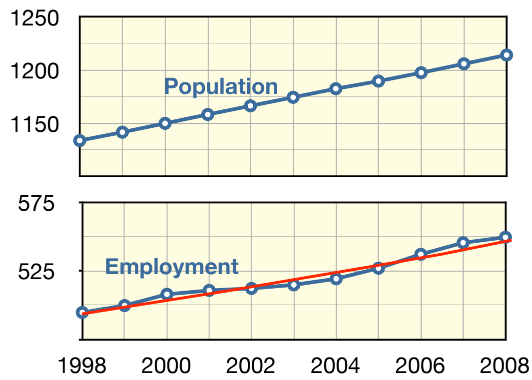

Fig.3: *Population and Employment in OECD from 1998 to 2008 (millions of people). The dots are annual empirical data from* `stat.oecd.org`*. The red curve represents the superimposed LS estimate of the balanced path.*

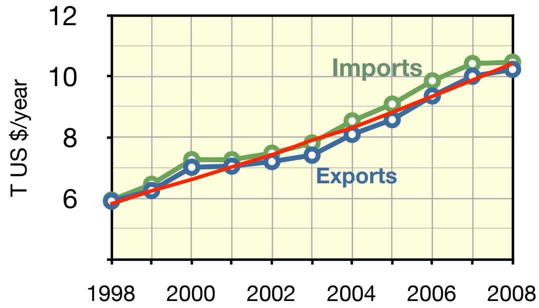

Fig.4: *Imports and Exports in OECD from 1998 to 2010 (constant prices 2005). The dots are annual empirical data from* `stat.oecd.org`*. The red curve represents the superimposed LS estimate of the balanced path.*

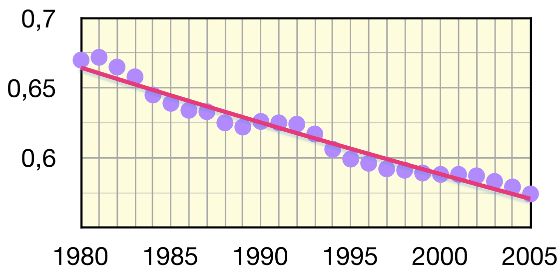

Fig.5: *Labour share in GDP in OECD from 1980 to 2005. The dots are empirical data from* `stat.oecd.org`*. The red curve represents an exponential LS estimate fitted on the empirical data.*

As mentioned above, it is assumed that imports equal exports $X(t) = M(t)\ \forall t$ along the balanced path, see Fig.4. The empirical data for $\alpha(t)$ are shown in Fig.5 and the following exponentially decreasing function is fitted on the data:

$$\alpha(t) = \alpha(0)e^{-at} \quad \text{with} \quad a \simeq 0.0061. \tag{12}$$

The initial time ($t = 0$) for the numerical simulations of the benchmark economy is the year 2000.

On the balanced path represented in Fig.2-3-4-5, we have directly the initial values

$$Y(0) = 31.60, \quad C(0) = 24.96, \quad L(0) = 495, \quad X(0) = M(0) = 6.8, \quad \alpha(0) = 0.67.$$

Therefore

$$K(0) = \frac{Y(0) - C(0)}{g + \delta} = 63.24, \quad A(0) = \frac{Y(0)}{L(0)} = 0.0638, \quad c = \frac{Y(0)}{K(0)} = 0.49 \text{ and } \gamma = g - \mu = 0.017.$$

The initial values are collected in Table 3 (see Appendix) where the corresponding units are also given.

4. Carbon dioxide

As in [Nordhaus, 2008], we assume that CO_2 emissions are representative of total GHG emissions. The CO_2 emission rates for OECD and non-OECD countries during the period 1970-2008 are shown in Fig.6. In non-OECD countries CO_2 emissions are steadily increasing proportionally to GDP. In contrast, the CO_2 emissions of OECD countries are slowly increasing and even almost constant over the last ten years. Assuming that the CO_2 emissions are related to the economic production, there is no loss of generality in writing

$$E(t) = h(t)Y(t) \tag{13}$$

where $E(t)$ is the CO_2 emission flow and $h(t)$ is the carbon intensity of the economic production $Y(t)$. The OECD empirical data for $h(t)$ are shown in Fig.7 and the following exponentially decreasing function can be fitted on the data:

$$h(t) = h(0)e^{-\varepsilon t} \;\; kg\ CO_2/US\$ \text{ with } \varepsilon \simeq 0.021. \tag{14}$$

By differentiating equation (13), we obtain:

$$\frac{dE}{dt} = \frac{dh}{dt}Y + h\frac{dY}{dt} = \left(\frac{1}{Y}\frac{dY}{dt} + \frac{1}{h}\frac{dh}{dt}\right)E = (g_Y(t) - \varepsilon)\,E. \tag{15}$$

An important point here is obviously that $\varepsilon \simeq 0.021 < g_Y \simeq 0.028$ which means that the efficiency of CO_2 abatement is not sufficient to compensate for GDP growth: the decoupling between growth and greenhouse gas emissions is relative but not absolute ([Jackson, 2009, p.53]).

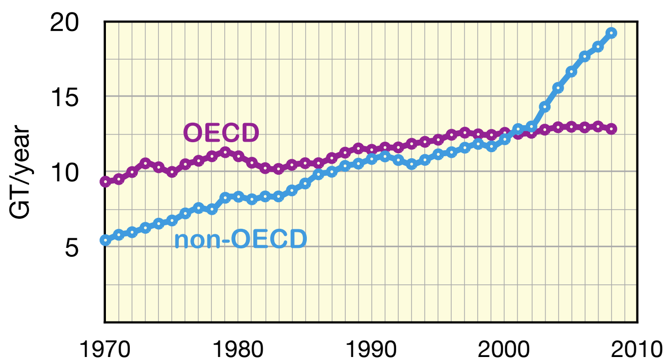

Fig.6: *CO_2 emissions (Data from World Bank Development Indicators.)*

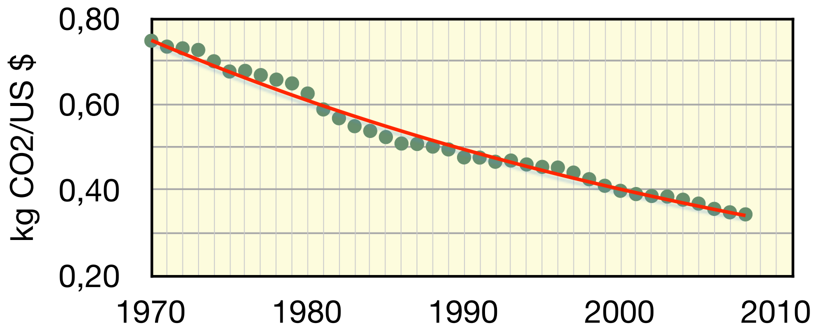

Fig.7: *CO_2 intensity in OECD countries computed with data from Fig.1 and Fig.6. (The exponential function $h(t)$ is represented by the solid line.)*

