# On the Role of Scheduling in Simulation-Based Security*

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**Abstract.** In a series of papers, Küsters et al. investigated the relationships between various notions of simulation-based security. Two main factors, the placement of a "master process" and the existence of "forwarder processes", were found to affect the relationship between different definitions. In this extended abstract, we add a new dimension to the analysis of simulation-based security, namely, the scheduling of concurrent processes. We show that, when we move from sequential scheduling (as used in previous studies) to task-based nondeterministic scheduling, the same logical statement gives rise to incomparable notions of security. Under task-based scheduling, the hierarchy based on placement of "master process" becomes obsolete, because no such designation is necessary to obtain meaningful runs of a system. On the other hand, the existence of "forwarder processes" remains an important factor.

## 1 Introduction

In simulation-based security, the behavior of a multi-party protocol $\rho$ executing in the "real world", often in the presence of an adversary $Adv$, is compared against the behavior of a simulator $Sim$ interacting with an ideal process $\phi$ (also called a functionality). Intuitively, if the behaviors of these two systems are indistinguishable, then $\rho$ is at least as secure as $\phi$.

This notion of simulation traces back to the early works of Micali et al. on *zero-knowledge proof systems* [GMR85] and *secure function evaluation* [GMW87]. Much progress was made during the 1990's [GL90,Bea91,MR91,PW94,Can95], leading to the general definitions of *reactive simulatability (RSIM)* [PW01] and *universally composable (UC) security* [Can01]. Many related definitions soon followed, including *strong simulatability (SS)* [DKM$^+$04].

Using informal notations of parallel composition $||$ and indistinguishability $\approx$, some major variants of simulation-based security can be formulated as follows:

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– *Reactive Simulatability*: $\rho \leq_{RS} \phi$ iff $\forall Adv\ \forall Env\ \exists Sim : \rho \| Adv \| Env \approx \phi \| Sim \| Env$.
– *UC Security*: $\rho \leq_{UC} \phi$ iff $\forall Adv\ \exists Sim\ \forall Env : \rho \| Adv \| Env \approx \phi \| Sim \| Env$.
– *Black-Box Simulatability*: $\rho \leq_{BB} \phi$ iff $\exists Sim\ \forall Adv\ \forall Env : \rho \| Adv \| Env \approx \phi \| Adv \| Sim \| Env$.
– *Strong Simulatability*: $\rho \leq_{SS} \phi$ iff $\exists Sim\ \forall Env : \rho \| Env \approx \phi \| Sim \| Env$.

Note that the indistinguishability condition is stated relative to an environment process *Env*, which takes on the role of a "distinguisher". Moreover, in the last three definitions, the simulator must be specified before the environment. This essentially guarantees composability of the security definition, because the simulation must be successful regardless of the environment in which the protocol $\rho$ is executed.

The relationships between some of these variants can be deduced simply by examining their logical structures. For example, strong simulatability implies black-box simulatability, which in turn implies both reactive simulatability and UC security. The other implications are less obvious and are investigated by Datta et al. in [DKM$^+$04,DKMR05], which present a hierarchy of simulation-based security definitions using two criteria: (i) the identity of the master process (which may be the environment, adversary or simulator) and (ii) the definability of a *forwarder* process that is able to forward an unbounded number of messages. In particular, it is shown that strong simulatability is equivalent to UC security if and only if forwarder processes are definable.

The notion of a master process used in the first criterion is common in modeling frameworks with *sequential activation*: given a system of machines/processes executing in parallel, at most one machine is active at any given point in time, and, when the active machine produces a message, the intended recipient is the next active machine. A designated *master process* is triggered if for whatever reason the chain of activation is broken.

Sequential activation is implemented in many frameworks, including the Interactive Turing Machine (ITM) model in [Can01,Küs06], the Reactive System (RS) model[1] of [PW01,BPW04] and the Sequential Probabilistic Process Calculus (SPPC) of [DKM$^+$04,DKMR05]. Since machines are activated via message delivery, one need not specify a separate scheduler to resolve nondeterminism (as is typical in traditional models of concurrency).

Although less common, non-sequential activation is also found in crypto-oriented frameworks, including the Probabilistic Polynomial-time Process Calculus (PPC) of [LMMS98,MMS03,DKM$^+$04,MRST06] and the Task-PIOA model of [CCK$^+$06a,CCK$^+$06b]. Here, nondeterminism is resolved using schedulers, which are state-dependent functions (or Markov chains) in PPC and oblivious task sequences in Task-PIOA.

[1] In general, the high-level scheduling in RS need not be sequential: messages are not delivered immediately; instead, they are stored in buffers that may be triggered by a component other than the sender. However, sequential scheduling is typically implemented in actual cryptographic protocol analysis [BPW03].

Since scheduling is an integral part of the semantics of concurrent processes, it is natural to ask whether the same definition of security would have different meanings when we move between sequential and non-sequential activation. Many in the community have argued that the two styles of scheduling are semantically equivalent, because sequential scheduling can be emulated in a non-sequential framework, and vice versa. In this paper, we show that such claims are misleading, because there exist protocols that are secure under one type of scheduling but insecure under the other. This shows scheduling is in fact an important aspect in simulation-based security.

In our first example (Section 3.1), we show that sequential scheduling can "create" correlations that are not present in machine specifications. As a result, two protocols are UC-indistinguishable even though one of them implements an input/ouput correlation explicitly while the other one does not. In contrast, oblivious task-based scheduling does not allow the possibility to forge correlations between dynamically chosen values, because a scheduler is a sequence of tasks that are chosen nondeterministically in advance. The same two protocols are therefore UC-distinguishable under oblivious scheduling.

In our second example (Section 3.2), we show that sequential scheduling can give the distinguisher environment additional power, because it can control the ordering of events elsewhere in the system by timing its own messages. (This holds even if the environment is *not* the master scheduler, because the master scheduler kicks in only if the activation chain is broken.) This allows the environment to distinguish two protocols that are indistinguishable under oblivious scheduling.

Finally, we observe that the "forwarder" criterion of [DKM$^+$04,DKMR05] remains meaningful and important when we move from sequential to non-sequential activation. (The "master process" criterion is no longer meaningful, because there is no designation of master processes in a non-sequential framework.) We prove that strong simulatability is equivalent to UC security in the Task-PIOA framework. The proof rests upon the fact that forwarder processes are definable as task-PIOAs.

*Roadmap* In Section 2, we briefly review the task-PIOA framework and our general modeling paradigm for cryptographic protocol analysis. Then, in Section 3, we use two examples to show that sequential and non-sequential activation give rise to incomparable notions of security. In Section 4, we prove that forwarders are definable in Task-PIOA, and hence strong simulatability is equivalent to UC security.

## 2 Security Modeling with Task-PIOAs

Our basic framework is that of task-PIOAs [CCK$^+$06a], which provides a partial-information scheduling mechanism suitable for cryptographic protocol analysis.

