

Joint seminar UCL - KU Leuven

March - April 2015

C*-simplicity (after Breuillard, Kalantar, Kennedy, Ozawa)

Web site of the seminar

Schedule

Wednesday March 18	Monday March 30	Monday April 20	Monday April 27
Leuven Room B.02.18 Map to building B	Louvain-la-Neuve Room Cycl 08 Map to building CYCL	Leuven Room S.00.03 Map to building S	Louvain-la-Neuve Room Cycl 08 Map to building CYCL
10h00-11h00: Talk 1.a 11h30-12h30: Talk 1.b	14h00-15h00: Talk 2.a 15h30-16h30: Talk 2.b	14h00-15h00: Talk 3.a 15h30-16h30: Talk 3.b	14h00-15h00: Talk 4.a 15h30-16h30: Talk 4.b

Talk 1.a : An introduction to C*-algebras and C*-simplicity

Speaker: Stefaan Vaes

For more background, see [3, Chapter VIII], [1, Appendix F] and [5, Sections 1 and 2].

- C*-algebras, spectral theory, commutative C*-algebras.
- The full and reduced C*-algebra of a locally compact group G .
- Weak containment of representations.
- Amenability.
- C*-simplicity viewed as follows: every representation that is weakly contained in the regular representation is weakly equivalent to the regular representation.

Talk 1.b : The Powers property and C*-simplicity

Speaker: François Le Maître

See [4, Sections 1 and 2] and [5, Section 3].

- The Powers property for a discrete group G .
- If a discrete group G has the Powers property, then $C_r^*(G)$ is simple and has a unique trace.
- Examples, including free groups and free products.

Talk 2.a : The Furstenberg boundary of a locally compact group

Speaker: Pierre-Emmanuel Caprace

- Basic definitions and examples: minimality, (strong) proximality.
- Existence and uniqueness of the Furstenberg boundary.
- The “fundamental lemma” (see e.g. [7, Lemma 4]).
- Compact convex G -spaces contain G -boundaries (see e.g. [7, Proposition 5]).
- The kernel of the action of G on its Furstenberg boundary is the amenable radical of G .

Talk 2.b : The Hamana boundary of a discrete group

Speaker: Stefaan Vaes

Sections 2 and 4 in [7], only for discrete groups.

- G -injective envelopes of G -operator systems.
- Identification of the Furstenberg boundary and the Hamana boundary.
- The C^* -algebra $C_r^*(G)$ has a unique trace if and only if G has a trivial amenable radical.

Talks 3.a and 3.b : The abstract C^* -simplicity criteria

Speakers: Tim de Laat and Peter Verraedt

Section 5 and Theorem 18 in [7].

- Enough background material on crossed product C^* -algebras.
- Let G be a discrete group. The C^* -algebra $C_r^*(G)$ is simple if and only if G admits a topologically free boundary action.
- Let G be a discrete group. If $C_r^*(G)$ is not simple, then G has an amenable subgroup H such that $\bigcap_{g \in F} gHg^{-1}$ is nontrivial for every finite subset $F \subset G$.

Talk 4.a : C^* -simplicity, bounded cohomology, ℓ^2 -Betti numbers

Speaker: David Hume

Section 3.1 in [2]. If G is a discrete group with trivial amenable radical such that either

- G has non-trivial bounded cohomology, or
- G has a non-vanishing ℓ^2 -Betti number.

Then $C_r^*(G)$ is simple.

Talk 4.b : C^* -simplicity for linear groups

Speaker: Phillip Wesolek

Section 3.2 in [2]. A linear group is C^* -simple if and only if its amenable radical is trivial.

References

- [1] B. Bekka, P. de la Harpe and A. Valette, Kazhdan's property (T). *New Mathematical Monographs* **11**, Cambridge University Press, Cambridge, 2008. Preprint version available here.
- [2] E. Breuillard, M. Kalantar, M. Kennedy and N. Ozawa, C^* -simplicity and the unique trace property for discrete groups. *Preprint*. arXiv:1410.2518
- [3] J.B. Conway, A course in functional analysis. Second edition. *Graduate Texts in Mathematics* **96**, Springer-Verlag, New York, 1990.
- [4] P. de la Harpe, Reduced C^* -algebras of discrete groups which are simple with a unique trace. In *Operator algebras and their connections with topology and ergodic theory (Buşteni, 1983)*. *Lecture Notes in Mathematics* **1132**, Springer, Berlin, 1985, pp. 230-253.
- [5] P. de la Harpe, On simplicity of reduced C^* -algebras of groups. *Bull. Lond. Math. Soc.* **39** (2007), 1-26. Download.
- [6] M. Kalantar and M. Kennedy, Boundaries of reduced C^* -algebras of discrete groups. *Preprint*. arXiv:1405.4359
- [7] N. Ozawa, Lecture on the Furstenberg boundary and C^* -simplicity. *Lecture notes*. Available here.