5. First simulation : Business as Usual

In this first simulation, we assume that the economy continues to follow the balanced path that we have identified above. The balanced path is a solution of the following set of differential and algebraic equations:

$$\frac{dL}{dt} = \mu L, \qquad \frac{d\alpha}{dt} = -a\alpha, \qquad \frac{dK}{dt} = (\mu+\gamma)K, \qquad \frac{dE}{dt} = (\mu+\gamma-\varepsilon)E, \tag{16}$$

$$I = (\mu+\gamma+\delta)K, \qquad Y = cK, \qquad C = Y - I, \qquad w = \frac{\alpha Y}{L}, \qquad r = c(1-\alpha). \tag{17}$$

For the population dynamics we adopt the medium projection of the United Nations (see [UN, 2004]) such that the population increases until about 2050 and then stabilizes for a while as shown in Fig.8.

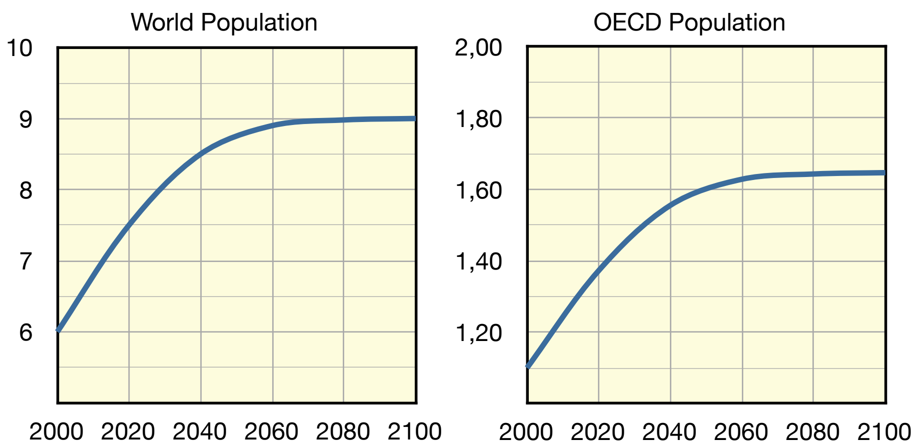

Fig.8: *Evolution of the population from 2000 to 2100 (milliards of people) in the benchmark model.*

The exogenous specific growth rate $\mu(t)$ is computed accordingly. The employment is supposed to be a constant fraction of the population. The model is initialized in 2000 with the values of Table 3. To model the technical progress, we assume a constant labour productivity growth rate $\gamma = 0.017$ as computed in Section 3. For the other constant parameters needed for the simulation, we adopt the values computed above : $a = 0.0061$, $c = 0.49$, $\varepsilon = 0.021$. The model equations are

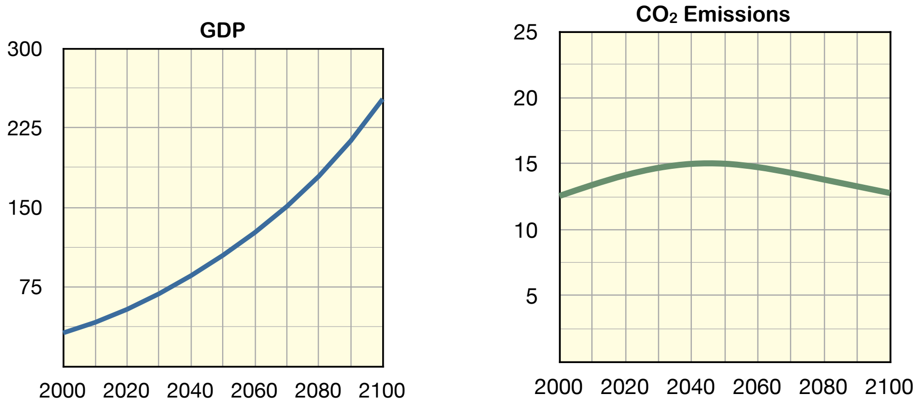

Fig.9: *Business as usual. Left: GDP (T US \$/year); Right: CO_2 emissions (GT/year).*

encoded in Matlab-Simulink.

The results of the simulation experiment are illustrated in Fig.9. As it can be expected, the economy keeps growing exponentially and does not significantly reduce the level of CO_2 emissions. There is a slight decrease of CO_2 emissions during the second half of the century which is due to the conjugate effects of population stabilization and carbon intensity decrease. But, at the end of the century, the CO_2 emission per capita is about 6T/year. Extended to the whole planet, such an emission rate per capita would make the CO_2 atmospheric concentration reaching unsupportable values in 2100 (over 800 ppm, see e.g.[Nordhaus, 2010]).

6. Green Growth

Despite the capitalist propension to efficiency and despite a significant decrease of carbon intensity (50% since 1970), it can clearly be suspected from the results of the previous section that the current economic trend will not succeed in reaching a sustainable economy. Vigorous new public policies are most probably needed to modify this trend in the desired direction. In this section, we investigate a so-called “green growth” public policy. For this purpose we extend the model by introducing the additional assumption that a share of the total investment is funded by the government and explicitly allocated to the development of “Novel Green Technical Knowledge”. These innovations are pure public goods that are both non-rival and non-excludable. In other words they are freely made available to all producers in order to further reduce the greenhouse gas emissions.

Therefore, we now consider an economy with two sectors:

1) A conventional sector with an accounting identity

$$Y_{\text{CS}} = w_{\text{CS}} L_{\text{CS}} + r_{\text{CS}} K_{\text{CS}}. \tag{18}$$

The conventional sector is endowed with the dynamics, the parameter values and the initial conditions of the benchmark model of the previous section.