*PIOAs* A *probabilistic I/O automaton (PIOA)* $\mathcal{A}$ is a tuple $\langle Q, \bar{q}, I, O, H, \Delta \rangle$, where: (i) $Q$ is a countable set of *states*, with *start state* $\bar{q} \in Q$; (ii) $I$, $O$ and $H$

are countable and pairwise disjoint sets of actions, referred to as *input, output and internal actions*, respectively; (iii) $\Delta \subseteq Q \times (I \cup O \cup H) \times \mathsf{Disc}(Q)$ is a *transition relation*, where $\mathsf{Disc}(Q)$ is the set of discrete probability measures on $Q$. An action $a$ is *enabled* in a state $q$ if $\langle q, a, \mu \rangle \in \Delta$ for some $\mu$. The set $Act := I \cup O \cup H$ is called the *action alphabet* of $\mathcal{A}$. If $I = \emptyset$, then $\mathcal{A}$ is said to be *closed*. The set of *external* actions of $\mathcal{A}$ is $I \cup O$ and the set of *locally controlled* actions is $O \cup H$. We assume that $\mathcal{A}$ satisfies the following conditions.

– **Input Enabling:** For every $q \in Q$ and $a \in I$, $a$ is enabled in $q$.
– **Transition Determinism:** For every $q \in Q$ and $a \in A$, there is at most one $\mu \in \mathsf{Disc}(Q)$ such that $\langle q, a, \mu \rangle \in \Delta$.

Parallel composition for PIOAs is based on synchronization of shared actions. Two PIOAs $\mathcal{A}_i$, $i \in \{1, 2\}$, are said to be *compatible* if $Act_i \cap H_j = O_i \cap O_j = \emptyset$ whenever $i \neq j$. In that case, we define their *composition* $\mathcal{A}_1 \| \mathcal{A}_2$ to be $\langle Q_1 \times Q_2, \langle \bar{q}_1, \bar{q}_2 \rangle, (I_1 \cup I_2) \setminus (O_1 \cup O_2), O_1 \cup O_2, H_1 \cup H_2, \Delta \rangle$, where $\Delta$ is the set of triples $\langle \langle q_1, q_2 \rangle, a, \mu_1 \times \mu_2 \rangle$ such that (i) $a$ is enabled in some $q_i$ and (ii) for every $i$, if $a \in A_i$ then $\langle q_i, a, \mu_i \rangle \in \Delta_i$, otherwise $\mu_i$ assigns probability 1 to $q_i$. A *hiding* operator is also available: given $\mathcal{A} = \langle Q, \bar{q}, I, O, H, \Delta \rangle$ and $S \subseteq O$, $\mathsf{hide}(\mathcal{A}, S)$ is the tuple $\langle Q, \bar{q}, I, O', H', \Delta \rangle$, where $O' = O \setminus S$ and $H' = H \cup S$. This prevents synchronizations of actions in $S$ with any other PIOA.

*Task-PIOAs* To resolve nondeterminism, we make use of the notion of tasks introduced in [CCK+06a]. Formally, a *task-PIOA* is a pair $(\mathcal{A}, \mathcal{R})$ such that (i) $\mathcal{A}$ is a PIOA and (ii) $\mathcal{R}$ is a partition of the locally-controlled actions. With slight abuse of notation, we use $\mathcal{A}$ to refer to both the task-PIOA and the underling PIOA. The equivalence classes in $\mathcal{R}$ are referred to as *tasks*. Unless otherwise stated, we will use terminologies inherited from the PIOA setting.

The following axiom is imposed on task-PIOAs.

– **Action Determinism:** For every state $q \in Q$ and every task $T \in \mathcal{R}$, there is at most one action $a \in T$ that is enabled in $q$.

In case some $a \in T$ is enabled in $q$, we say that $T$ is *enabled* in $q$.

Given compatible task-PIOAs $\mathcal{A}_1$ and $\mathcal{A}_2$, we define their *composition* to be $\langle \mathcal{A}_1 \| \mathcal{A}_2, \mathcal{R}_1 \cup \mathcal{R}_2 \rangle$. The hiding operator for PIOAs also extends in the obvious way: given a set $S$ of output actions, $hide(\langle \mathcal{A}, \mathcal{R} \rangle, S)$ is simply $\langle hide(\mathcal{A}, S), \mathcal{R} \rangle$.

Finally, a *task schedule* for a closed task-PIOA $\langle \mathcal{A}, \mathcal{R} \rangle$ is a finite or infinite sequence $\rho = T_1.T_2.T_3 \ldots$ of tasks in $\mathcal{R}$. This induces a well-defined (probabilistic) execution of $\mathcal{A}$ as follows:

(i) from the start state $\bar{q}$, we apply the first task $T_1$;
(ii) due to action- and transition-determinism, $T_1$ specifies at most one transition from $\bar{q}$;
(iii) if such transition exists, it is taken, otherwise nothing happens;
(iv) repeat with remaining $T_i$'s.

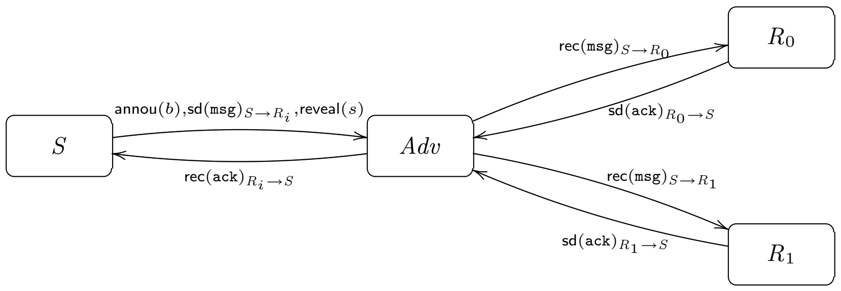
*Example: Adaptive Adversary* For cryptographic applications, we adopt the following modeling paradigm.

1. An adaptive adversary is modeled as a system component, for example, a message delivery service that can eavesdrop on network communications and

control the order of message delivery. Thus, the adversary resolves the so-called *high-level* nondeterminism.

2. *Low-level* nondeterminism is resolved by a task schedule chosen nondeterministically in advance. For example, in a typical protocol, many different parties make independent random choices, and it is inconsequential which of them does so first. A task schedule fixes a particular order in which the different coin tosses occur.

We illustrate this paradigm via an example. Consider a toy protocol in which a sender, $S$, and two receivers, $R_0$ and $R_1$, exchange messages via an adversary $Adv$ (Figure 1). The sender $S$ also chooses two random bits $b$ and $s$ independently. The first bit $b$ is announced to the adversary $Adv$ and the second bit $s$ is kept secret until $S$ receives an acknowledgment from either $R_0$ or $R_1$. If the acknowledgment from $R_b$ arrives before the acknowledgment from $R_{1-b}$, $S$ reveals $s$ to $Adv$, otherwise $s$ remains secret. A detailed description of $S$ is given in Appendix C, Figure 5.



