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Stable Coalition Structures with Open Membership and Asymmetric Firms

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I study games of coalition formation with open membership where firms form associations in order to decrease their costs before competing on the market. According to previous analyses, only the grand coalition forms at the Nash equilibrium of such games. I show that this result hinges on the assumption of symmetric firms. I therefore introduce asymmetric firms in a game where only two associations can form. I demonstrate that there exists a coalition-proof Nash equilibrium coalition structure in this game, and that when the equilibrium involves two associations, all the members of an association have a higher taste for this association than all nonmembers do. *Journal of Economic Literature* Classification Numbers: C70, C72, L13. © 2000 Academic Press

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1. INTRODUCTION

In many economic and social situations, agents prefer to operate in groups rather than on their own. Examples abound in various fields of economics such as industrial organization (firms form research joint ventures, cartels, associations, standardization committees, . . .), international trade (countries form custom unions or regional trading blocs, they regulate cooperatively cross-border pollution), and local public finance (individuals form communities in order to share the costs of production of local public goods).

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Despite their diversity, these situations share some common features: the coalitions formed are smaller than the grand coalition, and their formation creates important external effects both on members and on non-members. Because of the latter characteristic, i.e., the presence of *spillovers* between coalitions, the classical tools of cooperative game theory proved ill-suited.[1] This inadequacy has prompted a new strand of literature on the noncooperative theory of coalition formation, with the analysis of games where the worth of each coalition (and/or the payoffs of its members) is determined by the entire coalition structure and not just by the coalition in question [see the excellent recent survey by Bloch (1997) and the references therein].

Specifically, most of the models in this literature adopt a two-stage framework: in the first stage, players form coalitions and in the second stage, they engage in a noncooperative game given the coalition structure that emerged from the previous stage.[2] Assuming for simplicity that the second-stage equilibrium is unique for any coalition structure, the second-stage game can be reduced to a *valuation*, which assigns to each coalition structure a vector of individual payoffs. These models allow thus for the formation of multiple coalitions and are particularly well suited to compare equilibrium coalition structures under various rules of coalition formation, various representation of externalities across coalitions, and various stability requirements. Because such analysis involves significant complexities, these models are usually simplified by assuming that the players are *ex ante* identical; as a consequence, individual payoffs depend only on the *number and sizes* of the coalitions, and not on their *composition*.

Yet, there are several reasons why heterogeneity of players and composition of coalitions are worth considering with closer attention. First, empirical observations reveal that when forming coalitions, players care about the *identity* of other coalition members. For instance, Axelrod *et al.* (1995) estimate the choices of nine computer companies to join one of two alliances sponsoring competing Unix operating system standards in 1988. Their basic assumptions are that a firm prefers (i) to join a large standard-setting alliance in order to increase the probability of successfully developing and sponsoring a compatibility standard and (ii) *to avoid allying with rivals, especially close rivals*, in order to benefit individually from the alliance's efforts. The model they build on these two plausible assumptions provides a robust estimate of the existing alliance configuration. Second,

[1] As noted by Bloch (1997), cooperative game theory has focused mostly on the problem of the division of coalitional worth among coalition members, leaving aside the issues of endogeneous formation of coalitions and of competition between coalitions.

[2] See, e.g., Bloch (1995, 1996), Yi (1997), and Yi and Shin (1997).

recent research on the role of collaborative alliances in the innovation process argues that firms differ significantly in their ability to benefit from these collaborative relationships.[3] Finally, it can be argued (see Section 3) that some of the results derived in the literature on endogenous formation of coalition rely heavily on the assumption of symmetric players.

On account of these facts, the present paper examines what stable coalition structures emerge *when one relaxes the assumption of symmetric players*. Because of the complexity of this task, attention is confined to the class of *simultaneous games of coalition formation with open membership*—i.e., games where all players announce at the same time their decision to form coalitions, and where players are free to join or to leave any coalition. Specifically, I reconsider the model of Bloch (1995) where, in a first stage, firms form associations in order to decrease their production costs, and in a second stage, compete on an oligopolistic market. In this model, firms are *ex ante* identical and the benefits from cooperation increase linearly in the size of the association according to a parameter that is independent of which particular association is formed (i.e., coalitions are assumed to be symmetric as well). With open membership, this setting leads to the formation of the grand coalition as the unique (pure strategy) Nash equilibrium coalition structure. According to Bloch (1997) [and also to Yi (1997) who studies a very similar model], this result is attributable to a rather mild monotonicity property of the valuation, namely, to the fact that a member of a coalition becomes better off if she leaves her coalition to join another coalition of equal or larger size.

I generalize the game step by step. First, I consider the case with *symmetric firms and asymmetric associations*; that is, the benefits from cooperation are still assumed to be the same from one firm to the other but they are now association-specific. I show that the unique equilibrium of the game is still the formation of the grand coalition, despite the fact that the aforementioned monotonicity property is no longer satisfied by the valuation once associations are asymmetric. This finding demonstrates the key role played by the assumption of symmetric firms.

The second step consists in allowing firms to have different benefits from cooperation, independently of the association they belong to. The main results drawn from this case with *asymmetric firms and symmetric associations* are the following: (i) several associations might form at the Nash equilibrium of the game; (ii) a (pure strategy) Nash equilibrium coalition structure might fail to exist. The former finding reinforces the

[3] See, e.g., Arora and Gambardella (1994) who provide a conceptual framework for analyzing how in-house scientific and technological capabilities affect the ability and the willingness of a firm to derive value from collaborative R & D. See also Nelson (1990) and Teece (1986).

idea that the assumption of symmetric firms is crucial to have the formation of the grand coalition as the unique equilibrium coalition structure; the latter finding shows how firms asymmetry makes the game unstable and more complex to analyze.

Finally, the last—and most interesting—case to consider deals with *asymmetric firms and asymmetric associations*. Unfortunately, this case turns out to be hardly tractable without simplifying further the modeling framework. I therefore choose to restrict attention to an extension of Bloch's framework where at most two coalitions can form and where each coalition presents some distinguishing features for which the players have diverging tastes. In particular, it is assumed that the benefits from cooperation still increase linearly in the size of the association, but now according to a parameter that depends both on the identity of the firm and on the association in question.

