GENERALIZED CROSSED PRODUCTS.

by

Jean-Pierre TIGNOL*

Université Catholique de Louvain
B-1348 Louvain-la-Neuve, Belgium

Introduction :

Generalized crossed products, in which the Galois field extension is replaced by a central simple algebra over a Galois field extension, were defined by Teichmüller in 1940. In the present paper, a multiplication formula for generalized crossed products is proved. This multiplication formula turns out to be useful to investigate the structure of (inertially split) division algebras over Henselian valued fields, as Jacob and Wadsworth show in [3,§5]. Another application in the same vein is given in section 2 of the present paper, where the index and the exponent of generic abelian extensions of central simple algebras are determined.

The author wishes to thank A. Wadsworth for his keen interest in this work.

1. A multiplication formula for generalized crossed products.

1.1. Let G and M be (multiplicative) groups. Assume G acts by automorphisms on the center Z(M) of M. A *factor set of* G in M is a couple (ω, f) of maps :

$$\omega : G \to \mathrm{Aut}(M)$$
$$f : G \times G \to M$$

such that, denoting by Inn(m) the inner automorphism $x \to mx\,m^{-1}$:

$$\omega_\sigma(z) = \sigma(z) \quad \text{for} \quad \sigma \in G,\ z \in Z(M)$$

$$\omega_\sigma . \omega_\tau = \mathrm{Inn}(f(\sigma,\tau)).\omega_{\sigma\tau} \qquad \text{for } \sigma,\tau \in G$$

$$\omega_\sigma(f(\tau,\nu))f(\sigma,\tau\nu) = f(\sigma,\tau)f(\sigma\tau,\nu) \quad \text{for } \sigma,\tau,\nu \in G.$$

* Supported in part by the FNRS.

If G acts trivially on Z(M), then the set F(G,M) of factor sets of G in M contains the trivial element (I,1), where I_σ is the identity on M for all $\sigma \in G$ and $1(\sigma,\tau) = 1$ for all $\sigma,\tau \in G$. If the action of G on Z(M) is not trivial, then F(G,M) may be empty.

Factor sets (ω,f) and (η,g) are said to be *cohomologous* if there exists in M a family $(m_\sigma)_{\sigma\in G}$ such that

$$\eta_\sigma = \mathrm{Inn}(m_\sigma).\omega_\sigma \qquad \text{for } \sigma \in G$$

and

$$g(\sigma,\tau) = a_\sigma\omega_\sigma(a_\tau)f(\sigma,\tau)a_{\sigma\tau}^{-1} \qquad \text{for } \sigma,\tau \in G.$$

It is readily verified that cohomology is an equivalence relation on F(G,M). The quotient of F(G,M) by this relation is denoted by $H(G,M)$. These definitions are classical : they date back at least to O. Schreier [10] who showed that the sets $H(G,M)$, for the various actions of G on Z(M), classify the group extensions of M by G. If M is abelian, then $H(G,M) = H^2(G,M)$.

1.2. Henceforth, we focus on the following special case, first considered by O. Teichmüller [11,p.145] : G is the Galois group of a (finite) extension of fields K/F and $M = A^\times$ is the multiplicative group of invertible elements in a (finite-dimensional) central simple K-algebra A. Moreover, we consider only actions $\omega : G \to \mathrm{Aut}(A^\times)$ of G on $A^\times$ which arise from actions of G on A by *ring*-automorphisms. If $F(G,A^\times)$ is not empty, then for each factor set (ω,f) a generalized crossed product $(A,G,(\omega,f))$ is constructed as follows :

$$(A,G,(\omega,f)) = \oplus_{\sigma\in G} Az_\sigma$$

where the z_σ's are indeterminates subject to the relations :

$$z_\sigma.a = \omega_\sigma(a).z_\sigma \qquad \text{for } a \in A \text{ and } \sigma \in G$$

$$z_\sigma.z_\tau = f(\sigma,\tau).z_{\sigma\tau} \qquad \text{for } \sigma,\tau \in G.$$

It is easily checked that $(A,G,(\omega,f))$ is an associative F-algebra with identify element $f(I,I)^{-1}z_I$. The algebra A can be identified to a subalgebra of $(A,G,(\omega,f))$ by mapping $a \in A$ onto $a.f(I,I)^{-1}.z_I$, and the same arguments as in the commutative case (where A = K) (see for instance [4,§8.4]) yield the following results :