$S$
$Adv$
$R_0$
$R_1$
$\mathsf{annou}(b), \mathsf{sd}(\mathtt{msg})_{S\to R_i}, \mathsf{reveal}(s)$
$\mathsf{rec}(\mathsf{ack})_{R_i\to S}$
$\mathsf{rec}(\mathtt{msg})_{S\to R_0}$
$\mathsf{sd}(\mathsf{ack})_{R_0\to S}$
$\mathsf{rec}(\mathtt{msg})_{S\to R_1}$
$\mathsf{sd}(\mathsf{ack})_{R_1\to S}$

**Fig. 1.** A Toy Protocol

The adversary $Adv$ delivers the messages from $S$ to $R_i$ whenever they are available, while the acknowledgments from $R_i$ to $S$ are buffered until $S$ announces $b$. Then $Adv$ delivers $\mathtt{ack}_b$ before $\mathtt{ack}_{1-b}$. A detailed description of $Adv$ is given in Appendix C, Figure 6. Finally, a receiver $R_i$ simply accepts the message from $S$ and responds with an acknowledgment. This is described in Appendix C, Figure 7.

In this protocol, the ordering between $\mathsf{rec}(\mathsf{ack})_{R_0\to S}$ and $\mathsf{rec}(\mathsf{ack})_{R_1\to S}$ is a good example of high-level nondeterminism. Once these acknowledgments are placed onto the network (via actions $\mathsf{sd}(\mathsf{ack})_{R_i\to S}$), the adversary controls their transit delays. In particular, the adversary described above waits until he learns the value of $b$, and then he delivers the acknowledgment from $R_b$. This ensures that $S$ will reveal $s$ if the task $\mathsf{reveal}(*)$ is scheduled subsequently. In fact, it is easy to check that the following task scheduler allows $Adv$ to learn $s$ with probability 1. This shows $Adv$ is *adaptive*, since $b$ is randomly generated during execution.

$\mathsf{choose}.\ \mathsf{annou}(*).\ \mathsf{sd}(\mathtt{msg})_{S\to R_0}.\ \mathsf{sd}(\mathtt{msg})_{S\to R_1}.\ \mathsf{rec}(\mathtt{msg})_{S\to R_0}.\ \mathsf{rec}(\mathtt{msg})_{S\to R_1}.$
$\mathsf{sd}(\mathtt{ack})_{R_0\to S}.\ \mathsf{sd}(\mathtt{ack})_{R_1\to S}.\ \mathsf{rec}(\mathtt{ack})_{R_0\to S}.\ \mathsf{rec}(\mathtt{ack})_{R_1\to S}.\ \mathsf{reveal}(*)$

We now turn to low-level nondeterminism. For instance, the ordering between $\mathsf{sd}(\mathtt{msg})_{S\to R_0}$ and $\mathsf{sd}(\mathtt{msg})_{S\to R_1}$ is inessential in the security analysis, provided they are both performed by $S$. Similarly, the ordering between $\mathsf{annou}(*)$ and $\mathsf{sd}(\mathtt{msg})_{S\to R_i}$ is also inessential. All of these are examples of low-level nondeterministic choices and represent implementation freedom in $S$. That is, an actual implementation of $S$ may perform $\mathsf{annou}(*)$, $\mathsf{sd}(\mathtt{msg})_{S\to R_0}$ and $\mathsf{sd}(\mathtt{msg})_{S\to R_1}$ in any order. We capture all these possibilities in our formal semantics by quantifying over all possible task schedules.

*Implementation* The formal semantics of a closed task-PIOA is given in terms of the *trace distributions* induced by task schedules. As we described earlier, each task schedule induces a probabilistic run, from which a trace distribution is obtained by abstracting away state information. That is, a trace distribution contains only information about actions taken during the run.

For a possibly open task-PIOA $\mathcal{A}$, the semantics is given relative to closing environments: a task-PIOA $\mathit{Env}$ is an *environment* for $\mathcal{A}$ if it is compatible with $\mathcal{A}$ and $\mathcal{A}\|\mathit{Env}$ is closed. The *external behavior* of $\mathcal{A}$ is then the mapping that takes each environment $\mathit{Env}$ to the set of trace distributions of $\mathcal{A}\|\mathit{Env}$ (denoted $\mathsf{TrDists}(\mathcal{A}\|\mathit{Env})$).

We also define an implementation relation between task-PIOAs with the same I/O interface, expressing the idea that every possible behavior of one automaton in a particular environment is also a possible behavior of another automaton in the same environment. Formally, $\mathcal{A}_1$ and $\mathcal{A}_2$ are said to be *comparable* if $I_1 = I_2$ and $O_1 = O_2$. In that case, $\mathcal{A}_1$ is said to *implement* $\mathcal{A}_2$, denoted $\mathcal{A}_1 \leq_0 \mathcal{A}_2$, if $\mathsf{TrDists}(\mathcal{A}_1\|\mathit{Env}) \subseteq \mathsf{TrDists}(\mathcal{A}_2\|\mathit{Env})$ for all environments $\mathit{Env}$ for both $\mathcal{A}_1$ and $\mathcal{A}_2$.

The subscript 0 in $\leq_0$ refers to the requirement that every trace distribution in $\mathsf{TrDists}(\mathcal{A}_1||\mathit{Env})$ must have an identical match in $\mathsf{TrDists}(\mathcal{A}_2||\mathit{Env})$. For cryptographic protocol analysis, we define a variation based on the probability that $\mathit{Env}$ produces a special "accept" output. An *approximate* implementation, denoted $\leq_{\mathsf{neg,pt}}$, is then defined for task-PIOA families, which allows "negligible" discrepancies between acceptance probabilities. Details concerning $\leq_{\mathsf{neg,pt}}$ can be found in Appendix A.

## 3 Separation between Sequential Scheduling and Oblivious scheduling

We use two simple examples to illustrate the effect of different scheduling schemes on the semantics of security definitions. In Section 3.1, we exhibit two systems that are indistinguishable under sequential scheduling, but they do not implement each other under oblivious scheduling. In Section 3.2, we present two systems in the converse situation.

## 3.1 Sequential indistinguishability

Consider two variants of the system depicted in Figure 2. In the first variant, the environment interacts with two task-PIOAs $A_0$ and $A_1$, which simply answer requests. That is, task-PIOA $A_i$ answers each $Hello_i$ input from the environment with the output action $i$. In the second variant, the environment interacts with two task-PIOAs $B_0$ and $B_1$, which behave like beacons. That is, each $B_i$ spontaneously and persistently produces the output action $i$. These beacons also accept $Hello_i$ inputs, but they have no effect.

The codes for $A_i$ and $B_i$ are given in Appendix C, Figure 8.

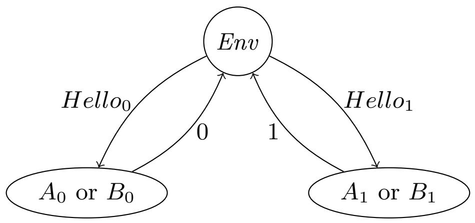


**Fig. 2.** Diagram for the *Answer* and *Beacon* automata

We claim that, under sequential activation, no environment can distinguish the system $Answer := A_0 \| A_1$ from the system $Beacon := B_0 \| B_1$. Indeed, the only way to activate $A_i$ (or $B_i$) is the $Hello_i$ action, after which both $A_i$ and $B_i$ will respond with $i$.

