A ROBUST AND EFFICIENT TECHNIQUE FOR DEALING WITH TIME-VARYING INSTRUMENTAL BIAS IN LINEAR FILTERING

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Abstract

A unifying approach is formulated for estimating zero bias errors on input signals of linear discrete dynamic systems in real time applications.
Generally, these parameters are treated by extending the state vector, hence introducing eigenvalues on the unit circle and uncontrollable modes. These two effects cause convergence difficulties of the Ricatti Equation (RE). In the paper, a technique has been developed to make the system maximally controllable with minimal system modification. Hereby, 'good' convergence behaviour of the RE is assured and linear forgetting is established automatically for the bias parameters. This allows to deal with constant or slowly time-varying zero bias errors.
Furthermore, a new implementation of the 'square root covariance filter' algorithm is described. It uses 'condensed forms' (Hessenberg or Schur) of the system structure and exploits the modelling of the bias terms. This leads to substantial savings in computing time compared to the conventional Kalman Filter implementation, with numerical superiority.
The resulting technique is demonstrated by simulation on the flight-path reconstruction problem that occurs in a two-step identification method. Here the system model is inherently 'condensed'.

1. Introduction

The system model used in INS (Inertial Navigation Systems) or in the flight-path reconstruction problem of a two-step aircraft model identification method is accurately known. For a given stationary flight reference condition the model is linear, time-invariant with known system matrices [1,2]. This mathematical system description is ideally suited for Kalman-Bucy recursive filter techniques [3] to reconstruct the aircraft's flight path (or the state of the system). In practical applications, it is well known that the filter estimates are only useful over a restricted period of time, depending on the accuracy of the inertial measurement system used. One of the main reasons is the presence of accelerometer zero bias errors and gyro drifts. To reduce the estimation errors caused by these parameters ,it is a common practice to extend the state of the original system with the bias errors, assuming the latter parameters are constant. In this case, the filter will estimate both the states of the original system and the bias terms.
Until now, a lot of effort is spent in the development of efficient computational techniques [4] to solve the extended filtering problem. These techniques are based on the generalized partitioning estimation method of Lainiotis [5], to separate the estimation of the state variables from the bias terms.
However, the influence of this modelling on the numerical behaviour of the discrete RE is not yet discussed. But simulations studies have already shown that it may lead to severe deterioration of the estimation results, especially in real-time applications.

In this paper, the influence of extending the state vector by the input zero bias errors on the convergence of the RE as well as on the accuracy of the estimation results will be analyzed.
In section 2, the problem of extending the state description to include the zero bias errors on the input signals is formulated. Also the special features of the mathematical model used in the simulation to reconstruct the aircraft's flight-path are discussed in this section. The method to make the corresponding modes of the zero bias terms controllable is given in section 3, as well as the effect on the convergence of the Ricatti Recursion (RR). An efficient implementation of the results of section 3 in a more reliable Square Root Covariance Filter (SRCF) will be given in section 4. In section 5, we will demonstrate the good performance of the derived algorithm in the case of simulated constant and linear time-varying bias errors. Section 6, presents the conclusions of this analysis.

2. The extended state space description

Let us consider the following discrete time-invariant system, that also appears in the flight-path reconstruction model [1,2]:

$$\text{process:}\quad x_{k+1} = A\,x_k + F\,u_k + B\,[w_k + b_k] \tag{1}$$

$$\text{observation:}\quad y_k = C\,x_k \tag{2}$$

where $x_k \in R^n$, $w_k \in R^m$ and $v_k \in R^p$. The deterministic input signal is given by u_k. The sequence w_k and v_k are Gaussian uncorrelated white noise with zero mean and covariances Q_k and R_k, respectively. The input is contaminated with bias errors b_k R^m assumed to be constant, i.e.

$$b_{k+1} = b_k \tag{3}$$

The inclusion of these bias errors in an extenoed state vector, gives rise to the following model:

$$\begin{bmatrix} b \\ x \end{bmatrix}_{k+1} = \begin{bmatrix} I_m & 0 \\ B & A \end{bmatrix} \begin{bmatrix} b \\ x \end{bmatrix}_k + \begin{bmatrix} 0 \\ B \end{bmatrix} w_k \tag{4}$$

$$y = [0 \quad C] \begin{bmatrix} b \\ x \end{bmatrix}_k + v_k \tag{5}$$

or,

$$x_e(k+1) = A_e\; x_e(k) + B_e\, w_k \tag{6}$$

$$y_k = C_e\; x_e(k) \tag{7}$$

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where I_m is an (m,m) unit matrix
and the deterministic input signal is left out for the sake of brevity.