1.3. **THEOREM** :

a) *The generalized crossed product* $(A,G,(\omega,f))$ *is a central simple* F*-algebra of degree* $[K : F]$ deg A. *The algebra* A *is the centralizer of* K *in* $(A,G,(\omega,f))$.

b) *If a central simple* F*-algebra* $\mathcal{A}$ *contains* K, *then, denoting by* A *the centralizer of* K *in* $\mathcal{A}$, *there exists a factor set* $(\omega,f) \in F(G,A^\times)$ *such that*

$$\mathcal{A} \simeq (A,G,(\omega,f)).$$

c) *Factor sets* (ω,f) *and* (η,g) *in* $F(G,A^\times)$ *are cohomologous if and only if*

$$(A,G,(\omega,f)) \simeq (A,G,(\eta,g)).$$

1.4. **COROLLARY** : *The map* $(\omega,f) \to [(A,G,(\omega,f))] \in Br(F)$ *defines a 1-1 correspondence between* $H(G,A^\times)$ *and the set of similarity classes* $[\mathcal{A}] \in Br(F)$ *such that* $[\mathcal{A} \otimes_F K] = [A]$.

<u>Proof</u> : The only assertion which does not readily follow from the preceding theorem is that if a class $[\mathcal{A}]$ satisfies $[\mathcal{A} \otimes K] = [A]$, then it is of the form $[(A,G,(\omega,f))]$. In order to prove that, consider a maximal subfield L of the division algebra D which is similar to A. Since $\mathcal{A}$ is split by L, it is similar to an algebra $\mathcal{B}$ containing L as a maximal subfield. If $A \simeq M_r(D)$, then $M_r(\mathcal{B})$ contains K and

$$[M_r(\mathcal{B}) : F] = r^2 [L : F]^2 = [A : K][K : F]^2.$$

Therefore, the centralizer C of K in $M_r(\mathcal{B})$ has the following properties : $[C : K] = [A : K]$ and $[C] = [M_r(\mathcal{B}) \otimes_F K] = [\mathcal{A} \otimes_F K] = [A]$.
It follows that $C \simeq A$, hence part (a) of the preceding theorem shows that $M_r(\mathcal{B})$ has the form $(A,G,(\omega,f))$ for some $(\omega,f) \in F(G,A^\times)$.

1.5. Let now A and B be central simple K-algebras and let $(\omega,f) \in F(G,A^\times)$ and $(\eta,g) \in F(G,B^\times)$. A factor set $(\omega \otimes \eta,\ f \otimes g) \in F(G,(A \otimes_K B)^\times)$ is then defined as follows : for $\sigma \in G$, $(\omega \otimes \eta)_\sigma = \omega_\sigma \otimes \eta_\sigma$ is the automorphism which maps $a \otimes b$ onto $\omega_\sigma(a) \otimes \eta_\sigma(b)$, and for $\sigma,\tau \in G$,

$$(f \otimes g)(\sigma,\tau) = f(\sigma,\tau) \otimes g(\sigma,\tau).$$

1.6. **THEOREM** : *Letting* $\sim$ *denote similarity of central simple algebras,*

$$(A,G,(\omega,f)) \otimes_F (B,G,(\eta,g)) \sim (A \otimes_K B,G,(\omega \otimes \eta,\ f \otimes g)).$$

<u>Proof</u> : Denote

$$\mathcal{A} = (A,G,(\omega,f)) = \oplus_{\sigma\in G} A.x_\sigma \qquad \mathcal{B} = (B,G,(\eta,g)) = \oplus_{\sigma\in G} B.y_\sigma$$

and

$$\mathcal{C} = (A \otimes_K B,\ G,(\omega \otimes \eta, f \otimes g)) = \oplus_{\sigma\in G} (A \otimes_K B).z_\sigma .$$

Let I be the left ideal of $\mathcal{A} \otimes_F \mathcal{B}$ generated by all the elements of the form

$$(a.k) \otimes b - a \otimes (b.k) \quad \text{for } a \in \mathcal{A},\ b \in \mathcal{B} \text{ and } k \in K.$$

The quotient $V = (\mathcal{A} \otimes_F \mathcal{B})/I$ is a left $\mathcal{A} \otimes_F \mathcal{B}$ - module on which $\mathcal{C}$ acts on the right by :

$$(v + I).(\Sigma_\sigma (a_\sigma \otimes b_\sigma) z_\sigma) = v.(\Sigma_\sigma\, a_\sigma\, x_\sigma \otimes b_\sigma\, y_\sigma) + I \quad \text{for } v \in A \otimes B, \; a_\sigma \in A \text{ and } b_\sigma \in B.$$

(That an action of C on V is well-defined by this formula is a tedious but straightfoward verification.)