On the other hand, we observe that $Answer \not\leq_0 Beacon$, and $Beacon \not\leq_0 Answer$. Consider an environment $\mathcal{E}$ that has one single output task $Hello = \{Hello_0, Hello_1\}$, where the activation of $Hello_0$ or $Hello_1$ is decided through some internal coin flipping, performed as the unique action in the task $Flip = \{flip\}$ of $\mathcal{E}$. Consider now the task schedule $\rho := Flip.Hello.Out_0$. In system $Answer \| \mathcal{E}$, the resulting trace distribution will be $Hello_1$ with probability $\frac{1}{2}$ and $Hello_0$ with probability $\frac{1}{2}$. This trace distribution cannot be matched by any task schedule for $Beacon \| \mathcal{E}$, because task schedules are chosen nondeterministically in advance. More precisely, a task schedule for $Beacon \| \mathcal{E}$ either schedules $Out_0$ after $Flip.Hello$ or it does not. In the first case, 0 occurs with probability 1 and in the second with probability 0.

Using exactly the same argument, we see that the trace distribution resulting from applying $\rho$ to $Beacon \| \mathcal{E}$ cannot be matched by any task schedule for $Answer \| \mathcal{E}$, because no task schedule can ensure that 0 occurs with probability 1 in $Answer \| \mathcal{E}$.

This example can be easily extended to the security setting, where the input and output actions of $A_i$'s and $B_i$'s are treated as protocol inputs and outputs. In this case, we observe that $Answer \leq_{UC} Beacon$ when we have sequential scheduling, while this relation does not hold with task-based scheduling. Indeed, since *Answer* and *Beacon* do not send any message on the network, the adversary cannot observe anything, and the best we can do is to choose the simulator as a copy of the adversary. Eventually, the UC security property comes down to the capacity of the environment to distinguish *Answer* from *Beacon*.

As a result, we see that using a sequential scheduling, like the one considered in [Can01,BPW03,DKMR05,Küs06], might introduce some constraints in the ordering of events that do not necessarily reflect the actual network behavior, and might hide characteristics that could be exploited by an attacker in the practice.

## 3.2 Task-based indistinguishability

In our second example, we consider two variants of the system described in Figure 3. In the first variant, an automaton *Box* selects a random $k$-bit string $x$, and transmit it to the environment. Then, it can receive bits 0 and 1 and, every time *Box* receives such a bit, it stores it into a $k$-bits buffer. Eventually, if the content of the buffer is equal to the secret $x$, *Box* can perform an *ok* output action.

The second variant has the same task-PIOAs except that the *Box* automaton is replaced by the $Box_s$ automaton, which is the same as *Box*, except that the *ok* action is now always disabled, that is, $Box_s$ does not care about the sequence of bits it receives, and remains silent once it sent the $x$ bit string.

The *Box* and $Box_s$, are actually defined as task-PIOA families $\overline{Box} = \{Box_k\}_{k\in\mathbb{N}}$ and $\overline{Box_s} = \{(Box_s)_k\}_{k\in\mathbb{N}}$, where the code for the $Box_k$ and $(Box_s)_k$ automaton is given in Appendix C, Figure 9. The index $k$ simply refers to the length of the $x$ bit string that these automata transmit.

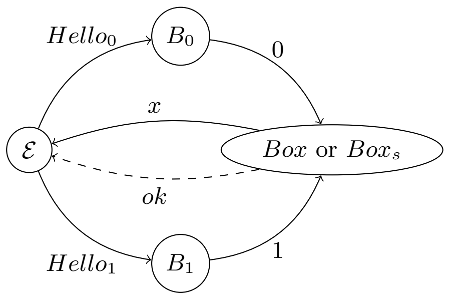


**Fig. 3.** Diagram for the *Secret* and $Secret_s$ automata

We first observe that the $\overline{Box}$ and $\overline{Box_s}$ families can be efficiently distinguished by an environment when a sequential scheduling is adopted. Also, $\overline{Box} \not\leq_{\mathsf{neg,pt}} \overline{Box_s}$ with oblivious scheduling.

Suppose now that we define the $\overline{Secret} = \{Secret_k\}_{k\in\mathbb{N}}$ and $\overline{Secret_s} = \{(Secret_s)_k\}_{k\in\mathbb{N}}$ task-PIOA families as follows: $Secret_k = hide(B_0\|B_1\|Box_k, \{0,1\})$, and $(Secret_s)_k = hide(B_0 \| B_1 \| (Box_s)_k, \{0,1\})$. This means that, now, the environment is separated from $Box$ and $Box_s$ by the automata $B_0$ and $B_1$ that produce bits 0 and 1 independently of the behavior of the environment, as represented in Figure 3.

We claim that an environment can still efficiently distinguish the $\overline{Secret}$ family from the $\overline{Secret_s}$ family when a sequential scheduling is used. Indeed, if the environment activates the $Box_k$ automaton in order to receive the $x$ bit string, then performs the $Hello_0$ and $Hello_1$ actions in the order corresponding to the bits of $x$, the $Box_k$ automaton will eventually perform the *ok* action with high probability. However, this last action will never occur in the $\overline{Secret_s}$ system since it has the *ok* action always disabled.

However, we claim that the following relation holds: $\overline{Secret} \leq_{\mathsf{neg,pt}} \overline{Secret_s}$. Indeed, the only way an environment can distinguish these two systems is by enabling the *ok* action. However, this will only happen if the $Out_0$ and $Out_1$ tasks are performed according to the order defined by the $x$ bit-string, which is randomly selected at execution time. So, for a given parameter $k$, the probability that the task-scheduler schedules the $Out_0$ and $Out_1$ in the desired order is bounded by $2^{-k}$. This probability is not null however and, therefore, the relation $\overline{Secret} \leq_0 \overline{Secret_s}$ does not hold.

Again, we can easily transpose this example to security definitions, by considering the $Hello_0$, $Hello_1$, *ok* and $x$ actions as protocol inputs and outputs. In that case, and as opposed to our previous example, we obtain that $Secret \leq_{UC} Secret_s$ when we have oblivious scheduling, while this relation does not hold with sequential scheduling.

The central point of this example is that, when oblivious scheduling is adopted, no system component (environment, adversary, protocol party, ...) knows the order in which activation will take place, which is not the case when sequential scheduling is considered.

While our previous example showed that adopting oblivious scheduling can give more distinguishing power to the environment, this example illustrates that oblivious scheduling can also weaken the environment. We believe this weakening is realistic as it reflects the fact that, in a distributed system, the ordering of the actions performed by honest (non-corrupted) components cannot be fixed by the adversary.

## 4 Equivalence between UC and SS

In this section, we prove that UC security and strong simulatability are equivalent in our framework. Essentially, we show that time-bounded task-PIOAs satisfy the forwarder axiom of [DKMR05].