2) A “green technology” public sector that produces the public green technical knowledge denoted H. The production flow of H is denoted

$$\frac{dH}{dt} = Y_{\text{GS}}. \tag{19}$$

with the accounting identity

$$Y_{\text{GS}} = w_{\text{GS}} L_{\text{GS}} + r_{\text{GS}} K_{\text{GS}},$$

where K_{GS} and L_{GS} are the capital and the labour allocated to the public research in green technical knowledge. The dynamics of the capital stock K_{GS} are represented by the equation

$$\frac{dK_{GS}}{dt} = -\delta K_{GS} + I_{GS}, \tag{20}$$

where I_{GS} denotes the green investment.

For simplicity, we will assume that the two sectors have identical labour productivities:

$$\frac{Y_{CS}}{L_{CS}} = \frac{Y_{GS}}{L_{GS}} = A,$$

but this could be relaxed to some degree. We consider equilibrium economic paths with competitive factor markets. This implies that, along the economic path, the wage rates and the rental rates are equal in the two sectors:

$$w_{CS}(t) = w_{GS}(t) = w(t), \qquad r_{CS}(t) = r_{GS}(t) = r(t). \tag{21}$$

The two sectors are aggregated by defining the total capital $K = K_{CS} + K_{GS}$, the total investment $I = I_{CS} + I_{GS}$ and the total output $Y = Y_{CS} + Y_{GS}$. It is then readily checked that:

$$\frac{dK}{dt} = -\delta K + I. \tag{22}$$

From these conditions, we have that the total output satisfies a global accounting identity of the form

$$Y = Y_{CS} + Y_{GS} = wL + rK. \tag{23}$$

Hence, the *structure* of the economy is not modified with respect to business as usual. But the *nature* of the production is different since the representative output Y is now partly composed of the public green knowledge Y_{GS} (in addition to the on-going private green technologies that are already incorporated in the conventional production).

Let us now turn to the issue of the sustainable transition. As it can be observed from the data of Fig.6, the current level of world CO_2 emissions (in 2008) is about 30 GT/Year with 45% for OECD and 55% for the rest of the world. We assume that **the objective of the transition to a sustainable economy is to guarantee equitable emissions over the planet in 2100 with a total value less than 24 GT/Year**. The target value is therefore set at:

$$E_w^* = 23.8 \text{ GT/Year}.$$

Hence, the sustainable challenge is to decrease the global emissions with respect to the present situation while ensuring progressively a fair distribution with the same emissions per capita everywhere in the world. This implies strongly reducing the OECD emissions while still allowing for a moderate increase in non-OECD countries. Since the ratio of OECD to world population is 0.183, the target for OECD emissions in 2100 must be (at most)

$$E^* = 0.183 \times E_w^* = 0.183 \times 23.8 = 4.66 \text{ GT/Year}.$$

In order to achieve this goal, the model of CO_2 emissions is extended to incorporate the effect of green technologies as follows:

$$E(t) = h(t)Y(t)e^{-\eta H(t)}.$$

With this model we thus now assume that E is not only linearly increasing with final output production as above but also exponentially decreasing with the level of public green technical knowledge H. The parameter η is an elasticity coefficient. The function $h(t)$ is given by expression (14) and represents the current decrease of CO_2 intensity. Obviously the elasticity η is a key parameter in this model since it determines how much can be achieved in CO_2 abatement per unit of time with a given investment. The answer to this question has given rise to an abundant literature but is still, nevertheless, a widely open question. Depending of the assumptions, the estimates of the cost of achieving 50% reduction in CO_2 emissions in 2050 span a very wide range,

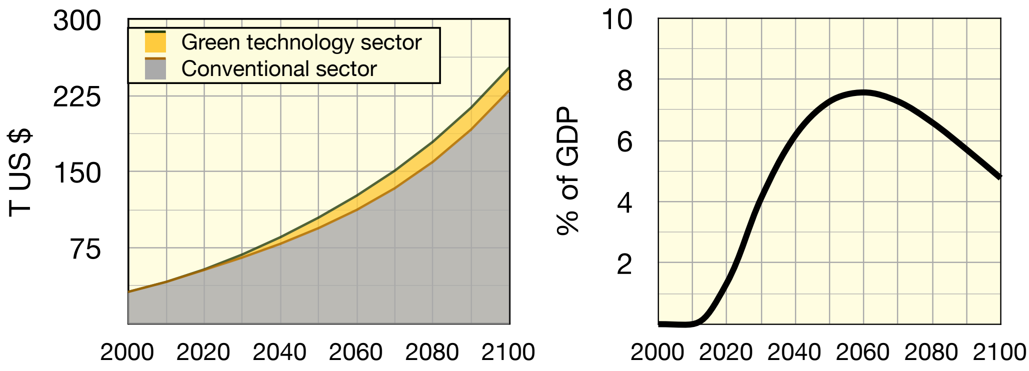

Fig.10: *Green growth in the OECD benchmark model. Left: GDP; Right: Public cost of green growth as a percentage of GDP.*

from 1% to 8% of GDP (see e.g. [Bréchet et al., 2011, Section 6]). In our simulation, we set $\eta = 0.001$ which provides a cost in this range. All the other constant parameters needed for the simulation have been given previously (see also Table 2). In order to achieve the goal of CO_2 abatement, an endogenous feedback investment policy is applied to the system from 2014. The public green investment I_{GS} is simply assumed to change proportionally to the excess of CO_2 emissions with respect to the target E^*:

$$\frac{dI_{\text{GS}}}{dt} = \theta_0(E - E^*). \tag{24}$$

The constant parameter θ_0 is adjusted by trial and error at the value $\theta_0 = 0.0075$.