The main drawback of such simplification is that it exogenously imposes the number of equilibrium coalitions instead of letting this number be endogenously determined. However, several advantages make up for this drawback. The main advantage is that two significant results, in terms of existence and of characterization, are drawn from the model: the *existence result* is that, despite players heterogeneity, there exists at least one coalition-proof Nash equilibrium (CPNE) coalition structure; the *characterization result* is that equilibrium might involve the formation of the two associations and when it does, all the members of a particular association have a higher taste for this association than all nonmembers do. The restriction to two coalitions presents the additional benefit of *de facto* transforming the game into a *game without spillovers* among different coalitions.[4] The contribution of the paper can thereby be put into perspective with the literature studying this class of games, namely, with articles by Konishi *et al*. (1997a, b, c).

The remainder of the paper is organized as follows. Section 2 outlines the general model. Section 3 analyzes the three intermediary cases where either firms or coalitions (or both) are symmetric. Section 4 considers a simplified version of the case with asymmetric firms and coalitions where at most two coalitions can form. Section 5 provides some concluding remarks and directions for further research.

[4] Konishi *et al*. (1997b) define the *no spillovers* condition as follows: "if players i and j choose different strategies, x^i and x^j, the payoff of player i would not be affected if player j will switch her strategy to $\tilde{x}^j$ which is different from x^i." It is obvious that this condition is vacuously satisfied in the case where the players can only choose among two alternatives.

2. THE GENERAL MODEL

Different versions of the following general model, inspired from Bloch (1995), will be studied throughout the paper. In this model, n firms ($i \in N$) first form associations and then compete on the market. Demand is assumed to be linear and given by $P = \alpha - \sum_{i=1}^{n} q_i$. Firms have a marginal cost of production that is independent of the quantity produced and decreasing in the size of the association they belong to. The cost of a firm i in an association A_l of size a_l is given by: $c_{il} = \lambda - \mu_{il} a_l$ (with $\mu_{il} > 0$). It is assumed that the parameter values α, λ and μ_{il} are such that, for any coalition structure, there exists a unique interior Cournot equilibrium on the market. For a coalition structure $\pi = \{A_1, A_2, \ldots, A_m\}$, it can be shown that the profit of firm i belonging to association A_l is a monotonically increasing function of the following valuation:

$$\begin{aligned} v_{il}(\pi) &= (n+1)(\mu_{il} a_l - \lambda) - \sum_{k=1}^{m} \sum_{j \in A_k} (\mu_{jk} a_k - \lambda) \\ &= (n+1)\mu_{il} a_l - \left(\sum_{k=1}^{m} \sum_{j \in A_k} \mu_{jk} a_k \right) - \lambda. \end{aligned} \tag{1}$$

It is readily checked that this valuation exhibits *spillovers* (i.e., the formation of an association by external firms has an effect on the payoff of a firm); whether these spillovers are negative or positive depends on which coalition is formed and by which firms.

For the analysis to follow, it proves useful to express the payoff differential a firm will get from switching associations in a given coalition structure. Suppose that in the initial coalition structure π, firm i belongs to association A_l and then moves to association A_k; let π' denote the new coalition structure resulting from i's move. Using (1), it is easy to compute the difference in payoffs as

$$v_{ik}(\pi') - v_{il}(\pi) = n(\mu_{ik}(a_k + 1) - \mu_{il} a_l) + \sum_{s \in A_l \setminus \{i\}} \mu_{sl} - \sum_{t \in A_k} \mu_{tk}. \tag{2}$$

The payoff differential is composed of three terms. The first term expresses the *own-cost effect*; it is either positive or negative depending on the associations' sizes and on i's preferences. The second and third terms express two *competition effects*, a positive and a negative one stemming, respectively, from an increase in the cost of i's former partners and from a decrease in the cost of i's new partners.

A similar payoff differential can be expressed for some firm j moving from A_k to A_l:

$$v_{jl}(\pi'') - v_{jk}(\pi) = n\big(\mu_{jl}(a_l + 1) - \mu_{jk}a_k\big) + \sum_{t \in A_k \setminus \{j\}} \mu_{tk} - \sum_{s \in A_l} \mu_{sl}, \tag{3}$$

where π'' is the coalition structure obtained from π by moving j from A_k to A_l.

A *Nash equilibrium coalition structure* is such that no firm wishes to unilaterally switch associations. Therefore, for associations A_k and A_l to be both nonempty in a Nash equilibrium coalition structure, it must be the case that (i) $\forall i \in A_l$, $v_{ik}(\pi') - v_{il}(\pi) \leq 0$, and (ii) $\forall j \in A_k$, $v_{jl}(\pi'') - v_{jk}(\pi) \leq 0$. Using (2) and (3), it is easy to show that joint satisfaction of this set of inequalities requires that

$$n(a_k + 1)(\mu_{jk} - \mu_{ik}) - n(a_l + 1)(\mu_{jl} - \mu_{il}) > (n-1)(\mu_{jk} + \mu_{il}), \quad \forall i \in A_l, \quad \forall j \in A_k. \tag{4}$$

Building from Eq. (2), we can also express the conditions under which a single association A_l forms at the Nash equilibrium:

$$n\mu_{il} - \mu_{ik} \geq (1/n) \sum_{j \in N \setminus \{i\}} \mu_{jl}, \forall i \in N, \quad \forall k \neq l. \tag{5}$$

I shall now consider different versions of the general game and derive existence and characterization results about its equilibria, under different stability requirements. Before considering in depth a simplified version of the case with asymmetric firms and coalitions, I briefly address the three intermediary cases where either firms or coalitions (or both) are symmetric.

3. SYMMETRIC FIRMS AND/OR COALITIONS

The three versions of the game examined in this section demonstrate the crucial role played by the assumption of symmetric firms, as summarized by the following proposition.

PROPOSITION 3.1. *With symmetric firms* (*and with either symmetric or asymmetric coalitions*), (*a*) *the game admits at least one Nash equilibrium coalition structure, and* (*b*) *the Nash equilibrium always involves the formation of the grand coalition. With asymmetric firms and symmetric coalitions,* (*c*) *the game might admit no Nash equilibrium coalition structure, and* (*d*) *the Nash equilibrium might involve the formation of several coalitions.*

The proof of statement (b) is straightforward. With symmetric firms, $\mu_{ik} = \mu_{jk}$ for any firms i, j and for any association A_k. This means that inequality (4) cannot be met or, in other words, that no more than one coalition can form at the Nash equilibrium. Note that Bloch (1997) and Yi (1997) establish the same result in the case with symmetric firms *and* coalitions; their proof relies on a property of the valuation, named "Individual Monotonicity," according to which a member of a coalition becomes better off if he leaves his coalition to join another coalition of equal or larger size.[5] The present analysis shows that it is not the latter property that should be made responsible for the result but the firm symmetry itself. Indeed, when coalitions are asymmetric, one can easily come up with sets of parameters for which individual monotonicity no longer holds. However, the formation of the grand coalition remains the only possible Nash equilibrium coalition structure.