At this point, it should be remarked that prior to solving any problem it should be verified that the extended state space model only includes those bias errors that are observable. The number of observable bias errors is $nb \leq m$. From the extended state description (6-7), the following observations can be drawn directly:

1. the inclusion of the bias terms introduces a set of nb eigenvalues on the unit circle.
2. the state matrices (Ae,Be) clearly have nb uncontrollable modes corresponding to the bias parameters.

On the other hand, the model (6-7) can be used to estimate the state quantity x_e, that is the original state and the zero bias terms, by linear filtering [3].
It is the aim of this paper to investigate the implications of the observations (1) and (2) on this method and to formulate efficient and reliable implementations for the RR.
The flight-path reconstruction model of a two-step aircraft model identification method [1] will be used in the experiments to evaluate the results.
This model describes the perturbation of motion from a stationairy flight condition, based on Newton's second law, when accelerometer and rate gyro signals are defined as input signals [1].
First, this model allows to reconstruct the flight path by linear filtering of a stochastic process. Secondly, the system structure of this model given as in eq. (1-2) is inherently condensed, i.e. the state matrix A is in lower Schur form:

$$A = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -g\sin\gamma_0\Delta t/V_0 & 1 & 0 & 0 \\ -g\cos\gamma_0\Delta t & 0 & 1 & 0 \\ V_0\cos\gamma_0\Delta t & -V_0\cos\gamma_0\Delta t & \sin\gamma_0\Delta t & 1 \end{pmatrix} \quad (8)$$

where the quantities γ_0 and V_0 and the state quantities giving rise to this model are defined in figure 1 and table 1, and Δt is the discretization period. The state in this table also includes those bias parameters that are observable, referring to the extended model (6-7).

	quantity	symbol	unit
input	acceleration along X-axis	A_x	m/s^2
	acceleration along Z-axis	A_z	m/s^2
	rate of pitch	q	rad/s
state	angle of pitch	θ	rad
	angle of attack	α	rad
	speed along X-axis	u	m/s
	altitude deviation	Δh	m
	bias on A_x	λ_x	
	bias on A_z	λ_z	
output	speed along X-axis	u	m/s
	altitude deviation	Δh	m

Table 1: System state variables of the mathematical model for flight-path reconstruction.

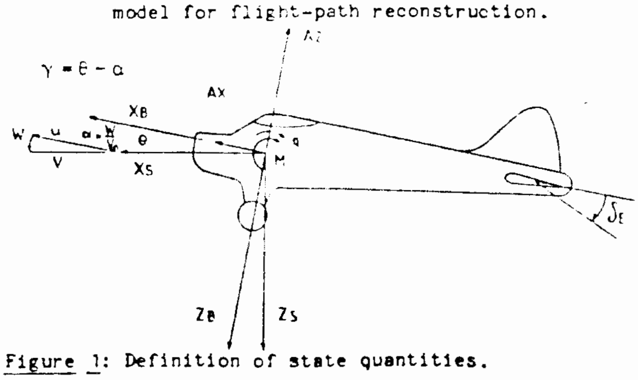

Figure 1: Definition of state quantities.

3. The algorithm to make the bias terms controllable

In [6] it has been shown that the combination of eigenvalues on the unit circle of the system model used in linear filtering and uncontrollability of the corresponding modes has disastruous effect on the RR. First, it makes the recursion very sensitive to numerical round-off, model errors and discretization errors, causing divergence of the RR when using conventional KF techniques. And secondly, when convergence is achieved by using more reliable SRCF techniques, the rate of convergence is very slow. This makes the method not useful for real-time applications.
A general solution to these convergence problems is obtained by making the extended state space model completely controllable.

Since, the only modes we have to check for controllability are equal to one, we can use the Popov-Belevitch-Hautus (PBH) eigenvector test [8] at $\lambda = 1$ only, to verify controllability.

The algorithm can now be summarized into the following two steps:

STEP 1: Calculate a basis Y of all left eigenvectors of the system matrix A_e in the model (6-7), with eigenvalue 1. This can be done by a SVD (Singular Value Decomposition) of the matrix $(A_e - I)$, given as:

$$SVD(A_e - I) = [\, U \mid Y \,] \begin{pmatrix} \Sigma & \\ \hline & 0 \end{pmatrix} V^T \quad (9)$$

STEP 2: Calculate the perturbation of the input distribution matrix B_e, i.e. ΔX according the following theorem.