As above, a balanced path is defined as the special case where the ouput-capital ratio is constant and identical in the two sectors:

$$\frac{Y_{\text{CS}}}{K_{\text{CS}}} = \frac{Y_{\text{GS}}}{K_{\text{GS}}} = c.$$

The balanced path is the solution of the following set of differential and algebraic equations:

$$\frac{dL}{dt} = \mu L, \qquad \frac{d\alpha}{dt} = -a\alpha, \qquad \frac{dA}{dt} = \gamma A, \qquad \frac{dK}{dt} = (\mu + \gamma)K,$$

$$I = (\mu + \gamma + \delta)K, \qquad Y = cK, \qquad C = Y - I, \qquad w = \alpha A, \qquad r = c(1 - \alpha).$$

$$\frac{dE}{dt} = (\mu + \gamma - \varepsilon - \eta c K_{\text{GS}})\, E, \qquad \frac{dK_{\text{GS}}}{dt} = -\delta K_{\text{GS}} + I_{\text{GS}}, \qquad \frac{dI_{\text{GS}}}{dt} = \theta_0(E - E^*). \tag{25}$$

$$Y_{\text{GS}} = cK_{\text{GS}}, \qquad L_{\text{GS}} = \frac{Y_{\text{GS}}}{A},$$

$$K_{\text{CS}} = K - K_{\text{GS}}, \qquad Y_{\text{CS}} = Y - Y_{\text{GS}}, \qquad I_{\text{CS}} = I - I_{\text{GS}}, \qquad L_{\text{CS}} = L - L_{\text{GS}}.$$

The three differential equation (25) describe the dynamics connecting the public green investment I_{GS} to the CO_2 emissions E. The first of these equations is a modification of (15) which accounts for the influence of H.

The model is initialized in 2000 with the values of Table 3 for the conventional sector and with zero initial conditions for the ecological sector. The green growth policy is activated in 2014. The result of the simulation experiment is illustrated in Fig.10 and 11. It must be clearly understood that, in this result, the conventional sector involves the "usual" technical progress towards CO_2 abatement at the rate ε which is not sufficient to reach sustainability. In addition, the green technology sector produces supplementary free public innovations that are used to further accelerate CO_2 abatement in order to reach the sustainable target. In Fig.10 the cost $(I_{\text{GS}} + wL_{\text{GS}})$ of this public policy is also represented as a percentage of GDP.

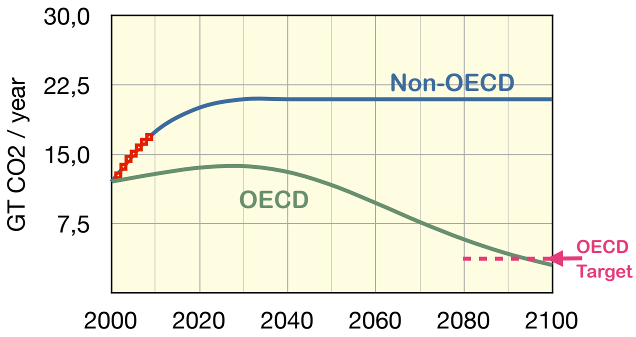

Fig.11: *CO_2 emissions for the period 2000-2100: simulation result for the OECD benchmark and highest admissible growth projection for non-OECD countries. The red dots are empirical data.*

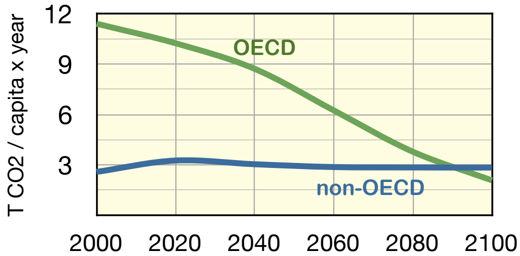

Fig.12: *CO_2 emissions per capita for the period 2000-2100*

In order to estimate the impact of this policy on the planet atmospheric CO_2 concentration, we need also to have a scenario for CO_2 emissions in non-OECD countries. The future effective evolution of CO_2 emissions in non-OECD countries depends on many factors such as the international trade, the extent of exported emissions [Davis and Caldeira, 2010] or the efficiency of international negociations (Kyoto, Copenhagen, Doha ...). In any case, in order to decrease the total world emissions, it is clear that the highest admissible projection of sustainable CO_2 emissions for non-OECD is given in Fig.11, because higher emissions would be too large with respect to the target of 24 GT/Year in 2100. The corresponding evolution of emissions per capita in both subregions is shown in Fig.12. In the non-OECD subregion, the increase of CO_2 may not exceed the population growth. It is interesting to notice that, as shown in Fig.13, the CO_2 emissions scenario has a magnitude close to the IPCC RCP4.5 scenario which predicts a temperature increase likely in the range [+1.1°C , + 2.6°C] by 2100 with respect to 2000 [IPCC, 2013, page 12-3].

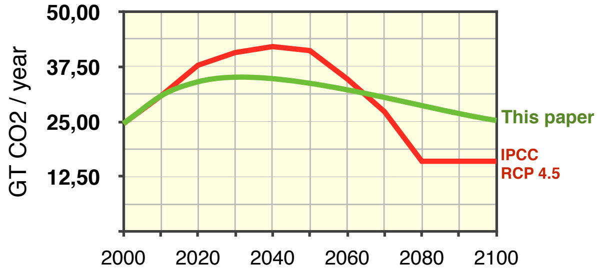

Fig.13: *Total CO_2 emissions: Comparison with IPCC RCP4.5 scenario for the period 2000-2100*

There are however many major objections that can be invoked against the feasibility of green growth. A very fundamental objection is that green growth relies essentially on a blind faith into the technological progress. Indeed it seems as well reasonable to believe that the required massive technological breakthrough is in fact out of reach. For this reason, a sound principle is to consider also alternatives like the low-growth strategy defended for instance by Tim Jackson [Jackson, 2009] and The Club of Rome [Meadows et al., 2004].

7. Low Growth

The principle of a low growth public policy is to foster a structural shifting of the economy composition towards activities which have low (or even zero) carbon intensity. Such activities are by nature labour intensive and far less subject to productivity growth (see e.g. [Jackson and Victor, 2011]). The simplest case is to consider an economy with two sectors:

1) A conventional sector with accounting identity

$$Y_{\text{CS}} = w_{\text{CS}} L_{\text{CS}} + r_{\text{CS}} K_{\text{CS}}. \tag{26}$$

The conventional sector is endowed with the dynamics, the parameter values and the initial conditions of the benchmark model of Section 4. The conventional sector output is taken as numeraire for the global economy.

2) An "ecological" sector of activities having zero carbon intensity and a constant labour productivity, with an accounting identity

$$\pi Y_{\text{ES}} = w_{\text{ES}} L_{\text{ES}} + r_{\text{ES}} K_{\text{ES}}, \tag{27}$$

where π is the relative price of the ecological sector output.