To prove statement (a), it suffices to rewrite condition (5) in the case where firms are symmetric: $n\mu_l - \mu_k \geq (1/n)(n-1)\mu_l \Leftrightarrow \mu_l \geq n/(n^2 - n + 1)\mu_k (\forall k \neq l)$. Since $n/(n^2 - n + 1) < 1$, there is always at least one μ_l that satisfies this condition.

I now construct examples to prove statements (c) and (d). Let me first develop Eq. (4) for the case where firms are asymmetric and coalitions are symmetric. Some easy manipulations lead to the following condition:

$$\left[n(a_k - a_l) - (n-1)\right]\mu_j > \left[n(a_k - a_l) + (n-1)\right]\mu_i, \qquad \forall i \in A_l, \quad \forall j \in A_k.$$

It immediately follows from the latter condition that, in this specific case, *it is impossible to have equal-sized coalitions as parts of a Nash equilibrium coalition structure*. Suppose now there are four firms in the industry. We derive from the previous finding that the Nash equilibrium coalition structure will involve either the formation of the grand coalition, or the formation of two coalitions, one composed of three firms and the other of one firm. Ordering the firms so that $\mu_1 \leq \mu_2 \leq \mu_3 \leq \mu_4$ (with at least one strict inequality to guarantee firms asymmetry), and using Eq. (2), (3) and (5), it is easy to derive the following conditions: if $12\mu_1 \geq \mu_2 + \mu_3 + \mu_4$, the formation of the grand coalition is the unique Nash equilibrium coalition structure (*case* 1); otherwise, either $\mu_1 + 4\mu_2 \geq \mu_3 + \mu_4$ and

[5] For a proof of this statement, see Proposition 3.1 in Bloch (1997) or Proposition 4-1 in Yi (1997). Note that Bloch (1997) also shows that when membership is *exclusive* rather than open (i.e., when firms are not free to join an existing coalition), when the formation of coalitions is *sequential* rather than simultaneous, or when one uses *cooperative* rather noncooperative stability concepts, the solution of the game is (at one exception) the following: a dominant association grouping around three quarters of the industry forms, and the remaining firms form a smaller association.

$\{\{1\}, \{2, 3, 4\}\}$ is the unique Nash equilibrium coalition structure (*case* 2), or this condition is not met and there is no Nash equilibrium (*case* 3). Setting $\mu_1 = 1$; $\mu_3 = 5$; $\mu_4 = 6$, we are in *case* 3 with $\mu_2 = 2$ and in *case* 2 with $\mu_2 = 3$, which, respectively, proves statement (c) and (d).

The conclusion that can be drawn from Proposition 3.1 is twofold. On the one hand, statements (b) and (d) establish that the formation of the grand coalition at the Nash equilibrium of the game crucially hinges on the assumption of symmetric firms; it seems thus extremely interesting to investigate further what happens when one relaxes this assumption. On the other hand, as shown by statement (c), firm asymmetry makes the game unstable insofar as a Nash equilibrium coalition structure might fail to exist; this finding suggests that additional conditions will have to be imposed in order to guarantee the existence of a stable coalition structure when firms are asymmetric. The next section takes a first step in this direction.

4. ASYMMETRIC FIRMS AND COALITIONS

When one tries to deal with asymmetric firms and coalitions, one quickly acknowledges that significant complexities appear that can only—it seems—be dealt with through the simplification of other parts of the model. In this respect, the price to pay here is the restriction on the number of possible associations to two. Such restriction might, nevertheless, fit with some empirical observations;[6] for examples, regarding the competition between standard-setting alliances, Axelrod *et al*. (1995) note the following:

> Such competition is often limited to two alliances rather than a large number of coalitions because the chance of successfully creating and sponsoring a standard declines as the number of designs increases. (...) More than two alliances sometimes form, and the number of alliances that might form is limited only by the number of firms, but our limit [to two possible alliances] is consistent with many empirical instances.

In the remainder of this section, a modified version of Bloch (1995)'s model is proposed where heterogeneous firms might form at most two associations; the properties of the valuation derived in this modified model are then studied, and finally, instructive results about existence and characterization of equilibrium coalition structures are stated.

[6] This restriction is also sometimes resorted to in the literature. For instance, in her analysis of stable cartel formation, Thoron (1998) assumes that only a single cartel can be formed, which implies that firms' strategies have a binary form (to cooperate or not).

4.1. *The Modified Model*

Consider the following setup adapted from Belleflamme (1998). Suppose (as above) that firms have a marginal cost of production that is independent of the quantity produced and decreasing with the size of the association they belong to. The n firms play the following two-stage game, called G. In the first stage, they choose simultaneously among two possible associations: they join either an association of type a or an association of type b; in the second stage, they compete on the market. Firms are assumed to have different preferences for the two types of associations. These preferences are represented by a value of θ between 0 and 1; more precisely, the parameter μ (i.e., the slope of the marginal cost function) differs across firms and across associations: for firm i, it is equal to $1 - \theta_i$ when i joins a type-a association, and to θ_i when it joins a type-b association.[7] Letting $\pi = \{A, B\}$ denote the coalition structure resulting from the firms' choices, and a denotes the number of firms in the type-a association A (leaving $n - a$ firms in B), marginal costs are given by the following expression:

$$c_i^A(\pi) = \lambda - a(1 - \theta_i), \quad \forall i \in A; \tag{6}$$

$$c_j^B(\pi) = \lambda - (n - a)\theta_j, \quad \forall j \in B. \tag{7}$$

One observes that with equal-sized associations, firm k with $\theta_k <$ [resp. $>$]0.5 prefers association A [resp. B]. Without any loss of generality, let us rank the firms by increasing values of θ and assume that $0 \leq \theta_1 \leq \theta_2 \leq \cdots \leq \theta_n \leq 1$; this means that firm 1 is intrinsically the strongest "type-a lover" and firm n the strongest "type-b lover."[8] Before proceeding further, I introduce one more piece of notation: let $E(\pi)$ denote the sum of the reductions in marginal costs for a given coalition structure π; that is,

$$E(\pi) \equiv a \sum_{i \in A} (1 - \theta_i) + (n - a) \sum_{j \in B} \theta_j. \tag{8}$$