Theorem:

The perturbation ΔX,

$$\Delta X = Y\, [\, U' \begin{pmatrix} \sigma_1' & & & \\ & \ddots & & 0 \\ & & \sigma_{nb}' & \end{pmatrix} V' \,] \quad (10)$$

with σ_1' to $\sigma_{nb}' \geq$ sigma

$U'(nb,nb), V'(m,m)$ given by a SVD of $Y^T B_e = U' \Sigma' V'^T$

of the input distribution matrix B_e of the model (6-7) ensures that $\|Y^T(B_e + \Delta X)\|_2 >$ sigma (where $\|.\|_2$ stands for the 2-norm). Proof:(left out here)

Corollary 1: The perturbation ΔX guarantees full column rank for the matrix $Y^T(B_e + \Delta X)$ that appears in the PBH test of which its smallest singular value is greater than the specified value sigma.

Corollary 2: Based on the definition of a general measure of controllability of the system (A,B), i.e. $\mu(A,B)$ [8], given by:

$\mu(A,B) = \min(\|\Delta A\|_2, \|\Delta B\|_2)$ such that the system defined by $(A+\Delta A, B+\Delta B)$ is uncontrollable.

the ΔX gives exactly the 'distance' of the controllable system $(A_e, B_e+\Delta X)$ to any uncontrollable system, and therefore also to (A_e, B_e). The above construction of ΔX thus gives a "minimal" perturbation achieving "maximal" controllability.

Corollary 3: The procedure of making the system controllable corresponds to inserting more input noise to the zero bias errors. This allows the corresponding entries of the state error covariance matrix to increase linearly. (selective linear forgetting)

The influence of this perturbation on the convergence rate of the RR in the SRCF (to be described in section 4) is illustrated in figure 2.
Let σ_{min} be the smallest singular value of the controllability matrix of the original system (A,B),

namely,

$$[B \quad A B \quad \dots \quad A^{n-1} B]$$

Then, for $|\Delta X|_2 < \sigma_{min}$, (1e-3 in our flight-path recontruction problem) the rate of convergence is slowed down compared to the rate that occured when only the original system was considered (see figure 2 for $|\Delta X|_2$ = 1e-3).
With larger perturbations, the rate of convergence is increased compared to the original rate. See figure 2 for $|\Delta X|_2$ = 1e-1.
Moreover, a similar improvement on the accuracy of the computed estimates is obtained as we will describe in section 5.

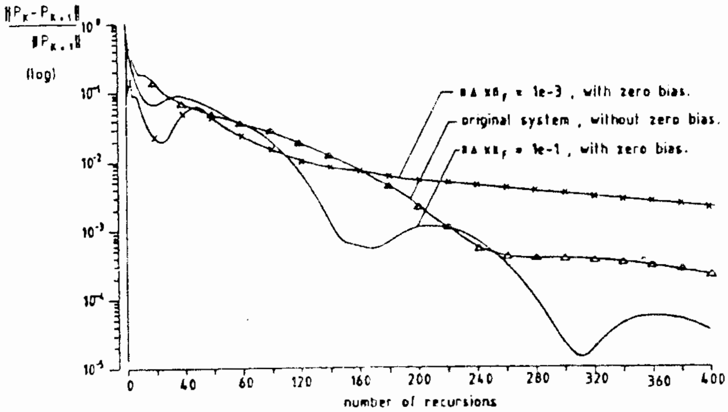

Figure 2: Rate of convergence of the Discrete Riccati Equation for different perturbations of B_e.

4. An efficient Square Root Covariance Filter implementation

The algebraic RE that appears in the linear optimal filtering problem formulates a recursion for the one-step predicted state covariance matrix, denoted by P_k.
The SRCF propagates the square root of P in time. If the square roots S, $R^{1/2}$ and $Q^{1/2}$ are chosen to be lower triangular:

$$P_k = S_k S_k^T \tag{11}$$

$$R = R^{1/2} R^{T/2} \tag{12}$$

$$Q = Q^{1/2} Q^{T/2} \tag{13}$$

then the computational scheme for the SRCF is summarized below:

$$T \begin{pmatrix} R^{T/2} & 0 \\ 0 & Q^{T/2}B^T \\ S_k^T C & S_k^T A^T \end{pmatrix} = \begin{pmatrix} R_k^{e\,T/2} & K_k^T A^T \\ 0 & S_{k+1}^T \\ 0 & 0 \end{pmatrix} \tag{14}$$