The actions of $A \otimes_F B$ and C on V define an embedding :

$$A \otimes_F B \otimes_F C^{op} \subset \mathrm{End}_F(V). \tag{1}$$

To complete the proof, it suffices to show :

$$\dim_F V = \deg A.\deg B.\deg C,$$

i.e. $$\dim_F V = [A : F].[B : F].[K : F]^{-1},$$

since then $$A \otimes_F B \otimes_F C^{op} = \mathrm{End}_F(V) \sim F,$$

hence $$A \otimes_F B \sim C.$$

In fact, the embedding (1) already yields :

$$\dim_F V \geq \deg A.\deg B.\deg C,$$

so it suffices to prove the reverse inequality.

Let (a_i) (resp. (b_j)) be a basis of A (resp. B) as a right vector space over K, and let (k_α) be a basis of K over F. Then $(a_i k_\alpha \otimes b_j\, k_\beta)$ is a basis of $A \otimes_F B$ over F, and since

$$a_i k_\alpha \otimes b_j\, k_\beta \equiv a_i \otimes b_j . k_\alpha\, k_\beta \quad \mod I,$$

it follows that $(a_i \otimes b_j\, k_\alpha + I)$ generates V as a vector space over F.

Therefore, $$\dim_F V \leq [A : F].[B : F].[K : F]^{-1},$$

and the proof is complete.

1.7. __Remark.__ In the commutative case, i.e. when $A = B = K$, theorem 1.6 amounts to the well-known multiplication formula for crossed products. The proof above is copied from the proof of that formula in [2,pp.94-95], for that matter. A. Wadsworth pointed out that the classical proof of the multiplication formula (see e.g. [4,Theorem 8.9]) can also be adapted to generalized crossed products : since A and B both contain K, the tensor product $A \otimes_F B$ contains

$$K \otimes_F K = \oplus_{\sigma \in G}\; K.e_\sigma$$

where $(e_\sigma)_{\sigma\in G}$ are orthogonal idempotents such that

$$k \otimes 1.e_\sigma = 1 \otimes \sigma(k).e_\sigma \quad \text{for } \sigma \in G,\ k \in K.$$

Then $\mathbf{A} \otimes \mathcal{B} \sim e_I.\mathbf{A} \otimes \mathcal{B}.e_I$ and one verifies that

$$e_I.\ \mathbf{A} \otimes \mathcal{B}.e_I \simeq C.$$

Details are left to the reader.

1.8. **COROLLARY** : *If* $H(G,A^\times)$ *is not empty, then for any factor set* $(\omega,f) \in F(G,A^\times)$ *there is a bijection* $H^2(G,K^\times) \to H(G,A^\times)$ *which maps any 2-cocycle* c *to* $(\omega,f.c)$.

<u>Proof</u> : It suffices to show that the map is onto, since injectivity is readily checked. Let $(\eta,g) \in F(G,A^\times)$. By theorem 1.3,

$$(A,G,(\eta,g)) \otimes_F (A,G,(\omega,f))^{op} \otimes_F K \sim A \otimes_K A^{op} \sim K\ ;$$

therefore, $(A,G,(\eta,g)) \sim (A,G,(\omega,f)) \otimes_F (K,G,c)$

for some cocycle $c \in Z^2(G,K^\times)$. By theorem 1.6, the right-hand side is similar to

$$(A \otimes_F K,\ G,(\omega \otimes I,f \otimes c)) \simeq (A,G,(\omega,f.c)),$$

hence, by theorem 1.3, the factor sets (η,g) and $(\omega,f.c)$ are cohomologous.

2. <u>Generic abelian extensions of central simple algebras</u>.