[7] One can think of the formation of an association as the common adoption of some technology exhibiting *network externalities*: the more such technology is adopted, the more it benefits its users (for a recent review of the literature on network externalities, see Matutes and Régibeau, 1996). The positive relationship between users' utility and network size is generally modeled as the addition of two components: a stand-alone benefit (which reflects the advantages from the immediate use of the technology) and a network benefit (which results from the fact that the users' willingness to pay increases with the number of users who own the same technology). The formulation adopted here assumes that the technologies vary according to network benefits they offer and that the firms are heterogeneous regarding the way they value these network benefits; Belleflamme (1998) vindicates this assumption.

[8] It is still assumed that the parameter values are such that for any coalition structure, all firms are active in the second-stage Cournot equilibrium.

4.2. *Properties of the Valuation*

Using expressions (6)–(8), it is easily seen that the valuation (1) is changed into

$$\begin{cases} v_i^A(\pi) = (n+1)a(1-\theta_i) - E(\pi) - \lambda, & \forall i \in A; \\ v_j^B(\pi) = (n+1)(n-a)\theta_j - E(\pi) - \lambda, & \forall j \in B. \end{cases} \tag{9}$$

What are the useful properties of the valuation? First, because only two associations can be formed, the issue of the spillovers between coalitions becomes irrelevant. This allows me to relate the present analysis to the literature on *games without spillovers*, and in particular to the work of Konishi *et al*. (1997a, b, c; KLW henceforth). The main focus of their three papers is to examine the no spillover games where the valuations satisfy the *population monotonicity* property; this property implies that the payoff of a player changes monotonically when the size of the group of players choosing the same strategy increases; according to the sign of this change, they refer to games with *negative* or with *positive externalities*. Two of their results are particularly interesting for the present purposes: (i) in games with positive externalities and with two possible associations, the sets of strong Nash equilibria (SNE) and the coalition-proof Nash equilibria (CPNE; see definitions below) are equivalent and nonempty; (ii) in games with negative externalities, the same holds if the set of alternatives is finite and if the valuation satisfies the *anonymity* property (which requires that each player's payoff depends only on the number of players who choose the same strategy).[9]

Are any of these properties satisfied by valuation (9)? The answer is no. Because of the second-stage Cournot competition, the change in size of an association induces on its members' profits two simultaneous effects that work in opposite directions: an own-cost effect and a competition effect. As a result, a firm's payoff does not necessarily increase or decrease as some firm joins the association to which it belongs; that is, population monotonicity does not hold. Anonymity does not hold either because of firms asymmetry. More generally, there is no certainty about the sign of the change in firm i's payoff (i) when firm i switches from one association to the other, or (ii) when other firms join or leave the association to which firm i belongs.

Nevertheless, one observes that the firms' incentives to switch associations exhibit interesting properties. Lemma 4.1 shows that the following three axioms are satisfied by valuation (9).

[9] The first result combines the findings of Proposition 2.2 in KLW (1997a) and of Corollary 3.3 in KLW (1997b); the second result combines the findings of Theorem 4.2 in KLW (1997b) and of Proposition 3.3 in KLW (1997c).

Switching Incentive Anonymity ($\mathscr{SIA}$): A firm's incentive to switch unilaterally from one association to the other depends only on the number of firms in each association and not on their respective identity.

Switching Incentive Monotonicity ($\mathscr{SIM}$): A firm's incentive to switch unilaterally from one association to the other increases with the size of the association to be joined.

Switching Incentive Intensity ($\mathscr{SII}$): A firm's incentive to switch unilaterally from one association to the other increases with the firm's intensity of preference for the association to be joined.

LEMMA 4.1. *The valuation* (9) *satisfies axioms* $\mathscr{SIA}$, $\mathscr{SIM}$ *and* $\mathscr{SII}$.

The proof is straightforward. Because only the size of the associations matters, let me adopt the following notation; let $L_i(a)$ and $J_j(a)$ denote, respectively, the incentive for some firm i to *leave* and for some firm j to *join* a type-a association of size a. Building from (2) and (9), one obtains the following expressions which clearly satisfy the three axioms (recalling that θ_i [resp. $(1-\theta_i)$] represent firm i's intensity of preference for an association of type b [resp. type a]):

$$\begin{aligned} L_i(a) &= v_i^B(A \setminus \{i\}, B \cup \{i\}) - v_i^A(\pi) \\ &= (n-1)(n-a) + (n^2+n+1)\theta_i - (n^2-n+1) - n\bar{\theta}, \\ J_j(a) &= v_j^A(A \cup \{j\}, B \setminus \{j\}) - v_j^B(\pi) \\ &= (n-1)a + (n^2+n+1)(1-\theta_j) - (n^2-n+1) - n(1-\bar{\theta}), \end{aligned}$$

where $\bar{\theta}$ is the mean of θ's over N.

Two comments are in order with respect to the three axioms. First, it is straightforward to see that a direct consequence of $\mathscr{SIA}$ is that a firm's incentive to leave a type-a association of size a has the opposite value of its incentive to joint a type a-association of size $(a-1)$; that is, $L_i(a) = -J_i(a-1)$. Second, a parallel can be drawn with axioms defined by Milgrom and Shannon (1994) in their analysis of monotone comparative statics; precisely, there is a close connection between $\mathscr{SII}$ and the *single crossing property*, and $\mathscr{SIM}$ (coupled with $\mathscr{SIA}$) is a particular case of the *increasing differences property*. As a consequence of the latter finding, game G has the attributes of a *supermodular game*.[10]

[10] I thank an anonymous referee for having brought this parallel to my attention. For a thorough description of supermodular games, see Section 12.3 in Fudenberg and Tirole (1991) and the references therein.