Along an economic equilibrium path, the factor markets are competitive and therefore the remuneration of the factors are equal in the two sectors:

$$w_{\text{CS}}(t) = w_{\text{ES}}(t) = w(t), \qquad r_{\text{CS}}(t) = r_{\text{ES}}(t) = r(t) \qquad \forall t.$$

The labour shares in the sectorial added values are denoted :

$$\alpha = \frac{wL_{\text{CS}}}{Y_{\text{CS}}}, \qquad \beta = \frac{wL_{\text{ES}}}{\pi Y_{\text{ES}}}.$$

Since the conventional sector is supposed to be a replica of the benchmark model, we assume that the labour productivity A increases exponentially with a constant growth rate γ and that the labour share α in added value follows the dynamics of equation (12). In contrast, in the ecological sector, the labour productivity $B = Y_{\text{ES}}/L_{\text{ES}}$ is supposed to be constant and smaller than A. As in the previous sections, a balanced path is defined as the special case where the output-capital ratio is constant in both sectors:

$$\frac{Y_{\text{CS}}}{K_{\text{CS}}} = c, \qquad \frac{\pi Y_{\text{ES}}}{K_{\text{ES}}} = c\frac{1-\alpha}{1-\beta} = \text{ constant.}$$

It follows that, along a balanced trajectory, the labour shares α and β in the two sectors must satisfy the following differential equality:

$$\frac{1}{1-\beta}\frac{d\beta}{dt} = \frac{1}{1-\alpha}\frac{d\alpha}{dt}.$$

In this economy, the CO_2 emissions are proportional to Y_{CS} only:

$$E(t) = h(t) Y_{\text{CS}}(t) \tag{28}$$

with the carbon intensity function $h(t)$ given by (14).

The strategy for the transition to a sustainable economy is a sectorial shift to activities with zero CO_2 emissions. Hereafter we present a simulation of a low-growth scenario that produces, along time, the same CO_2 emissions as the green growth scenario of the previous section. Therefore the emission profile $E(t)$ of Fig.11 which has been computed in the green growth scenario is a reference E_{REF} which is used, in the simulation, as an exogenous driving variable to compute $Y_{\text{CS}}(t)$ from equation (28).

The balanced path is the solution of the following set of differential and algebraic equations:

$$\frac{dL}{dt} = \mu L, \qquad \frac{d\alpha}{dt} = -a\alpha, \qquad \frac{d\beta}{dt} = -a\alpha\frac{1-\beta}{1-\alpha} \qquad \frac{dh}{dt} = -\varepsilon h, \qquad \frac{dA}{dt} = \gamma A,$$

$$Y_{\text{CS}} = \frac{E_{\text{REF}}}{h}, \qquad K_{\text{CS}} = \frac{Y_{\text{CS}}}{c}, \qquad L_{\text{CS}} = AY_{\text{CS}}, \qquad r = (1-\alpha)c, \qquad w = \alpha A,$$

$$L_{\text{ES}} = L - L_{\text{CS}}, \qquad \pi = \frac{\alpha A}{\beta B}, \qquad Y_{\text{ES}} = \frac{wL_{\text{ES}}}{\pi\beta}, \quad K_{\text{ES}} = \frac{\pi Y_{\text{ES}} - wL_{\text{ES}}}{r}.$$

The constant labour productivity in the ecological sector is set to $B = 0.054$, i.e. 20% lower than the initial productivity in the conventional sector. The initial labour share in added value in the ecological sector is set to $\beta(0) = 0.8$ in order to have a unit initial relative price $\pi(0) = 1$. This means that the ecological sector is more labour intensive than the conventional sector by about 10 %. All the other constant parameters needed for the simulation have been given previously (see Table 2).

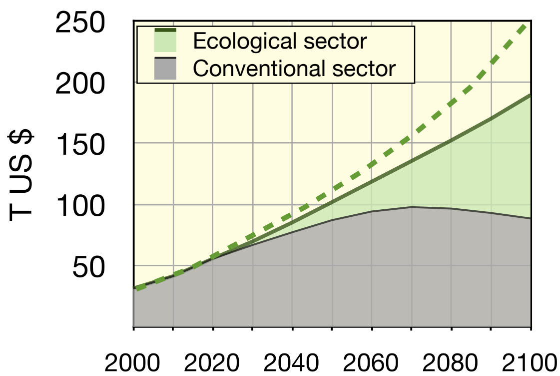

Fig.14: *GDP for the low growth scenario (The dashed line is the green growth GDP trajectory).*

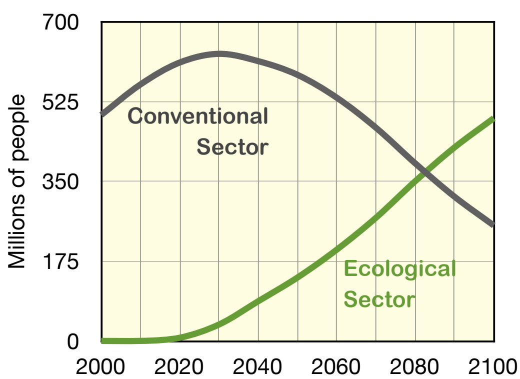

Fig.15: *Employment in the two sectors for the low growth scenario.*

The model is initialized in 2000 with the values of Table 3 for the conventional sector and with zero initial conditions for the ecological sector. The new low-growth policy is activated in 2014. The government strategy to foster the transition is to tax the conventional production (using e.g. carbon taxes) and subsidize the ecological sector in order to equalize the market price between the two sectors. The result of the simulation experiment is illustrated in Fig.14 and Fig.15. Naturally, in this case, a balanced economy means that capital and output vary at the same rate within each sector, but at different rates between the sectors because of the reallocation of labor and capital as illustrated in these figures. As expected, in this scenario, the economic growth in the conventional sector is drastically reduced (with even a small de-growth from 2060) and only partially compensated by the expansion of the ecological sector. This results in a global economic growth slowned down as compared to the green growth scenario. Fig.15 illustrates quantitatively the labour reallocation which is needed.

8. Modelling the limitation of material resources

So far we have implicitly assumed that the material resources needed for the production are not limiting. In a sustainable economy it is however an evidence that the material resources, even renewable, are limited. By "renewable resources", we mean either biotic resources that are biodegradable and naturally regenerated by the environment or a-biotic resources (such as minerals and metals) that are depletable but can be industrially recycled (see e.g. [Fagnart and Germain, 2011] and the references therein).