*Structures* First we define the notion of structures in the spirit of [PW01]: a *structure* $\Pi$ is a pair $\langle \mathcal{A}, EAct \rangle$, where $\mathcal{A}$ is a task-PIOA and $EAct$ is a subset of the external actions of $\mathcal{A}$, called the *environment actions*. The set of *adversary actions* is defined to be $AAct := (I \cup O) \setminus EAct$. We also have: (i) $EI := EAct \cap I$

(*environment inputs*), (ii) $EO := EAct \cap O$ (*environment outputs*), (iii) $AI := AAct \cap I$ (*adversary inputs*) and (iv) $AO = AAct \cap O$ (*adversary inputs*).

Two structures $\Pi_1$ and $\Pi_2$ are said to be *comparable* if $EI_1 = EI_2$ and $EO_1 = EO_2$. They are *compatible* if $\mathcal{A}_1$ and $\mathcal{A}_2$ are compatible task-PIOAs and $Ext_1 \cap Ext_2 = EAct_1 \cap EAct_2$. That is, every shared action must be an environment action of both automata. Composition is straightforward: given compatible $\Pi_1$ and $\Pi_2$, their *composition* $\Pi_1 \| \Pi_2$ is the structure $\langle \mathcal{A}_1 \| \mathcal{A}_2, EAct_1 \cup EAct_2 \rangle$.

As the names suggest, an adversary interact with a protocol via adversary actions. Formally, a task-PIOA $Adv$ is an *adversary* for the structure $\Pi$ if: (i) $Adv$ is compatible with $\mathcal{A}_\Pi$, (ii) $Act_{Adv} \cap Act_\Pi \subseteq AAct_\Pi$, and (iii) $AI_\Pi \subseteq O_{Adv}$. The last condition requires that $Adv$ provides all adversary inputs of $\Pi$.

Finally, we consider hiding for structures: given a structure $\langle \mathcal{A}, EAct \rangle$ and a set $S$ of output actions of $\mathcal{A}$, we define $hide(\langle \mathcal{A}, EAct \rangle, S)$ to be the structure $\langle hide(\mathcal{A}, S), EAct \setminus S \rangle$. That is, the newly hidden actions can no longer be environment actions.

All of the definitions above can be formulated easily in the setting with time bounds, as well as for families of structures. Some details are provided in Appendix B.

*Secure Emulation* We have now enough machinery to formulate UC security and strong simulatability.

**Definition 1 (UC-Security).** *Suppose $\rho$ and $\phi$ are comparable structure families. We say that $\rho$* UC-emulates *$\phi$ (denoted $\rho \leq_{UC} \phi$) if, for every polynomial time-bounded adversary family $Adv$ for $\rho$, there is a polynomial time-bounded adversary family $Sim$ for $\phi$ such that:*

$$hide(\rho || Adv, AAct_\rho) \leq_{\mathsf{neg,pt}} hide(\phi || Sim, AAct_\phi).$$

**Definition 2 (Strong Simulatability).** *Suppose $\rho$ and $\phi$ are comparable structure families with $AAct_\rho \cap AAct_\phi = \emptyset$. We say that $\rho$* strongly simulates *$\phi$ (denoted $\rho \leq_{SS} \phi$) if there is a polynomial time-bounded adversary family $Sim$ for $\phi$ such that:*

$$\rho \leq_{\mathsf{neg,pt}} hide(\phi || Sim, AAct_\phi).$$

**Theorem 1.** *Definitions 1 and 2 are equivalent.*

*Proof.* Suppose $\rho$ and $\phi$ are defined as in the hypotheses. We first prove that Definition 2 implies Definition 1.

Suppose there exists a polynomial time-bounded adversary family $Sim$ for $\phi$ such that $\rho \leq_{\mathsf{neg,pt}} hide(\phi || Sim, AAct_\phi)$, and suppose $Adv$ is a polynomial time-bounded adversary family for $\rho$. Without loss of generality, we assume that all internal actions of $Adv$ have new names. Since $Adv$ is polynomial time-bounded and is compatible with $\rho$, we have that $\rho || Adv \leq_{\mathsf{neg,pt}} hide(\phi || Sim, AAct_\phi) || Adv$. Since $Adv$ is compatible with $hide(\phi || Sim, AAct_\phi)$, the relation $H_{hide(\phi || Sim, AAct_\phi)} \cap Act_{Adv} = \emptyset$ must hold. Therefore, $hide(\phi || Sim, AAct_\phi) || Adv = hide(\phi || Sim || Adv, AAct_\phi)$. Now, using the hiding property of the $\leq_{\mathsf{neg,pt}}$ relation, we have that

$hide(\rho||Adv, AAct_\rho) \leq_{\mathsf{neg,pt}} hide(\phi||Sim||Adv, AAct_\phi \cup AAct_\rho)$. Since $AAct_\rho \cap AAct_\phi = \emptyset$ and $AAct_\rho \cap EAct_\phi = AAct_\rho \cap EAct_\rho = \emptyset$, we also have that $AAct_\rho \cap Ext_\phi = \emptyset$.
Therefore, $hide(\phi||Sim||Adv, AAct_\phi \cup AAct_\rho) = hide(\phi||hide(Sim||Adv, AAct_\rho), AAct_\phi)$. Eventually, if we define $Sim' = hide(Sim||Adv, AAct_\rho)$, we proved that, for every polynomial time-bounded adversary family $Adv$ for $\rho$, there is a polynomial time-bounded adversary family $Sim'$ for $\phi$ such that $hide(\rho||Adv, AAct_\rho) \leq_{\mathsf{neg,pt}} hide(\phi||Sim', AAct_\phi)$, which is as needed.

Now, we prove that Definition 1 implies Definition 2. Suppose that, for every polynomial time-bounded adversary family $Adv$ for $\rho$, there is a polynomial time-bounded adversary family $Sim$ for $\phi$, such that $hide(\rho||Adv, AAct_\rho) \leq_{\mathsf{neg,pt}} hide(\phi||Sim, AAct_\phi)$. Consider the specific adversary $Adv$ defined in Figure 4, which just forwards the adversary external actions between $\rho$ and the environment, using a renaming function $f$ such that $f(AAct_\rho)$ only contains new actions names.

**Automaton:** $Adv(\rho, f)$:
**Signature:**
Input: $AO_\rho \cup f(AI_\rho)$
Output: $f(AO_\rho) \cup AI_\rho$
**State:**
$in \in AAct_\rho \cup f(AAct_\rho) \cup \bot$, initially $\bot$
**Transitions:**
$a \in I_{Adv(\rho)}$
  Effect: $in := a$
$a \in O_{Adv(\rho)}$
  Precondition: $in = a$
  Effect: $in := \bot$
**Task:**
$Forward = f(AO_\rho) \cup AI_\rho$

**Fig. 4.** $Adv$ automaton

We now define $\rho'$ as $\rho$ except that every output actions $a \in AAct_\rho$ is renamed to $f(a)$. We observe that $\rho' \leq_{\mathsf{neg,pt}} \rho||Adv$. Indeed, for every environment family $Env$ for $\rho'$ $\rho||Adv$, there is a 2-bounded simulation relation from each task-PIOA of $\rho'||Env$ to the corresponding task-PIOA of $\rho||Adv||Env$:[2] we can just map every task $T$ of $\rho'||Env$ on the same task, followed by the $Forward$ task of $Adv$. As a result, if the execution of $T$ involves executing an output action $a$ of $\rho'$ that is in $f(AAct_\rho)$, the action $f^{-1}(a)$ will be executed by $\rho$, followed by its forwarding by $Adv$. In a similar way, if the execution of $T$ involves executing an output action $a$ of $Env$ that is in $f(AAct_\rho)$, this action will be executed by $Env$, and immediately forwarded to $\rho$ by $Adv$.