2.1. Let $\mathbf{A}$ be a (finite-dimensional) central simple algebra over a field F. Assume $\mathbf{A}$ contains a Galois extension K of F with *abelian* Galois group G, and denote by A the centralizer of K in $\mathbf{A}$. A generic abelian extension of A (with respect to a basis of G) can then be constructed by mimicking the construction of $\mathbf{A}$ from A and replacing generators of $\mathbf{A}$ by indeterminates (see 2.2 below for a precise definition). This was first done by Amitsur and Saltman [1] in the special case where $A = K$. Their construction was subsequently generalized by Rowen [7] (see also his construction in [8]) to the case where K is contained in a Galois maximal subfield of $\mathbf{A}$.

In this section, we define a generic abelian extension of A and determine its (Schur) index and exponent in full generality. This is done by discussing simultaneously a power series counterpart first considered by B.H. Neumann [5], to which recent results of Jacob and Wadsworth on

Henselian valuations on division algebras can be applied.

2.2. For each $\sigma \in G$, pick $z_\sigma \in A^\times$ such that

$$z_\sigma k = \sigma(k) z_\sigma \qquad \text{for all } k \in K ;$$

let $\omega_\sigma \in \mathrm{Aut}(A)$ be the restriction of $\mathrm{Inn}(z_\sigma)$ to A and let

$$f(\sigma,\tau) = z_\sigma z_\tau z_{\sigma\tau}^{-1} \in A^\times \text{ for all } \sigma,\tau \in G ;$$

then $(\omega,f) \in F(G,A^\times)$ and $\mathcal{A} = (A,G,(\omega,f))$.
For simplicity, we assume henceforth that $z_I = 1$; then $\omega_I = I$ and $f(\sigma,\tau) = 1$ whenever σ or τ is I.
Now, fix a surjective homomorphism

$$\varepsilon : \mathbb{Z}^n \to G$$

and consider formal series

$$s = \Sigma_{\alpha\in \mathbb{Z}^n} a_\alpha Z_\alpha ,$$

where Z_α is an indeterminate and $a_\alpha \in A$ for $\alpha \in \mathbb{Z}^n$. The ***support*** of such a series is defined as usual :

$$\mathrm{supp}(s) = \{\alpha \in \mathbb{Z}^n \mid a_\alpha \neq 0\}.$$

Let then $P_\varepsilon(A,G,(\omega,f))$ (resp. $S_\varepsilon(A,G,(\omega,f))$) (denoted simply by P and S in the sequel) be the set of all formal series whose support is finite (resp. well-ordered with respect to some (fixed) total ordering on $\mathbb{Z}^n$ compatible with its group structure). Multiplication is defined on P and S by the following relations :

$$Z_\alpha . a = \omega_{\varepsilon(\alpha)}(a).Z_\alpha$$

$$Z_\alpha . Z_\beta = f(\varepsilon(\alpha),\varepsilon(\beta))\, Z_{\alpha+\beta}.$$

The centers of P and S are easily determined : letting $\Gamma = \ker \varepsilon$, we have

$$Z(P) = \{\Sigma_{\gamma\in\Gamma} f_\gamma Z_\gamma \in P \mid f_\gamma \in F\}$$

$$Z(S) = \{\Sigma_{\gamma\in\Gamma} f_\gamma Z_\gamma \in S \mid f_\gamma \in F\}.$$

The ring $Z(P)$ is a domain which can be identified to the group ring of Γ over F, while $Z(S)$ is a field of formal power series in n indeterminates with coefficients in F.

Let R (= $R_\varepsilon(A,G,(\omega,f))$) be the ring of central quotients of P. The center of R is the ring of fractions of $Z(P)$, which we denote by F'. We also denote

$$\hat{F} = Z(S) ;$$

thus $\qquad F' \subset \hat{F}, \qquad P \subset R \subset S$

and it is easily verified that

$$S = R \otimes_{F'} \hat{F}.$$

The ring R is a generic abelian extension of A, as defined in [7,§2].

2.3. **THEOREM** : *R and S are central simple algebras over F' and $\hat{F}$ respectively. Their degree, exponent and (Schur) index are*

$$\deg R = \deg S = \deg \mathbf{A} \quad (= [K : F].\deg A).$$

$$\exp R = \exp S = \ell.c.m.\ (\exp G, \exp \mathbf{A}).$$

$$\operatorname{ind} R = \operatorname{ind} S = [K : F].\operatorname{ind}(A \otimes_F K) \quad (= [K : F].\operatorname{ind}(A)).$$

*In particular, if one of the **algebras** $\mathbf{A}$, R or S is a division algebra, then the other two are also division **algebras**.* [compare [12,Theorem 2.7] and [13, Proposition 2.4]]