We denote F_a the flow of a-biotic material incorporated in the consumption goods and the physical capital:

$$F_a = m_a Y. \tag{29}$$

In this expression, m_a represents the a-biotic material intensity, i.e. the average amount of a-biotic material embodied in one unit of output production.

Similarly, we denote F_b the flow of incorporated biotic material:

$$F_b = m_b Y \tag{30}$$

with m_b the biotic material intensity.

The balance of renewable biotic material resources is described by the equation

$$\frac{dM_n}{dt} = F_n - F_b$$

where M_n is the stock of biodegradable resources available for production and F_n is the rate of natural regeneration. Using the vocabulary of the Global Footprint Network (GFN), the flows F_b and F_n may also be interpreted as being proportional to the "ecological footprint" (EF) and the "biocapacity" (BC), see [Ewing et al., 2010] and Table 1 for OECD values.

The time evolution of a-biotic and biotic material intensities in OECD are shown in Fig.16. From these data, it can be computed that the decay rates of both m_a and m_b are quasi-identical with an average value (sometimes called "dematerialization" rate)

$$\sigma = \frac{-1}{m_a}\frac{dm_a}{dt} = \frac{-1}{m_b}\frac{dm_b}{dt} \approx 0.021 \tag{31}$$

which is remarkably close to the decay rate $\varepsilon \approx 0.021$ of CO_2 intensity.

Moreover, a part of the a-biotic material embodied in production is supposed to be recycled at the rate:

$$R = (1 - \upsilon) m_a Y$$

	1990	2008	Units
Ecological Footprint		6229	million gha
Biocapacity		3507	million gha
Ecological Footprint per capita ↗	5.02	5.12	gha per person
Biocapacity per capita ↘	3.23	2.88	gha per person
Deficit of biocapacity per capita ↗	1.79	2.24	gha per person

Table 1: *OECD ecological footprint and biocapacity. Approximate values computed from the empirical data of the Ecological Footprint Atlas 2012*

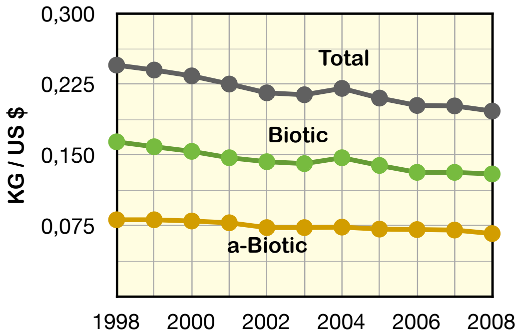

Fig.16: *Material intensities in OECD from 1998 to 2008. The dots are empirical data from* `stat.oecd.org`

where $1-\upsilon$ is the material fraction which is recycled and υ is the material fraction which is discarded as wastes. The evolution of the recycled fraction $1-\upsilon$ of municipal wastes in OECD is given in Fig.17. From these data, we can compute a waste discarding decay rate

$$\rho = \frac{-1}{\upsilon}\frac{d\upsilon}{dt} \approx 0.0095. \tag{32}$$

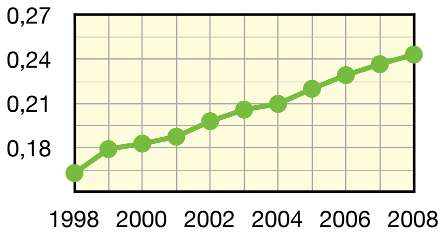

Fig.17: *Recycling rate of municipal wastes in OECD from 1998 to 2008. The dots are empirical data from* `stat.oecd.org`

Business as usual

The "business as usual" balanced path that we have presented in Section 4 is a solution of equations (16) and (17). Along this path, the trajectories of GDP and CO_2 emissions have been illustrated in Fig.9. Assuming that the dematerialization continues to decrease at the same rate for another century, the trajectories of the material flows F_a and F_b along the balanced path are given in Fig.18. Exactly as for CO_2, the efficiency of the current dematerialization trend (about 2% per year) is unfortunately not sufficient to achieve sustainability: the decoupling is relative but not absolute. The evolution of Fig.18 is clearly not acceptable because of an insufficient decrease of both the exploitation of depletable resources (here represented by F_a) and of the ecological footprint (here represented by F_b).

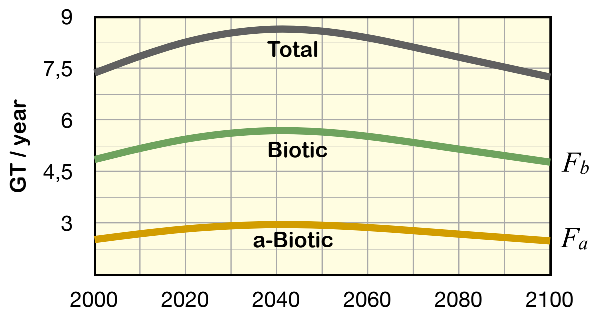

Fig.18: *Evolution of material flows along the "business-as-usual" balanced path*

A green-low growth scenario

Our model can be extended in various ways in order to deal with the material resource limitation in the transition to sustainability. The purpose of this section is to present one possible modeling option which combines the green growth policy to enhance the recycling of abiotic resources with a low growth policy for the abatement of the ecological footprint.

We thus now consider an economy with three sectors:

1. A conventional sector as described in Sections 5 and 6.

2. A green technology sector, as described in Section 5, which produces public technical knowledge to enhance both CO_2 abatement and recycling of a-biotic materials.