4.3. *Nash Equilibrium*

Building on the previous findings, we can establish the existence of a Nash equilibrium coalition structure in the open membership game, and even characterize the set of possible equilibrium coalition structures. In order to state formally these results, let us first introduce some more notation. Let $\mathscr{P} \equiv \{P = \{A, B\} | A \in 2^N, B = N \setminus A$, and $\forall i \in A, \forall j \in B, \theta_i < \theta_j\}$; in words, an element of $\mathscr{P}$ determines a coalition structure such that every firm in the type-a association has a lower value of θ than every firm in the type-b association. Let P_k denote an element of $\mathscr{P}$ where the type-a association has cardinality k; that is, $P_k = \{A_k, N \setminus A_k\} = \{\{1, \ldots, k\}, \{k + 1, \ldots, n\}\}$ (with the first subset denoting the type-a association). Let also $\mathscr{NE}(G)$ denote the set of the Nash equilibrium coalition structures of game G. The next proposition summarizes the results.

PROPOSITION 4.1. *If the valuation of game G satisfies* $\mathscr{SIA}$ *and* $\mathscr{SII}$, *then* $\mathscr{NE}(G) \neq \emptyset$; *moreover, if the valuation satisfies* $\mathscr{SIM}$ *as well, then* $\mathscr{NE}(G) \subset \mathscr{P}$.

This proposition (which is proven in the Appendix) says that (i) if the valuation satisfies switching incentives anonymity and intensity, there exists at least one Nash equilibrium coalition structure in the game, and (ii) if the valuation further satisfies switching incentives monotonicity, then, in the cases where the equilibrium coalition structure involves two associations, all firms in the type-a association have a strictly lower θ than all firms in the type-b association. As a corollary, one can also give an operational expression of the conditions for a Nash equilibrium in game G (using the assumption that the firms are ranked by increasing values of θ):

$$
\begin{aligned}
P_k \in \mathscr{NE}(G) &\Leftrightarrow J_k(k-1) \geq 0 \text{ and } J_{k+1}(k) \leq 0 \\
&\Leftrightarrow \theta_k + \frac{n-1}{n^2+n+1} \leq \frac{(n-1)k + n(1+\bar{\theta})}{n^2+n+1} \leq \theta_{k+1}.
\end{aligned}
\tag{10}
$$

Using the latter expression, we can easily formulate (i) a necessary and sufficient condition under which the formation of the grand coalition (on either type of association) cannot be a Nash equilibrium outcome of the game, and (ii) a sufficient condition under which the formation of the grand coalition is the unique Nash equilibrium outcome of the game.

PROPOSITION 4.2. (*i*) P_0 *and* $P_n \notin \mathscr{NE}(G)$ *if and only if* $\theta_1 < (n + n\bar{\theta})/(n^2 + n + 1)$ *and* $\theta_n > 1 - [(n + n(1 - \bar{\theta}))/(n^2 + n + 1)]$. (*ii*) $\mathscr{NE}(G) \subseteq \{P_0, P_n\}$ *if* $\forall k = 1 \ldots (n-1), \theta_{k+1} - \theta_k \leq [(n-1)/(n^2 + n + 1)]$.

This proposition is straightforwardly proven. The first part asserts that the equilibrium coalition structure involves necessarily two associations provided that the lowest and highest values of the preference parameter θ are close enough to the extremes; in other words, if there is a sufficient degree of heterogeneity between the firms, then the grand coalition cannot be a Nash equilibrium of game G. On the other hand, the second statement says that a single association will form at the Nash equilibrium of G if firms are sufficiently homogeneous (i.e., if differences between successive values of θ are not larger than some constant).

4.4. *Cooperative Refinements of Nash Equilibrium*

It must be stressed that multiple Nash equilibria might arise for wide constellations of parameters. Therefore, in order to obtain a sharper prediction about stable coalition structures, we move to stability concepts which allow deviations by a *group* of firms, not just *individual* deviations. Two such concepts are *coalition-proof Nash equilibria* (CPNE) and *strong Nash equilibrium* (SNE). They are formally defined in the Appendix (see Definitions 5.1 and 5.2). Very briefly, the concept of coalition-proofness was introduced by Bernheim *et al*. (1987); as they define it, ‘‘an agreement is coalition-proof if and only if it is Pareto-efficient within the class of self-enforcing agreements; in turn, an agreement is self-enforcing if and only if no proper subset (coalition) of players, taking the actions of its complement as fixed, can agree to deviate in a way that makes all of its members better off.’’ As KLW (1997b) point out, the novelty of the concept was highlighted by the fact that it is immune only to *credible* coalitional deviations, in contrast to SNE, introduced by Aumann (1959), which is immune to *any* coalitional deviations, including those which are not credible. The SNE stability requirement is thus more stringent than the CPNE requirement; no wonder then that a SNE may fail to exist in the present game (an example given in the Appendix proves this statement).

Though less stringent, the CPNE concept requires much more computations than the SNE concept does: the consistent application of the notion of self-enforceability involves a recursion, which means that when the deviation of a coalition is considered, deviations from the deviations by subcoalitions (and so forth) have also to be examined. Interestingly, in the present analysis, this recursive nature is all but a handicap: because the valuation satisfies $\mathcal{SIA}$, $\mathcal{SIM}$ and $\mathcal{SII}$, we can *a priori* discard an entire category of coalitions (see Lemma 4.2) and identify, among the remaining coalitions, the ones which are the most likely to deviate in a self-enforcing way from a given Nash equilibrium (see Lemma 4.3). The last step consists in showing that there always exists at least one Nash equilibrium coalition structure that is immune to credible coalitional deviations, which establishes the main result of the section: the existence of CPNE in the game.

There are in the literature some other results about the existence of CPNE and it is instructive to put them into perspective with the present one. First, I mentioned above the results of KLW (1997a, b, c) which rest on assumptions about the way the payoff of a player changes when additional players choose the same strategy; in particular, when the change is positive (*positive externalities*, $\mathscr{PE}$), the authors show that the sets of SNE and CPNE are equivalent and nonempty when only two associations can be formed. It is easy to show that if the valuation satisfies $\mathscr{PE}$, it necessarily satisfies $\mathscr{SIM}$ as well; however, as argued above, the reverse is not true. So, to some extent, the result provided here is more general (although the valuation has to meet other requirements and that the existence of SNE is no longer guaranteed). Second, Milgrom and Roberts (1996) provide a CPNE existence theorem for games with *strategic complementarities* (a class of games that includes supermodular games); a sufficient condition for the existence of CPNE is that every player's payoff be nondecreasing (or nonincreasing) in every other player's strategy; this condition has the flavor of the population monotonicity in KLW and is not met in the present setting. Third, Thoron (1998) proves the existence of a unique CPNE in a game of cartel formation in an oligopoly where firms' strategies have a binary form (to cooperate or not); in such a game, cartel formation generates a positive externality and firms that remain independent are free riders (since their profit increases as the cartel size increases). Finally, Kukushkin (1997) presents a theorem establishing a set of assumptions guaranteeing the existence of CPNE; some of these assumptions restrict the way in which one player's choice may affect another player's utility. It is worth noting that the conditions provided in these papers are about the way a player's payoff varies *when keeping her strategy fixed and changing other players' strategies*; in contrast, the conditions provided here are about the way a player's payoff varies *when changing her strategy and keeping the other players' strategies fixed*.