(pre-array) (post-array)

with the one-step predicted state estimate $\hat{x}_{k+1}$ given by:

$$\hat{x}_{k+1} = A\,[\hat{x}_k + K_k R_k^{e\,-1/2} (y_k - C\,\hat{x}_k)] \tag{15}$$

Here T is an arbitrary orthogonal transformation that triangularizes the pre-array. First, the special structure of the extended state space model (6-7) and the triangular structure of the original system (1-2) allows to construct the pre-array very efficiently, i.e. to save computing time in the calculation of the products $S_k^T C^T$, $S_k^T A_e^T$, $Q^T(B_e + \Delta X)$. The special forms of these matrices is visualized in the example given in eq. 16.
Secondly, this particular structure of the pre-array is exploited to derive an efficient triangularization. This is briefly outlined in the following example.

Example:

For arbitrary dimensions of the system quantities, resp. n=2, m=3, p=2 amd nb=2 (as appears in the flight-path reconstruction problem) the pre-array has the following form:

$$\left(\begin{array}{cc|cc|cc} x & x & 0 & 0 & 0 & 0 \\ & x & 0 & 0 & 0 & 0 \\ \hline & & x_1 & x & x & x \\ & & 0 & x_2 & x & x \\ & & 0 & 0 & x_3 & x \\ \hline x_7 & x_8 & x_1 & x_2 & x_3 & x_4 \\ x_5 & x_6 & & x_2 & x_3 & x_4 \\ x_3 & x_4 & & & x_3 & x_4 \\ x_1 & x_2 & & & & x_4 \end{array}\right) \tag{16}$$

where 'x' denote the non-zero entries
The anihillation of the (1,3) block in this array is done by Givens orthogonal transformation in the order denoted by the subscripts. In this way the special structure of the remaining blocks is preserved.
Further triangularization is then done by using Householder transformations that operates on low dimensional matrices in the order given by the subscripts in the blocks (2,2);(2,3);(3,2) and (3,3). Depending on the proportion of the system dimensions this 'new' filter implementation can lead to considerable reductions in the computational burden. Again in the flight-path reconstruction problem, discussed in section 2, a saving of 40% computation time resulted each recursion compared to the conventional KF technique.

5. Performance analysis

In section 3, we highlighted the positive effects of making the corresponding modes of the zero bias errors controllable on the rate of convergence of the RE.
The additional advantages of this method in real time application will be demonstrated in this section. In these circumstances the bias parameters may no longer be assumed constant, but they can vary slowly in time.

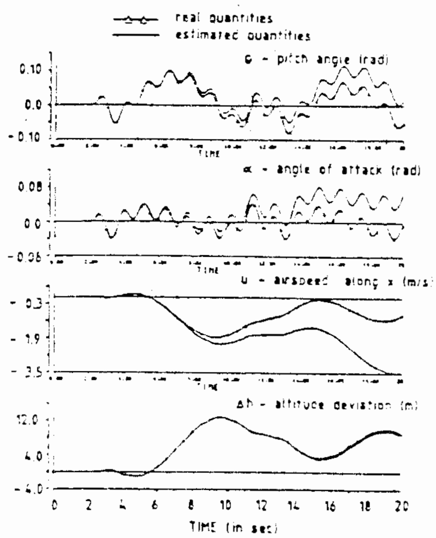

Fig.3 Estimated flight-path with $\|\Delta x B_f\| = 0$ for linear time varying bias errors λz, λq

Using the reliable SRCF technique, without application of the algorithm of section 3, leads to severe deterioration of the estimation results in time, when the simulated bias errors and vary slowly and linearly in time as indicated in figure 6. This is clearly confirmed by the reconstruction results in figure 3 and the norm of $(y_m - C\hat{x})$ (SSR=Sum of Square Roots) in figure 4.

