SOME FIRST STEPS TOWARDS 2-REPRESENTATION THEORY OF COXETER GROUPS

OR:

THE 'NEXT GENERATION' OF REPRESENTATION THEORY OF COXETER GROUPS!?

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ABSTRACT. This talk is an introduction as well as a survey about 2-representation theory of Coxeter groups. The motivation to study such 'higher' representation is as follows:

In groundbreaking work Chuang-Rouquier and, independently, Khovanov-Lauda introduced 2-representation theory of Lie algebras and their quantum analogs.

As a 'higher' version of classical representation theory of Lie groups, their ideas have already led to many successive works as well as applications throughout mathematics (and physics).

Historically speaking, what 'should have come first' is 2-representation theory of finite groups. Sadly the story of 'higher' representations of finite groups is not understood at all at the moment.

But, quite recently, Mazorchuk-Miemietz made very good progress towards a 'higher' analog of representations of finite-dimensional algebras (as e.g. groups rings of finite groups), and applied it quite successful to one of the most well-behaved family of finite groups: finite Coxeter groups. Already in this case a lot of interesting new phenomena show up, most of which are neither present in classical representation theory nor in the story of Chuang-Rouquier, Khovanov-Lauda, and which might lead to interesting connections and applications in the years to come.

However, in this talk we will focus on one particular (completely explicit) example, i.e. the first non-trivial family of examples given by the dihedral groups, where already several new phenomena are visible.