It remains to prove the result. Writing $\mathscr{CPNE}(G)$ for the set of CPNE of game G, one can state it formally as follows.

PROPOSITION 4.3. *If the valuation of game G satisfies* $\mathscr{SIA}$, $\mathscr{SIM}$ *and* $\mathscr{SII}$, *then* $\mathscr{CPNE}(G) \neq \emptyset$.

To facilitate the exposition, the proof of Proposition 4.3 (see the Appendix) is decomposed into a three-step procedure. It is first shown that if there is only one Nash equilibrium, this strategy profile is necessarily a CPNE; then, one considers the case where there are two Nash equilibria and proves that at least one of them is a CPNE; finally, the previous arguments are extended to cases where there are more than two Nash equilibria. As explained above, the proof relies on two important lemmas.

LEMMA 4.2. *For any $P_k \in \mathcal{NE}(G)$, no coalition, comprising firms of different associations is able to sustain a credible deviation from P_k.*

Note that the argument behind this result is reminiscent of the finding that in any Nash equilibrium coalition structure with two associations, all firms in the type-a association have a lower θ than all firms in the type-b association. Lemma 4.2 teaches us that any Nash equilibrium has only to be tested against deviations by coalitions exclusively composed of members of the same association. Next question which arises naturally is whether it is possible to state sufficient conditions to discard all such coalitions of a given size. Lemma 4.3 gives an affirmative answer to this question.

LEMMA 4.3. *For any $P_k \in \mathcal{NE}(G)$, (i) if $J_{k-s+1}(k-s) > 0$, then all coalitions of size s included in the type-a association are unable to sustain a credible deviation from P_k, and (ii) if $J_{k+s}(k+s-1) < 0$, then so are all coalitions of size s included in the type-b association.*

While the recursive nature of the self-enforceability concept was helpful to prove the existence of a CPNE, it is unfortunately detrimental to the *characterization* of the set of CPNE. The main reason is that valuation (9) does no longer exhibit any regularity when one assesses the change in the payoff of a firm which switches associations *together with other firms*; in particular, there is no clear relationship between a firm's switching incentive and the number of accompanying firms. It therefore proves really cumbersome to single out the regions of parameters where a unique CPNE obtains.[11] I am, nonetheless, in a position to provide necessary conditions for the existence of multiple CPNE in game G, and thereby sufficient conditions for uniqueness of CPNE, as stated in the next proposition and corollary.

PROPOSITION 4.4. *If P_k and $P_l \in \mathcal{CPNE}(G)$ (with $l > k+1$), then (i) $v_{k+1}^A(P_l) > v_{k+1}^B(P_k)$ and (ii) $v_l^A(P_l) < v_l^B(P_k)$.*

The proof is rather immediate. Simple manipulations show that the difference $v_i^A(P_l) - v_i^B(P_k)$ is a decreasing function of θ_i. This means, in particular, that if condition (i) is violated, all firms $j \geq k+1$ are better off in P_k than in P_l. Therefore, if (i) is violated, then coalition $s = \{k+1, \ldots, l\}$ can profitably deviate from P_l to P_k. Moreover, since $P_k \in \mathcal{CPNE}(G)$, such deviation is itself immune to credible subdeviations, which implies that $P_l \notin \mathcal{CPNE}(G)$, a contradiction. A similar argument holds when one starts from the violation of condition (ii).

COROLLARY 4.1. *If $P_k \in \mathcal{CPNE}(G)$, $v_k^A(P_k) > v_k^B(P_j)$, $\forall j < k-1$, and $v_{k+1}^B(P_k) > v_{k+1}^A(P_l)$, $\forall l > k+1$, then $\mathcal{CPNE}(G) = \{P_k\}$.*

[11] Even in the simple case with three firms, when the two Nash equilibria are the formation of the grand coalition on either type of association, one has to follow a complex algorithm to determine under which conditions one or the other is the unique CPNE.

5. CONCLUDING REMARKS

This paper takes a first step in the analysis of coalition formation between asymmetric players (i.e., players who benefit differently from being member of a particular coalition). Because of the complexities involved, this step has been made at the expense of several simplifications. First, only games with simultaneous moves and with open membership are considered; second, the players can choose among no more than two coalitions. Despite the simplicity of this framework, some instructive points emerge about the stability of the coalition structure (there exists a coalition-proof Nash equilibrium coalition structure in the game), and about the composition of coalitions (when two coalitions are formed at the equilibrium, all the members of a particular coalitions have a higher taste for this coalition than all nonmembers do).

Natural extensions of this analysis would be to increase the number of possible coalitions and to consider other rules of coalition formation (sequential moves, exclusive membership). In this respect, it would be very interesting to introduce heterogeneity of players in the model of Bloch (1995) where coalitions are formed in a sequential way, in the spirit of Rubinstein (1982)'s alternating-offers bargaining game.