3. An ecological sector as described in Section 6.

Along an economic equilibrium path, the factor markets are competitive and therefore the remuneration of the factors are equal in the three sectors:

$$w_{\text{CS}}(t) = w_{\text{GS}}(t) = w_{\text{ES}}(t) = w(t), \qquad r_{\text{CS}}(t) = r_{\text{GS}}(t) = r_{\text{ES}}(t) = r(t) \qquad \forall t.$$

The labour shares in the sectorial added values are denoted:

$$\alpha = \frac{wL_{\text{CS}}}{Y_{\text{CS}}} = \frac{wL_{\text{GS}}}{Y_{\text{GS}}}, \qquad \beta = \frac{wL_{\text{ES}}}{\pi Y_{\text{ES}}}.$$

Regarding the material intensity, it is assumed that the conventional sector follows the current dematerialization trend with exponentially decaying m_a and m_b at the rate σ. In contrast it is assumed that the ecological sector has lower constant a-biotic m'_a and biotic m'_b material intensities and therefore a lower ecological footprint. For simplicity we assume that the green technology sector production is totally dematerialized and has negligible CO_2 emissions. As we have mentioned above, it is also assumed that the public green technical knowledge contributes to enhance the recycled fraction of a-biotic materials in addition to the abatement of CO_2 emissions in the conventional sector. The model of R is therefore modified as

$$R = (1 - ve^{-\xi H})(m_a Y_{\text{CS}} + m'_a Y_{\text{ES}}).$$

The parameter ξ is an elasticity coefficient, similar to the CO_2 abatement elasticity η of Section 5.

The objective of the transition to a sustainable economy is **to reach $\mathbf{CO_2}$ emissions lower than the OECD target $\mathbf{E^* = 4.66}$ GT/year before 2100 AND to reach the GFN "rapid reduction target" of a zero biocapacity deficit by 2050**.

As in Section 5, the public investment I_{GS} in green technology is assumed to change proportionally to the excess of CO_2 emissions with respect to the target E^*:

$$\frac{dI_{\text{GS}}}{dt} = \theta_1(E - E^*). \tag{33}$$

Moreover, the employment shift from the conventional sector to the ecological sector is represented by the simple exponential dynamics:

$$\frac{d}{dt}\left(\frac{L_{\text{CS}}}{L_{\text{ES}}}\right) = -\theta_2\left(\frac{L_{\text{CS}}}{L_{\text{ES}}}\right).$$

The balanced path is the solution of the following set of differential and algebraic equations:

$$\frac{dL}{dt} = \mu L, \qquad \frac{d\alpha}{dt} = -a\alpha, \qquad \frac{d\beta}{dt} = -a\alpha\frac{1-\beta}{1-\alpha}, \qquad \frac{dA}{dt} = \gamma A,$$

$$w = \alpha A, \qquad r = c(1-\alpha).$$

$$\frac{dL_{\text{CS}}}{dt} = \psi L_{\text{CS}} \text{ with } \psi = \left[-\theta_2\left(1 - \frac{L_{\text{CS}}}{L - L_{\text{GS}}}\right) + \frac{1}{L - L_{\text{GS}}}\left(\mu L + (\delta+\gamma)L_{\text{GS}} - \frac{cI_{\text{GS}}}{A}\right)\right],$$

$$\frac{dE}{dt} = (\gamma + \psi - \varepsilon - \eta c K_{\text{GS}})\, E, \qquad \frac{dK_{\text{GS}}}{dt} = -\delta K_{\text{GS}} + I_{\text{GS}}, \qquad \frac{dI_{\text{GS}}}{dt} = \theta_1(E - E^*).$$

$$Y_{\text{GS}} = cK_{\text{GS}}, \qquad L_{\text{GS}} = \frac{Y_{\text{GS}}}{A}, \qquad Y_{\text{CS}} = AL_{\text{CS}}, \qquad K_{\text{CS}} = \frac{Y_{\text{CS}}}{c}, \qquad \pi = \frac{\alpha A}{\beta B_L},$$

$$L_{\text{ES}} = L - L_{\text{CS}} - L_{\text{GS}}, \qquad Y_{\text{ES}} = \frac{wL_{\text{ES}}}{\pi\beta}, \qquad K_{\text{ES}} = \frac{\pi Y_{\text{ES}} - wL_{\text{ES}}}{r}.$$

The model is initialized in 2000 with the values of Table 3 for the conventional sector. The initial value of the waste discarding fraction $\upsilon(0) = 0.818$ (see Fig.17). Since the ratio (EF/BC) $\approx$ 1.75 in 2000 (see Table 1), we take the value $F_n(0) = F_b(0)/1.75 = 2.75$ GT/year for initializing the biocapacity and we assume that it remains constant. All other initial values for the green technology sector and the ecological sector are set to zero. The green-growth and low-growth policies are activated in 2014. The green technology investment is assumed to be twice as less efficient than in the scenario of Section 5. The elasticity coefficient is therefore set to $\eta = 0.0005$. According to equation (31), the biotic material intensity in the conventional sector should be around $m_b \approx 0.02$ in 2100. In the ecological sector the biotic material intensity is lower by assumption and set to $m'_b = 0.015$. The parameter values $\theta_1 = 0.005$ and $\theta_2 = 0.07$ are selected by trial and error to produce a simulation which achieves the sustainability objective. All the other constant parameters needed for the simulation have been given previously (see also Table 2). The results of the simulation experiment are illustrated in Fig.19, Fig.20, Fig.21.

- From Fig. 19, we see that the de-growth of the conventional sector must be stronger in this scenario in order to reach a zero biocapacity deficit as early as 2060 as shown in Fig.20.
- CO_2 abatement is totally parallel to EF reduction since $\varepsilon = \sigma$ and reaches a value of 0.3 T CO_2 per capita and per year in the OECD region. Extended to the whole planet, such value would now produce a scenario closer to the IPCC RCP2.6 scenario which predicts a temperature increase in the range [+0.3°C , +1.7°C] by 2100. This would obviously also impose a reduction of emissions per capita in the non-OECD region.
- Finally Fig.21 gives the evolution of the recycling rate $1 - \upsilon e^{-\xi H}$ and the employment allocation to the three sectors.