<u>**Proof.**</u> In order to prove the first part, it suffices to represent R and S as generalized crossed products. Let

$$\hat{K} = \{\Sigma_{\gamma\in\Gamma}\ k_\gamma Z_\gamma \in S \mid k_\gamma \in K\}$$

and

$$K' = \hat{K} \cap R ;$$

let also

$$\hat{A} = \{\Sigma_{\gamma\in\Gamma}\ a_\gamma Z_\gamma \in S \mid a_\gamma \in A\}$$

and

$$A' = \hat{A} \cap R.$$

Then, $\hat{K}/\hat{F}$ and K'/F' are Galois extensions with Galois group G, and $\hat{A}$ (resp. A') is the centralizer of $\hat{K}$ (resp. K') in S (resp. R).

Now, choose a map

$$\rho : G \to \mathbb{Z}^n$$

such that $\varepsilon\rho = I$. For simplicity, we assume moreover $\rho(I) = 0$. The indeterminates $Z_{\rho(\sigma)}$, for $\sigma \in G$, generate S (resp. R) as a left vector space over $\hat{A}$ (resp. A') and are subject to the following relations :

$$Z_{\rho(\sigma)}.a = \omega_\sigma(a).Z_{\rho(\sigma)} \qquad \text{for } a \in \hat{A} \text{ and } \sigma \in G$$

$$Z_{\rho(\sigma)} . Z_{\rho(\tau)} = f(\sigma,\tau)c(\sigma,\tau)Z_{\rho(\sigma\tau)} \quad \text{for} \quad \sigma,\tau \in G,$$

where
$$c(\sigma,\tau) = Z_{\rho(\sigma)+\rho(\tau)-\rho(\sigma\tau)} \in F'.$$

Therefore,

$$S = (\hat{A},G,(\omega,fc)) \qquad \text{and} \qquad R = (A',G,(\omega,fc)).$$

This proves that S and R are central simple algebras of the same degree as A. Moreover, since

$$(\hat{A},G,(\omega,f)) \simeq A \otimes_F \hat{F} \quad \text{and} \quad (A',G,(\omega,f)) \simeq A \otimes_F F',$$

the isomorphisms above also yield, by theorem 1.6 :

$$S \sim A \otimes_F (\hat{K},G,c) \quad \text{and} \quad R \sim A \otimes_F (K',G,c). \qquad (2)$$

In order to determine the index and the exponent of S, observe that $\hat{F}$ is Henselian with respect to the valuation

$$v : \hat{F} \to \Gamma$$

defined by
$$v(s) = \min\ (\text{supp}(s)).$$

(See [9, Corollary, p.51] or [6, **Corollaire**, p. 103]).

2.4. **LEMMA** : *The algebra* $N = (\hat{K},G,c)$ *is a nicely semiramified division algebra ; the quotient of value groups* Γ_N/Γ *is naturally isomorphic to* G.

<u>Proof</u> : Let $N = \oplus_{\sigma \in G} \hat{K}\, x_\sigma$ where the indeterminates x_σ satisfy the relations

$$x_\sigma\, k = \sigma(k)\, x_\sigma \quad \text{for} \quad \sigma \in G \quad \text{and} \quad k \in \hat{K}$$

$$x_\sigma\, x_\tau = c(\sigma,\tau)\, x_{\sigma\tau} \quad \text{for} \quad \sigma,\tau \in G.$$

For $\alpha \in \mathbb{Z}^n$, define

$$X_\alpha = Z_{\alpha-\rho\varepsilon(\alpha)}\ x_{\varepsilon(\alpha)} \in N,$$

so that, in particular, $X_\gamma = Z_\gamma$ for $\gamma \in \Gamma$.

Then
$$X_\alpha\, X_\beta = X_{\alpha+\beta} \quad \text{for} \quad \alpha,\beta \in \mathbb{Z}^n$$

and
$$X_\alpha . k = \varepsilon(\alpha)(k).X_\alpha \ \text{for}\ \alpha \in \mathbb{Z}^n \text{ and } k \in K.$$

Every element in N can be regarded as a series with well-ordered support in the X_α's, hence N can be identified to the ring $S_\varepsilon(K,G,1)$, where $1 \in Z^2(G,K^\times)$ is the trivial cocycle. It then follows from a general result of B.H. Neumann [5,Theorem 5.7] that N is a division algebra. Moreover, N contains as maximal subfields the field $\hat{K}$, which is inertial over $\hat{F}$, and the field $\{\Sigma_{\alpha\in\mathbb{Z}^n} f_\alpha X_\alpha \in N \mid f_\alpha \in F\}$, which is totally ramified of radical type over $\hat{F}$; this shows that N is nicely semiramified. Finally, $\Gamma_N = \mathbb{Z}^n$ and ε induces an isomorphism $\Gamma_N/\Gamma \simeq G$.