APPENDIX

Proof of Proposition 4.1. (i) *Existence*. Existence follows directly from the fact that G is supermodular and that supermodular games always have pure-strategy Nash equilibria. Let me, nevertheless, develop the argument since it proves useful for other proofs. Consider first P_n; if $P_n \in \mathscr{NE}(G)$, we are done. Otherwise, $\exists i \in A_n$ s.t. $L_i(n) > 0$. From $\mathscr{SII}$ (and the assumption that $\theta_1 \le \theta_2 \le \cdots \le \theta_n$), we necessarily have that $L_n(n) > 0$. Consider then P_{n-1}; if $P_{n-1} \in \mathscr{NE}(G)$, we are done. Otherwise, there must be at least one firm wishing to switch associations. Since it cannot be firm n ($L_n(n) > 0 \Rightarrow J_n(n-1) < 0$), and from the previous argument, we have that $L_{n-1}(n-1) > 0$. We can repeat this procedure as long as no equilibrium coalition structure is found. In the worst-case scenario, we will have that $P_1 \notin \mathscr{NE}(G)$, meaning that $L_1(1) > 0$. But then, since $L_1(1) > 0 \Rightarrow J_1(0) < 0 \Rightarrow J_k(0) < 0$, $\forall k > 1$ (from $\mathscr{SII}$), $P_0 \in \mathscr{NE}(G)$. In any case, $\mathscr{NE}(G) \neq \emptyset$. (ii) *Characterization.* Suppose that $\pi = \{A, B\} \in \mathscr{NE}(G)$, with A and B nonempty. Take some firm $i \in A$ and some firm $j \in B$ such that $\theta_i \ge \theta_j$. Nash equilibrium means form firm j that $J_j(a) \le 0 \Leftrightarrow L_j(a+1) \ge 0$. Because $\theta_i \ge \theta_j$ and because the valuation satisfies $\mathscr{SII}$, $L_j(a+1) \ge 0 \Rightarrow L_i(a+1) \ge 0$. This, in turn, implies that $L_i(a) \ge 0$ because the valuation satisfies $\mathscr{SIM}$. But this inequality means that firm i

has an incentive to leave the type-a association, violating the Nash stability requirement; a contradiction. Hence, $\pi \in \mathcal{NE}(G) \Rightarrow \pi \in \mathcal{P}$. ■

Definitions of the Cooperative Refinements of Nash Equilibrium

I define first a strategic game G by the triple $(N, \{S_i\}_{i \in N}, \{u_i\}_{i \in N})$. Next, in order to introduce the notion of CPNE, I define a reduced game for each coalition C and each strategy profile σ as follows:

$$G_\sigma^C = \left(C, \{S_i\}_{i \in C}, \{\tilde{u}_i\}_{i \in C}\right),$$

where $\tilde{u}_i(s) = u_i(s_C, \sigma_{N \setminus C})$. In other words, the reduced game for C at σ is restricted to members of C only, assuming that firms outside of C still choose the strategies given by vector σ.

I am now in a position to offer the recursive definition of CPNE.

DEFINITION 5.1 (Bernheim *et al.*, 1987). A *coalition-proof Nash equilibrium* of game G is defined recursively. In a one-firm game, σ_i is a CPNE if and only if σ_i is a maximizer of u_i over S_i. Let $n > 1$ and assume that CPNE has been defined for games with fewer than n firms. Then,

1. For any game G with n firms, σ is *self-enforcing* in G if for all subsets C of $N, C \neq N$, $\sigma_C \in \mathcal{CPNE}(C_\sigma^C)$.

2. For any game G with n firms, $\sigma \in \mathcal{CPNE}(G)$ if σ is self-enforcing and there is no other self-enforcing vector of strategies $\tilde{\sigma}$ such that $u_i(\tilde{\sigma}) > u_i(\sigma)$ for every $i \in N$.

The set of all CPNE of game G is denoted $\mathcal{CPNE}(G)$.

DEFINITION 5.2 (Aumann, 1959). A *strong Nash equilibrium* of game G is a strategy profile σ for which there does not exist a coalition $C \subset N$ and a strategy profile σ'_C for players in C such that $u_i(\sigma'_C, \sigma_{N \setminus C}) > u_i(\sigma)$, $\forall i \in C$.

Nonexistence of a Strong Nash Equilibrium Coalition Structure

The following example proves that the set of SNE of game G might be empty. Suppose that there are five firms in the industry with the following preferences: $\theta_1 = 0.3$, $\theta_2 = 0.35$, $\theta_3 = 0.45$, $\theta_4 = 0.62$, $\theta_5 = 0.7$. Suppose further that $\lambda = 5$. For these values, one has that $J_1(0) = -1.88$, $J_2(1) = 0.57$, $J_3(2) = 1.47$, $J_4(3) = 0.2$, $J_5(4) = 1.72$, meaning that $\mathcal{NE}(G) = \{P_0, P_5\}$. Computing the payoffs of the firms for other coalition structures, one observes that firms 1 and 2 can improve upon their payoffs in the structure P_0 by deviating together: $v_1^A(P_2) = -4.61 > v_1^B(P_0) = -8.1$; $v_2^A(P_2) = -5.21 > v_2^B(P_0) = -6.6$. Similarly, firms 4 and 5 are better off if they deviate together from P_5: $v_4^B(P_3) = -5.9 > v_4^A(P_5) = -6.5$;

$v_5^B(P_3) = -4.94 > v_5^A(P_5) = -8.9$. This means that none of the Nash equilibria is a strong Nash equilibrium.

Proof of Lemma 4.2. Take $P_k = \{A, B\} \in \mathcal{NE}(G)$ (and a corresponding σ), with $k \neq 0$, and $k \neq n$. Let $S \subseteq N$ be composed of firms from the two associations; that is, $S = S^a \cup S^b$, where $S^a = S \cap A$, $S^b = S \cap B$, $S^a \neq \emptyset$, $S^b \neq \emptyset$. A deviation by the members of S would change the type-a association from A to $C \equiv (A \setminus S^a) \cup S^b$, with $\#C = c$. For such deviation to be self-enforcing, it must be a Nash equilibrium in the reduced game for S at σ. Formally, (C1) $J_i(c) \leq 0$, $\forall i \in S^a$, and (C2) $L_j(c) \leq 0 \Leftrightarrow J_j(c-1) \geq 0$, $\forall j \in S^b$. Take some i in S^a and some j in S^b; by the definitions of P_k, S^a and S^b, we have that $\theta_i < \theta_j$. From (C2), $J_j(c-1) \geq 0$. But, since the valuation satisfies $\mathcal{SII}$ and $\mathcal{SIM}$, $\theta_i < \theta_j \Rightarrow J_i(c-1) > 0 \Rightarrow J_i(c) > 0$, which contradicts condition (C1). ■

Proof of Lemma 4.3. *Claim* (*i*). Take $S \subseteq A_k$; deviation by S is not credible if $\exists i \in S$ s.t. $J_i(k-s) > 0$. From $\mathcal{SII}$, the condition is the most likely to be met by the firm in S with the lowest index. Take thus the firm with the lowest index in all conceivable size-s coalitions included in A_k and express the corresponding sufficient condition for a noncredible deviation. From $\mathcal{SII}$ again, we know that the most stringent among these conditions is expressed for the firm with the highest index among the subset of firms identified in the previous step; that is, this firm's index is equal to $k - s + 1$; the condition is thus $J_{k-s+1}(k-s) > 0$, which completes the proof of claim (i). The proof of claim (ii) follows exactly the same lines. ■

Proof of Proposition 4.3.