To conclude, we observe that the relation $\rho' \leq_{\mathsf{neg,pt}} \rho||Adv$ implies that $hide(\rho', AAct_\rho) \leq_{\mathsf{neg,pt}} hide(\phi||Sim, AAct_\phi)$. Now, since $\rho'$ has all actions in

[2] See [CCK+06a,CCK+06b] for more details about simulation relations for task-PIOAs. The 2-bound on the simulation relation means that at most two tasks of $\rho||Adv||Env$ are needed to match the execution of one task of $\rho'||Env$

$AAct_\rho$ renamed to new names, we have that $hide(\rho', AAct_\rho) = \rho'$, which provides the relation $\rho' \leq_{\mathsf{neg,pt}} hide(\phi||Sim, AAct_\phi)$, as needed.

## 5 Conclusions

In this paper, we investigate whether the underlying treatment of concurrency affects the meaning of simulation-based security. We give an affirmative answer, based on two examples showing that UC security under sequential scheduling is incomparable with UC security under oblivious scheduling.

This extends the analysis of Datta et al. [DKM$^+$04,DKMR05] with a new dimension, namely, the scheduling of concurrent processes. In fact, our separation result is of a slightly different character: rather than proving one definition is stronger/weaker than another, we show that the *same* definition has different meanings. This separation applies not only to security definitions (involving adversary and simulator), but also to the underlying notion of indistinguishability.

Our results seemingly contradict the common understanding that sequential and non-sequential scheduling schemes can, to a large extent, emulate each other. This is not a real contradiction, because indistinguishability and security definitions are never given with a layer of emulation. For example, one can emulate oblivious scheduling in a sequential framework by adding a scheduler machine and specifying all other machines in such a way that activation only takes place via the scheduler machine. However, when security definitions are given in a sequential framework (e.g., [Can01,BPW04,Küs06]), this pattern is never enforced. Therefore the existence of a scheduling emulation says little about how the meaning of a security definition changes with the underlying model of concurrency.

Aside from the issue of scheduling, we also consider the "forwarder" property of [DKM$^+$04,DKMR05]. We observe that forwarders are definable in Task-PIOA and, as expected, strong simulatability is equivalent to UC security. This implies all three notions (i.e., strong simulatability, black-box simulatability and UC security) are equivalent in the Task-PIOA framework.

Unbounded forwarders are definable in our framework because we do not place any *a priori* length restrictions on task schedules. In our implementation relation definitions (i.e., $\leq_0$ and $\leq_{\mathsf{neg,pt}}$), the bound on the length of schedules for the ideal system can depend on the corresponding bound for the real system. Since the real system includes the environment, we may choose a large enough schedule length bound for the ideal system so that the simulator can forward sufficiently many messages from the environment.

This is not the case anymore if we impose *a priori* bounds on the number of occurrences of tasks in task schedulers. In that case, we claim that strong simulatability and UC security are no longer equivalent. Essentially, we construct a protocol for which an unbounded simulator must be used in order to satisfy strong simulatability. We leave the details as future work.

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## A Time-Bounded Task-PIOAs

In order to carry out computational analysis, we restrict our attention to those task-PIOAs whose operations can be represented by a collection of Turing machines with bounded runtime. This is the time-bounded task-PIOA introduced in [CCK$^+$06b,CCK$^+$05].

We assume a standard bit-string representation for various parts of a task-PIOA, including states, actions, transitions and tasks. Let $\mathbb{R}^{\geq 0}$ denote the set of nonnegative reals and let $b \in \mathbb{R}^{\geq 0}$ be given. A task-PIOA $\mathcal{A}$ is said to be *b-bounded* just in case: (i) the bit-string representation of every automaton part has length at most $b$; (ii) there is a Turing machine that decides whether a given representation of a candidate automaton part is indeed an automaton part, and this machine runs in time at most $b$; (iii) there is a Turing machine that, given a state and a task of $\mathcal{A}$, determines the next action in time at most $b$; (iv) there is a probabilistic Turing machine that, given a state and an action of $\mathcal{A}$, determines the next state of $\mathcal{A}$ in time at most $b$. Furthermore, all these Turing machines can be described using a bit string of length at most $b$, according to some standard encoding of Turing machines.

Composing two compatible time-bounded task-PIOAs yields a time-bounded task-PIOA with a bound linear in the sum of the original bounds. Similarly, the hiding operator changes the time bound by a linear factor. Proofs for these claims can be found in [CCK$^+$05].

Finally, we say that a task schedule $\rho$ is $b$-*-bounded* if $|\rho| \leq b$, that is, $\rho$ is finite and contains at most $b$ tasks.

*Task-PIOA Families* We define families of task-PIOAs indexed by a security parameter $k$: a *task-PIOA family* $\overline{\mathcal{A}}$ is an indexed set $\{\mathcal{A}_k\}_{k\in\mathbb{N}}$ of task-PIOAs. The notion of time bound is also expressed in terms of the security parameter; namely, given $b : \mathbb{N} \to \mathbb{R}^{\geq 0}$, we say that $\mathcal{A}$ is *$b$-bounded* if every $\mathcal{A}_k$ is $b(k)$ -bounded.

The notions of compatibility and parallel composition are defined pointwise. Results for composition and hiding extends easily from those for time-bounded task-PIOAs. Again, detailed statements can be found in [CCK$^+$05].

*Approximate Implementation* Our approximate implementation relation compares acceptance probabilities of an environment (in the style of [Can01]), as opposed to trace distributions or to views (in the style of [BPW04]). Let $\mathcal{A}$ be a closed task-PIOA with a special output action $\mathsf{acc}$ and let $\rho$ be a task schedule for $\mathcal{A}$. The *acceptance probability* with respect to $\mathcal{A}$ and $\rho$ is defined to be:

$$\mathbf{P}_{\mathsf{acc}}(\mathcal{A},\rho) := \Pr[\beta \leftarrow_{\mathsf{R}} \mathsf{tdist}(\mathcal{A},\rho) : \beta \text{ contains } \mathsf{acc}],$$

where $\beta \leftarrow_{\mathsf{R}} \mathsf{tdist}(\mathcal{A},\rho)$ means $\beta$ is drawn randomly form the trace distribution induced by the task schedule $\rho$ on task-PIOA $\mathcal{A}$.