2.5. End of the proof of theorem 2.3 : The division algebra similar to $A \otimes_F \hat{F}$ is inertial over $\hat{F}$, hence it follows from (2), by theorem 5.13 of [3], that ind $S = [K : F]$.ind A and exp $S = \ell cm$ (exp G, exp A). In order to determine the index and the exponent of R, first note that from

$$S = R \otimes_{F'} \hat{F},$$

it readily follows that

ind S divides ind R and exp S divides exp R. (3)

If M is a maximal subfield of the division algebra similar to A, then $M' = M \otimes_F F'$ contains K' and splits A, hence also R, in view of (2). Therefore,

ind R divides $[M' : F'] = [K : F]$.ind A. (4)

On the other hand, if $e = \exp G$, then $e\rho(\sigma) \in \Gamma$ for all $\sigma \in G$, hence $Z_{e\rho(\sigma)} \in F'$ and

$$c(\sigma,\tau)^e = Z_{e\rho(\sigma)} \cdot Z_{e\rho(\tau)} \cdot Z^{-1}_{e\rho(\sigma\tau)}$$

is a coboundary in $H^2(G,K'^\times)$. Therefore,

exp(K',G,c) divides exp G

and from (2) it follows that

exp R divides ℓcm (exp G, exp A). (5)

Since ind $S = [K : F]$ ind A and exp $S = \ell cm$ (exp G, exp A), relations (3), (4) and (5) show that ind R = ind S and exp R = exp S, and complete the proof.

REFERENCES

++++++++++

[1.] AMITSUR S. and D. SALTMAN : Generic abelian crossed products and p-algebras, *J. Algebra* *51*(1978)76-87.

[2.] BLANCHARD, A. : *Les corps non commutatifs*, Presses Univ. France, Coll. Sup., Paris, 1972.

[3.] JACOB, B. and WADSWORTH A. : Division algebras over Henselian fields, to appear.

[4.] JACOBSON, N. : *Basic Algebra II*, Freeman, San Franscisco, 1980.

[5.] NEUMANN, B.H. : On ordered division rings, ***Trans. Amer. Math. Soc.*** *66*(1949)202-252.

[6.] RIBENBOIM, P. : *Théorie des valuations*, Presses Univ. Montréal, Montréal, 1968.

[7.] ROWEN, L. : Central simple algebras with involution viewed through centralizers, *J. Algebra* *63*(1980)41-55.

[8.] ROWEN, L. : Cyclic division algebras, *Israel J. Math.* *41*(1982)213-234 ; corr. : *ibid.* *43*(1982)277-280.

[9.] SCHILLING, O.F.G. : *The theory of valuations*, Math. Surveys 4, Amer. Math. Soc., Providence, R.I., 1950.

[10.] SCHREIER, O. : Ueber die Erweiterung von Gruppen I, *Monats. für Math. u. Phys.* *34*(1926)165-180.

[11.] TEICHMÜLLER, O. : Ueber die sogenannte nichtkommutative Galoissche Theorie und die Relation $\xi_{\lambda,\mu,\nu}\,\xi_{\lambda,\mu\nu,\pi}\,\xi^{\lambda}_{\mu,\nu,\pi} = \xi_{\lambda,\mu,\nu\pi}\,\xi_{\lambda\mu,\nu,\pi}$, *Deutsche Math.* *5* (1940) 138-149. (= *Gesammelte Abhandlungen - Collected Papers*. (Ahlfors, L. und F. Gehring, Herausg.), Springer, Berlin, 1982, pp. 543-553).

[12.] TIGNOL, J.P. : Cyclic and elementary abelian subfields of Malcev-Neumann division algebras, *J. pure appl. Algebra* *42*(1986)199-220.

[13.] TIGNOL, J.P. : Algèbres indécomposables d'exposant premier, to appear in : *Advances in Math.*