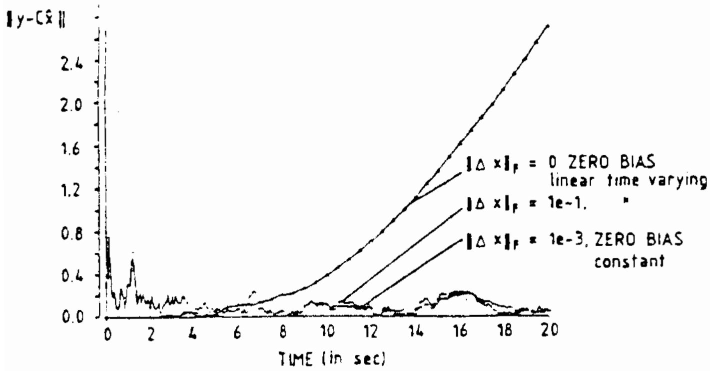

Fig.4: The norm of the residuals for different filter circumstances

On the other hand, applying the technique of section 3, where the sigma parameter can be adapted in relation to the SSR magnitude, enables the SRCF to correct the state reconstruction results very accurately. First, in the case of constant bias terms the SSR values decreases (see figure 4) with time. Secondly, in the case of linear variation of the bias errors the SSR does not increase.

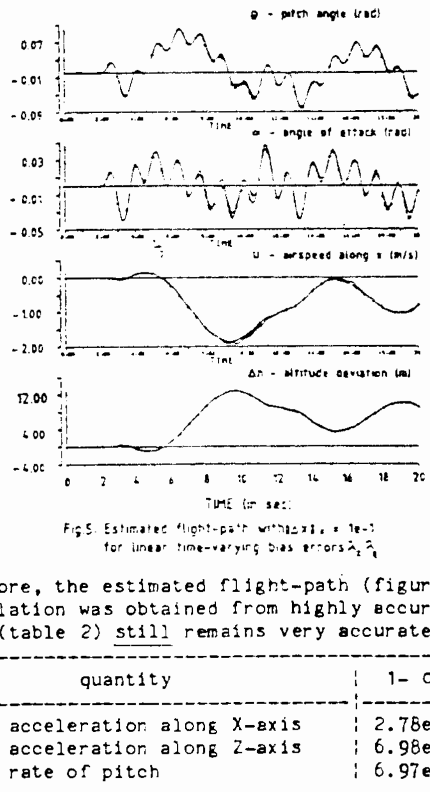

Fig.5: Estimated flight-path with‖Δx‖F = 1e-1 for linear time-varying bias errors λ_z λ_q

Furthermore, the estimated flight-path (figure 5) which in the simulation was obtained from highly accurate inertial signals (table 2) still remains very accurate.

quantity	1- σ	unit
acceleration along X-axis	2.78e-3	m/s^2
acceleration along Z-axis	6.98e-3	m/s^2
rate of pitch	6.97e-5	rad/s

Table 2: Measurement error statistics of the inertial signals.

Hereby, the applied method of section 3 not only allows the filter algorithm to follow small time variations of the observable bias errors (see figure 6) it also does not deteriorate the reconstructed state (i.e. flight-path) accuracy.

This is opposite when using exponential weighting [9]. In this case, already for small forgetting factors the estimation results were highly inaccurate and hence not useful for any further manipulation.

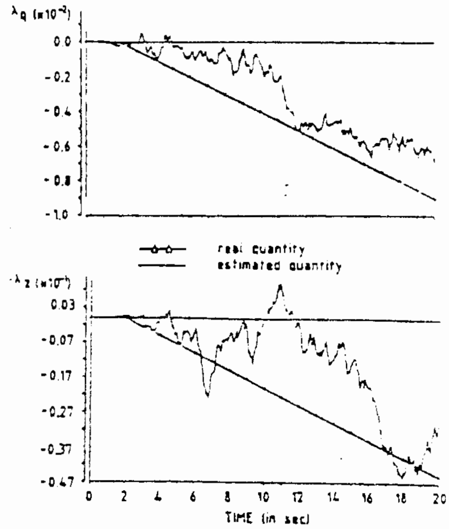

Fig.6: Estimation results of the Observable zero bias errors with‖Δx‖F = 1e-1

6. Conclusions

In this paper an algorithm is derived to calculate the perturbation of the input distribution matrix of the extended state space model.

This perturbation first improves the rate of convergence of the Discrete Riccati Equation and secondly allows the linear filtering procedure to estimate observable slowly time-varying bias errors on the input signals.

The spectral norm of the perturbation, specified by the value sigma can be related to the norm of the residuals of the linear filter. This in combination with the formulation of very time consuming and reliable SRCF implementation clearly make the proposed linear filter technique attractive for on-line application. This is demonstrated by simulations in a flight-path reconstruction problem.

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