We assume that every environment has $\mathsf{acc}$ as an output. Now let $\mathcal{A}_1$ and $\mathcal{A}_2$ be comparable task-PIOAs and let $\epsilon, p \in \mathbb{R}^{\geq 0}$ and $q_1, q_2 \in \mathbb{N}$ be given. (As a convention, we use variable $p$ for automata time bounds and variable $q$ for schedule length bounds.) We define $\mathcal{A}_1 \leq_{p,q_1,q_2,\epsilon} \mathcal{A}_2$ as follows: given any $p$-bounded environment *Env* for both $\mathcal{A}_1$ and $\mathcal{A}_2$ and any $q_1$-bounded task schedule $\rho_1$ for $\mathcal{A}_1 \| Env$, there is a $q_2$-bounded task schedule $\rho_2$ for $\mathcal{A}_2 \| Env$ such that

$$|\,\mathbf{P}_{\mathsf{acc}}(\mathcal{A}_1 \| Env, \rho_1) - \mathbf{P}_{\mathsf{acc}}(\mathcal{A}_2 \| Env, \rho_2)| \leq \epsilon.$$

In other words, from the perspective of a $p$-bounded environment, $\mathcal{A}_1$ and $\mathcal{A}_2$ "look almost the same" provided $\mathcal{A}_2$ can use $q_2$ many steps to emulate $q_1$ many steps of $\mathcal{A}_1$. We claim that $\leq_{p,q_1,q_2,\epsilon}$ is transitive and preserved under composition, with certain adjustments to errors and time bounds. Proofs can be found in [CCK$^+$05].

The relation $\leq_{p,q_1,q_2,\epsilon}$ can be extended to task-PIOA families in the obvious way. Let $\overline{\mathcal{A}}_1 = \{(\mathcal{A}_1)_k\}_{k\in\mathbb{N}}$ and $\overline{\mathcal{A}}_2 = \{(\mathcal{A}_2)_k\}_{k\in\mathbb{N}}$ be *comparable* task-PIOA families, that is, they are pointwise comparable. Let $\epsilon, p : \mathbb{N} \to \mathbb{R}^{\geq 0}$ and $q_1, q_2 : \mathbb{N} \to \mathbb{N}$ be given. We say that $\overline{\mathcal{A}}_1 \leq_{p,q_1,q_2,\epsilon} \overline{\mathcal{A}}_2$ provided $(\mathcal{A}_1)_k \leq_{p(k),q_1(k),q_2(k),\epsilon(k)} (\mathcal{A}_2)_k$ for every $k$.

Restricting our attention to negligible error and polynomial time bounds, we obtain a generic version of approximate implementation, namely, $\leq_{\mathsf{neg,pt}}$. Formally, a function $\epsilon : \mathbb{N} \to \mathbb{R}^{\geq 0}$ is said to be *negligible* if, for every constant $c \in \mathbb{R}^{\geq 0}$, there exists $k_0 \in \mathbb{N}$ such that $\epsilon(k) < \frac{1}{k^c}$ for all $k \geq k_0$. (In other words, $\epsilon$ diminishes more quickly than the reciprocal of any polynomial.) We say that $\overline{\mathcal{A}}_1 \leq_{\mathsf{neg,pt}} \overline{\mathcal{A}}_2$ if:

$$\forall p\ \forall q_1\ \exists q_2\ \exists \epsilon\ \overline{\mathcal{A}}_1 \leq_{p,q_1,q_2,\epsilon} \overline{\mathcal{A}}_2,$$

where $p, q_1, q_2$ are polynomials and $\epsilon$ is a negligible function.

Again, [CCK$^+$05] contains proofs that $\leq_{\mathsf{neg,pt}}$ is transitive and preserved under composition and hiding.

## B Time-Bounded Structures

Time-bounded structures are defined in a similar fashion as time-bounded task-PIOAs. First we need the notion of $b$-time recognizable set: given a set $B$ of binary strings and $b \in \mathbb{R}^{\geq 0}$, we say that $B$ is *$b$-time recognizable* if there is a probabilistic Turing machine $M$ that

- decides, in time at most $b$, if a binary string $a$ is in the set $B$ and
- has a description with fewer than $b$ bits according to some standard encoding.

If $\bar{B} = \{B_k\}_{k \in \mathbb{N}}$ is a family of sets of binary stings, we say that $\bar{B}$ is *polynomial time-recognizable* if there is a polynomial $p$ such that every $B_k$ is $p(k)$-time recognizable.

Now a structure $\Pi = (\mathcal{A}, \mathit{EAct})$ is said to be *$b$-bounded* if $\mathcal{A}$ is b-bounded and the set $\langle \mathit{EAct} \rangle$ of the representations of actions in $\mathit{EAct}$ is $b$-time recognizable by some Turing machine $M_{\mathit{EAct}}$. For a family $\overline{\Pi}$ of structures and a function $b : \mathbb{N} \to \mathbb{R}^{\geq 0}$, we say that $\overline{\Pi}$ is *$b$-bounded* if $\Pi_k$ is $b(k)$-bounded for every $k$. If $\overline{\Pi}$ is $p$-bounded for some polynomial $p$, then we say that $\overline{\Pi}$ is *polynomial time-bounded*.

We claim that, given any polynomial time-bounded family $\overline{\Pi}$ and a polynomial-time recognizable family $\bar{S}$ of sets of actions, the family $\mathsf{hide}(\overline{\Pi}, \bar{S})$ is again polynomial time-bounded. Moreover, the $\leq_{\mathsf{neg,pt}}$ relation is preserved by hiding.

## C Task-PIOA Codes

Consider the following task schedules for the task-PIOA $S \| \mathit{Adv} R_0 \| R_1$ of Section 2.

(1) $\rho_1 = \mathsf{sd}(\mathtt{msg})_{S \to R_0}.\mathsf{choose}.\mathsf{rec}(\mathtt{msg})_{S \to R_0}.\mathsf{sd}(\mathtt{msg})_{S \to R_1}.\mathsf{rec}(\mathtt{msg})_{S \to R_1}.$
$\mathsf{sd}(\mathtt{ack})_{R_1 \to S}.\mathsf{annou}(*).\mathsf{sd}(\mathtt{ack})_{R_0 \to S}.\mathsf{rec}(\mathtt{ack})_{R_0 \to S}.\mathsf{rec}(\mathtt{ack})_{R_1 \to S}.\mathsf{reveal}(*)$

(2) $\rho_2 = \mathsf{sd}(\mathtt{msg})_{S \to R_0}.\mathsf{choose}.\mathsf{rec}(\mathtt{msg})_{S \to R_0}.\mathsf{sd}(\mathtt{msg})_{S \to R_1}.\mathsf{rec}(\mathtt{msg})_{S \to R_1}.$
$\mathsf{sd}(\mathtt{ack})_{R_1 \to S}.\mathsf{annou}(*).\mathsf{sd}(\mathtt{ack})_{R_0 \to S}.\mathsf{rec}(\mathtt{ack})_{R_0 \to S}.\mathsf{reveal}(*).\mathsf{rec}(\mathtt{ack})_{R_1 \to S}$

It is interesting to note that $\mathit{Adv}$ learns the secret $s$ with probability 1 under $\rho_1$, but with probability $\frac{1}{2}$ under $\rho_2$. This is because $\mathsf{rec}(\mathtt{ack})_{R_0 \to S}$ causes no changes in state if $b = 1$, hence $\mathsf{reveal}(*)$ is not yet enabled. Nonetheless, $\mathit{Adv}$ is considered to have high advantage in learning $s$, because our semantics quantifies over all possible schedules.