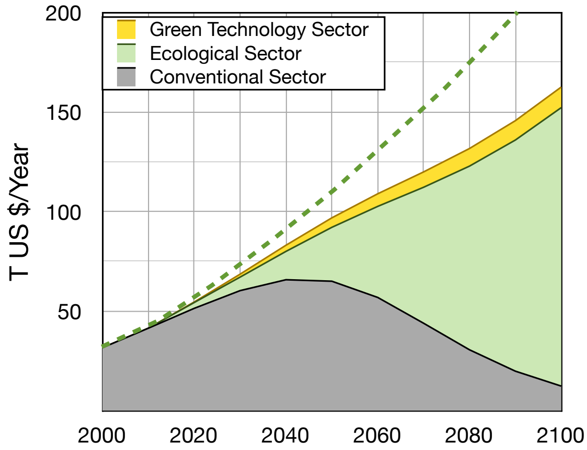

Fig.19: *GDP for the green-low growth scenario (The dashed line is the business-as-usual GDP trajectory).*

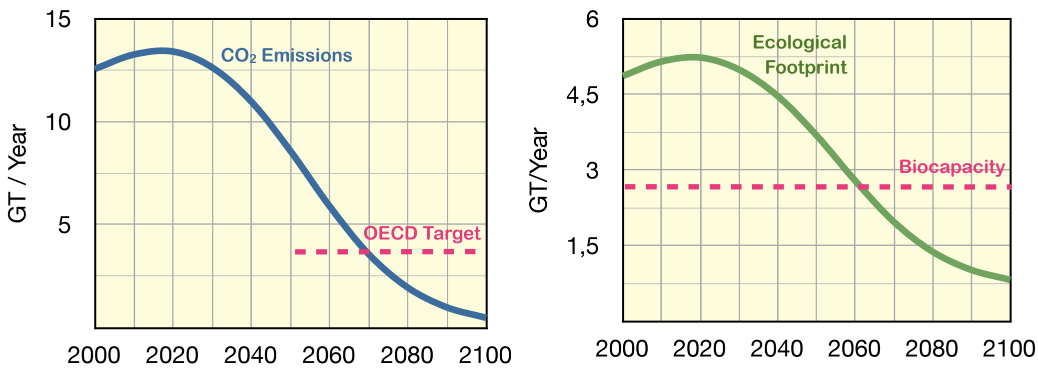

Fig.20: *OECD CO_2 emissions (left) and ecological footprint (right) for the green-low growth scenario.*

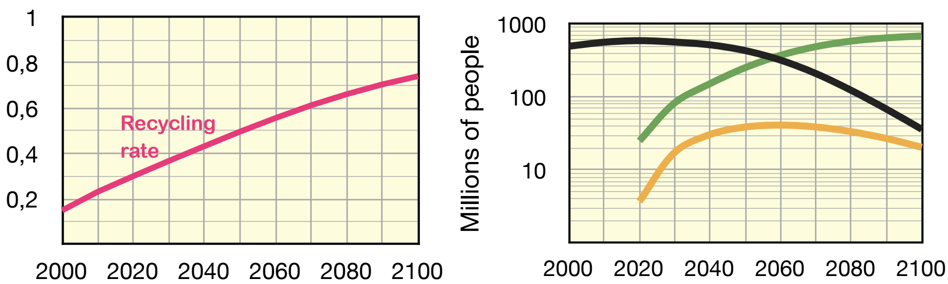

Fig.21: *Left: Recycling rate of a-biotic material. Right: Employment (black: conventional sector, green: ecological sector, yellow: green technology sector)*

9. Conclusions

This paper gives a contribution to the modelling of the transition to a sustainable economy. Three different options are investigated : a green growth option that focuses on accelerating the green technological change, a low growth option that focuses on shifting the structure of the economy towards zero carbon emission activities and a combined green-low growth option that focuses on the limitation of material resources and the abatement of the ecological footprint. The main results can be summarized as follows:

- With the green growth scenario, the economic trajectory reaches the target of CO_2 emission reduction with a specific public development of massive additional green technologies representing a cost up to 8% of GDP.
- With the low growth scenario, it is possible to achieve the same objective, within the same time horizon, without blind faith in technologies, by systematically subsidising a transition to low carbon and low capital intensive activities, leading to a sectoral shift from the conventional sector (from 100% to 45% of GDP) to the ecological sector (from 0% to 55% of GDP).
- With the green-low growth scenario, the objective is to jointly reach the CO_2 emissions target before 2100 and to reach the GFN target of a zero biocapacity deficit by 2050. The simulations show that this objective may be achieved at the price of a substantial de-growth of the conventional economic sector.

The model, as it has been set up in this paper, represents a rather narrow and limited perspective regarding the transition to sustainability. Many relevant aspects of the economic impact of global warming and resource overexploitation are ignored. Moreover the structure of the economy itself is quite simplified. Important related issues such as social inequalities or international finance unreliability are not addressed. However, we hope that our parsimonious modelling contributes to highlight some of the fundamental challenges in terms of economic policy. Moreover as we have mentioned in the introduction, the model can be easily extended to include more subregions and economic sub-sectors or explicit fiscal policies. One important issue which has been omitted relies on the modelling of the mechanisms that underly the public policy and their impact on the economy dynamics. This issue will be dealt with in a forthcoming paper.

Appendix : Tables of numerical values

Parameter		Value	Units
Capital depreciation rate	δ	0.08	1/year
Labour share decay rate	a	0.0061	1/year
Labour productivity growth rate	γ	0.017	1/year
Output/Capital ratio	c	0.49	1/year
Carbon intensity decay rate	ε	0.021	1/year
First atmospheric $C0_2$ coefficient	κ_0	0.16	ppm/GT
Second atmospheric $C0_2$ coefficient	κ_1	0.15	GT/ppm×year
Elasticity of CO_2 abatement vs investment (Sect. 5)	η	0.001	
Elasticity of CO_2 abatement vs investment (Sect. 7)	η	0.0005	
Elasticity of recycling vs investment	ξ	0.0005	
Material intensity decay rate	σ	0.021	1/year
Waste discarding decay rate	ρ	0.0095	1/year
	θ_0	0.0075	US \$/kgCO_2×year
	θ_1	0.005	US \$/kgCO_2×year
	θ_2	0.07	1/year

Table 2: *Contant parameter values used in the simulation experiments*

Variable		Value	Units
Capital	$K(0)$	64.21	T US dollars
Labour	$L(0)$	495	millions of people
Output rate	$Y(0)$	31.60	T US $/year
Consumption	$C(0)$	24.96	T US $/year
Labour share of conventional sector	$\alpha(0)$	0.67	
Labour share of ecological sector	$\beta(0)$	0.8	
Labour productivity	$A(0)$	63.8	10^3 US $/worker
CO_2 Emissions	$E(0)$	12.55	GT CO_2/year
CO_2 intensity	$h(0)$	0.4	kg CO_2/US $
a-Biotic material intensity	$m_a(0)$	0.080	kg/US $
Biotic material intensity	$m_b(0)$	0.154	kg/US $
Waste discarding fraction	$v(0)$	0.845	

Table 3: *Initial conditions for the year 2000 in the simulation experiments*

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