Claim 1: $\mathcal{NE}(G) = \{P_k\} \Rightarrow \mathcal{CPNE}(G) = \{P_k\}$. The conditions for $\mathcal{NE}(G) = \{P_k\}$ are: **(C1)** $J_k(k-1) \geq 0$ and $J_{k+1}(k) \leq 0$; **(C2)** $\forall l, 0 \leq l \leq k-2$, $J_{l+1}(l) > 0$; **(C3)** $\forall m$, $k+2 \leq m \leq n$, $J_m(m-1) < 0$. Lemma 4.2 rejects all coalitions composed of firms from the two associations; Lemma 4.3, together with condition (C2) [resp. (C3)], shows that no coalition of members of the type-a [resp. type-b] association can sustain a credible deviation. This implies that the unique Nash equilibrium is immune to coalitional deviation and is thus the unique CPNE of game G.

Claim 2: $\mathcal{NE}(G) = \{P_k, P_l\} \Rightarrow \mathcal{CPNE}(G) \neq \emptyset$. Suppose that $l > k + 1$. The proof of the claim is done in three steps. **Step 1**. Since $P_k \in \mathcal{NE}(G)$ and there is no Nash equilibrium with a smaller type-a association, we have that (i) $\forall j$, $1 \leq j \leq k$, $J_j(j-1) > 0$, and (ii) $J_{k+1}(k) < 0$. Similarly, because $P_l \in \mathcal{NE}(G)$ and there is no Nash equilibrium with a larger type-a association, we also have that (iii) $J_l(l-1) > 0$, and (iv) $\forall m, l+1 \leq m \leq n$, $J_m(m-1) < 0$. This implies, together with Lemma 4.3, that P_k [resp.

P_l] is immune to the deviations by all coalitions included in A_k [resp. in $N \setminus A_l$] and by the coalitions included in $N \setminus A_k$ [resp. A_l] with a size strictly larger than $l - k$. Hence, P_k [resp. P_l] still has to be tested against deviations by coalitions included in $N \setminus A_k$ [resp. A_l] with a size not larger than $l - k$. **Step 2**. Suppose that $P_k \notin \mathscr{CPNE}(G)$ because some coalition of size $s \leq l - k$ included in $N \setminus A_k$ is able to deviate in a self-enforcing way. The coalition which is the most likely to sustain such a credible deviation is $S = \{k + 1, k + 2, \ldots, k + s\}$. Because the deviation by S is self-enforcing, we have that $J_{k+s}(k + s - 1) \geq 0$, which implies (from the fact that $\mathscr{NE}(G) = \{P_k, P_l\}$ and from expression (10)) that $\forall j, s - 1 \leq j \leq l - k - 2, J_{k+j+1}(k + j) > 0$. Now, considering P_l and applying Lemma 4.3, it is easy to show that no coalition of size t included in A_l, with $2 \leq t \leq (l - k) - s + 1$ is able to deviate in a self-enforcing way from P_l. A similar argument holds when we start from $P_l \notin \mathscr{CPNE}(G)$. Hence, we have established that if a size-s coalition is able to deviate in a self-enforcing way from one Nash equilibrium, then the other Nash equilibrium is immune to deviations by coalitions of a size comprised between 2 and $(l - k) - s + 1$. **Step 3**. Continue to suppose that $P_k \notin \mathscr{CPNE}(G)$ because of the credible deviation by S. We know that the only coalitions that could possibly deviate in a self-enforcing way from P_l have a size strictly greater than $(l - k) - s + 1$; the best candidates are $V = \{k + s - v, k + s - v + 1, \ldots, l\}$, (with $v \geq 1$). Let $W = S \cap V$. Consider now the reduced game induced on S by the deviation from P_k; since the deviation by S is self-enforcing, it is a CPNE in the reduced game. Therefore, no single member of $W \subset S$ and no subcoalition of members of W is able to sustain a credible deviation from the initial common deviation from P_k. In other words, in the case where all members of W do not go along with the other firms in S in the deviation from P_k, we know that this situation is not stable in the sense that either some firm or a group of firms in W will have an incentive to deviate from it. But this situation is precisely the one that would result from the deviation by V from P_l. We have thus shown that the deviation by V cannot be self-enforcing since there are firms in W that can sustain a credible deviation from it. Again, a similar argument holds when we start from $P_l \notin \mathscr{CPNE}(G)$. This completes the proof of claim 2.

Claim 3: *$\mathscr{CPNE}(G) \neq \emptyset$, whatever its cardinality.* Let, e.g., $\mathscr{NE}(G) = \{P_k, P_l, P_m\}$ with $k < l < m$. Let us show that $P_k, P_l \notin \mathscr{CPNE}(G) \Rightarrow P_m \in \mathscr{CPNE}(G)$. Suppose first that $P_k \notin \mathscr{CPNE}(G)$. From claims 1 and 2, we know that there are two possible cases.

Case 1. P_k is disqualified by $S \subseteq (A_l \setminus A_k)$. From claim 2, this makes P_l immune to all coalitions in A_l. So, P_l can only be disqualified by some coalition included in $N \setminus A_l$. But then, from claim 2 again, P_m is immune to any coalitional deviation and we are done.

Case 2. P_k is disqualified by $S' = (A_l \setminus A_k) \cup T$, where $T \subseteq (A_m \setminus A_l)$; this implies that $P_l \notin \mathcal{CPNE}(G)$ (because T is able to sustain a credible deviation from P_l; if this were not true, the deviation by S' from P_k would not be credible). But then, from the arguments of claim 2, we have necessarily that $P_m \in \mathcal{CPNE}(G)$. Similar arguments can easily be replicated in order to show that whatever the number of Nash equilibria, it is always true that at least one of them is a CPNE. ■

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