**Task-PIOA $S$**

**Signature**

```
Input:
    $\mathsf{rec}(\mathtt{ack})_{R_0 \to S}$, $\mathsf{rec}(\mathtt{ack})_{R_1 \to S}$
Output:
    $\mathsf{annou}(x)$, $x \in \{0,1\}$
    $\mathsf{sd}(\mathtt{msg})_{S \to R_0}$, $\mathsf{sd}(\mathtt{msg})_{S \to R_1}$
    $\mathsf{reveal}(x)$, $x \in \{0,1\}$
Internal:
    choose
```

**Tasks**

$\{\mathsf{annou}(x) | x \in \{0,1\}\}$
$\{\mathsf{sd}(\mathtt{msg})_{S \to R_0}\}$, $\{\mathsf{sd}(\mathtt{msg})_{S \to R_1}\}$
$\{\mathsf{reveal}(x) | x \in \{0,1\}\}$, $\{\mathsf{choose}\}$

**States**

$b, s \in \{0, 1, \perp\}$, initially $\perp$
$c \in \{0, 1, 2\}$, initially $0$

**Transitions:**

```
choose
Precondition:
    $b = \perp \wedge s = \perp$
Effect:
    $b := \mathsf{random}(\mathsf{unif}(\{0,1\}))$;
    $s := \mathsf{random}(\mathsf{unif}(\{0,1\}))$

annou($b$)
Precondition:
    $b \neq \perp$
Effect:
    None

sd(msg)$_{S \to R_0}$
Precondition:
    True
Effect:
    None

sd(msg)$_{S \to R_1}$
Precondition:
    True
Effect:
    None

rec(ack)$_{R_0 \to S}$
Effect:
    if $b = 0 \wedge c = 0$ then $c := 1$
    else if $b = 1 \wedge c = 0$ then $c := 2$

rec(ack)$_{R_1 \to S}$
Effect:
    if $b = 1 \wedge c = 0$ then $c := 1$
    else if $b = 0 \wedge c = 0$ then $c := 2$

reveal($s$)
Precondition:
    $s \neq \perp \wedge c = 1$
Effect:
    None
```

**Fig. 5.** Code for Sender $S$

**Task-PIOA** *Adv*

```
Signature

Input:
    annou(x), x ∈ {0,1}
    sd(msg)_{S→R_0}, sd(msg)_{S→R_1}
    sd(ack)_{R_0→S}, sd(ack)_{R_1→S}
    reveal(x), x ∈ {0,1}
Output:
    rec(msg)_{S→R_0}, rec(msg)_{S→R_1}
    rec(ack)_{R_0→S}, rec(ack)_{R_1→S}
Internal:
    None

Tasks
{rec(msg)_{S→R_0}}, {rec(msg)_{S→R_1}}
{rec(ack)_{R_0→S}}, {rec(ack)_{R_1→S}}

States
b, s ∈ {0,1,⊥}, initially ⊥
c_0, c_1 ∈ {0,1,2,3}, initially 0

Transitions:

annou(x)
Effect:
    $b := x$

sd(msg)_{S→R_0}
Effect:
    if $c_0 = 0$ then $c_0 := 1$

sd(msg)_{S→R_1}
Effect:
    if $c_1 = 0$ then $c_1 := 1$

rec(msg)_{S→R_0}
Precondition:
    $c_0 = 1$
Effect:
    None

rec(msg)_{S→R_1}
Precondition:
    $c_1 = 1$
Effect:
    None

sd(ack)_{R_0→S}
Effect:
    $c_0 := 2$

sd(ack)_{R_1→S}
Effect:
    $c_1 := 2$

rec(ack)_{R_0→S}
Precondition:
    $(b = 0 \wedge c_0 = 2) \vee (b = 1 \wedge c_1 = 3)$
Effect:
    $c_0 := 3$

rec(ack)_{R_1→S}
Precondition:
    $(b = 1 \wedge c_1 = 2) \vee (b = 0 \wedge c_0 = 3)$
Effect:
    $c_1 := 3$

reveal(x)
Effect:
    $s := x$
```

**Fig. 6.** Code for Adaptive Adversary *Adv*

**Task-PIOA** $R_i$

**Signature**

Input:
    $\mathsf{rec}(\mathtt{msg})_{S \to R_i}$
Output:
    $\mathsf{sd}(\mathtt{ack})_{R_i \to S}$
Internal:
    None

**Tasks**

$\{\mathsf{rec}(\mathtt{msg})_{S \to R_i}\}$, $\{\mathsf{sd}(\mathtt{ack})_{R_i \to S}\}$

**States**

$c \in \{0, 1\}$, initially 0

**Transitions:**

```
$\mathsf{rec}(\mathtt{msg})_{S \to R_i}$
Effect:
    $c := 1$

$\mathsf{sd}(\mathtt{ack})_{R_i \to S}$
Precondition:
    $c \neq 0$
Effect:
    None
```

**Fig. 7.** Code for Receiver $R_i$

**Task-PIOA** $A_i$ and $B_i$

**Signature**

Input:
    $Hello_i$
Output:
    $i$
Internal:
    none

**Tasks**

$Out_i = \{i\}$

**States**

For $A_i$: $hello \in \{\bot, \top\}$, initially $\bot$
For $B_i$: none

**Transitions:**

```
$Hello_i$
Effect:
    For $A_i$: $hello := \top$
    For $B_i$: $none$

$i$
Precondition:
    For $A_i$: $hello = \top$
    For $B_i$: $none$
Effect:
    For $A_i$: $hello := \bot$
    For $B_i$: $none$
```

**Fig. 8.** Code for Task-PIOAs $A(i)$ and $B(i)$

**Task-PIOA** $Box_k$ and $(Box_s)_k$

**Signature**

Input:
    0, 1
Output:
    $out(x), x \in \{0,1\}^k$
    $ok$
Internal:
    choose

**Tasks**

$Out = \{out(*)\}$
$Ok = \{ok\}$

**States**

$x - val \in \{0,1\}^k \cup \perp$, initially $\perp$
$input$, a buffer of at most $k$ elements in $\{0,1\}$, initially empty

**Transitions:**

```
0
Effect:
    if $input$ is not full then
        add "0" to $input$

1
Effect:
    if $input$ is not full then
        add "1" to $input$

$choose$
Precondition:
    $x - val = \perp$
Effect:
    $x - val := random(\{0,1\}^k)$

$out(x)$
Precondition:
    $x = x - val \neq \perp$
    $input \neq x - val$
Effect:
    none

$ok$
Precondition:
    For $Box_k$: $input = x - val \neq \perp$
    For $(Box_s)_k$: $false$
Effect:
    $none$
```

**Fig. 9.** Code for Task-PIOAs $Box_k$ and $(Box_s)_k$