Joint seminar KU Leuven – UCLouvain
February - March 2016
Gromov's Polynomial Growth Theorem (after N. Ozawa and Y. Shalom)

Web site of the seminar

Main reference: [7]

Schedule

Monday February 29	Friday March 4	Monday March 14	Friday March 18
Louvain-la-Neuve Room Cycl 09 Map to building CYCL	Leuven Room B.02.18 Map to building B	Louvain-la-Neuve Room E.349 Map to building CYCL	Leuven Room B.02.18 Map to building B
14h00-15h00: Talk 1.a 15h30-16h30: Talk 1.b	14h00-15h00: Talk 2.a 15h30-16h30: Talk 2.b	14h00-15h00: Talk 3.a 15h30-16h30: Talk 3.b	14h00-15h00: Talk 4.a 15h30-16h30: Talk 4.b

Talk 1.a : The statement of Gromov's PGT and the original proof

Speaker: Phillip Wesolek

Reference: [5]

- General structure of the proof.
- Role of Hilbert's fifth problem in original argument.

Talk 1.b : The algebraic part

Speaker: Nicolas Radu

- PGT for soluble groups (following Wolf [12]).
- For PGT, it is enough to show that PG implies virtually indicable (following Tits [11]) — a group is *indicable* if it has an infinite cyclic quotient.

Talk 2.a : The Tits alternative

Speaker: Philip Dowerk

- Overview of the result and its proof ([10] and [6]).
- A finitely generated amenable group with a f.d. linear representation of infinite image is virtually indicable.

Talk 2.b : Reduced cohomology

Speaker: Yuki Arano

- Definitions (Chapter 3 in [1]).
- A compactly generated locally compact group without property (T) has non-vanishing $\overline{H^1}$ (Appendix in [7] or [8] or §3.2 in [1]).
- Harmonic 1-cocycles (§2 in [7] and [2]).

Talks 3.a : Shalom's property H_{FD}

Speaker: Peter Verraedt

- Weakly mixing representations.
- Overview of [9].
- A finitely generated amenable group with H_{FD} is virtually indicable (Theorem 4.3.1 in [9]).

Talks 3.b : Ozawa's proof

Speaker: Tobe Deprez

- §3 and proof of Main Theorem in [7].

Talk 4.a : Random walks

Speaker: Anna Krogager

- Overview of [4].

Talk 4.b : Controlled Følner sequences

Speaker: Adrien Le Boudec

- Overview of [3].
- Proposition in §4 of [7].

References

[1] Bachir Bekka, Pierre de la Harpe, and Alain Valette, *Kazhdan's property (T)*, New Mathematical Monographs, vol. 11, Cambridge University Press, Cambridge, 2008. MR2415834 (2009i:22001)

[2] Ionut Chifan and Thomas Sinclair, *On the ergodic theorem for affine actions on Hilbert space*, Bull. Belg. Math. Soc. Simon Stevin **22** (2015), no. 3, 429–446. MR3396994

[3] Yves de Cornulier, Romain Tessera, and Alain Valette, *Isometric group actions on Hilbert spaces: growth of cocycles*, Geom. Funct. Anal. **17** (2007), no. 3, 770–792, DOI 10.1007/s00039-007-0604-0. MR2346274 (2009g:22007)

[4] Anna Erschler and Anders Karlsson, *Homomorphisms to $\mathbb{R}$ constructed from random walks*, Ann. Inst. Fourier (Grenoble) **60** (2010), no. 6, 2095–2113 (English, with English and French summaries). MR2791651 (2012c:60018)

[5] Mikhael Gromov, *Groups of polynomial growth and expanding maps*, Inst. Hautes Études Sci. Publ. Math. **53** (1981), 53–73. MR623534 (83b:53041)

[6] Pierre de la Harpe, *Free groups in linear groups*, Enseign. Math. (2) **29** (1983), no. 1-2, 129–144. MR702736 (84i:20050)

[7] Narutaka Ozawa, *A functional anaylis proof of Gromov's polynomial growth theorem*, 2015. arXiv:1510.04223.

[8] Yehuda Shalom, *Rigidity of commensurators and irreducible lattices*, Invent. Math. **141** (2000), no. 1, 1–54, DOI 10.1007/s002220000064. MR1767270 (2001k:22022)

[9] ———, *Harmonic analysis, cohomology, and the large-scale geometry of amenable groups*, Acta Math. **192** (2004), no. 2, 119–185, DOI 10.1007/BF02392739. MR2096453 (2005m:20095)

[10] J. Tits, *Free subgroups in linear groups*, J. Algebra **20** (1972), 250–270. MR0286898 (44 #4105)

[11] Jacques Tits, *Appendix to: "Groups of polynomial growth and expanding maps" [Inst. Hautes Études Sci. Publ. Math. No. 53 (1981), 53–73] by M. Gromov*, Inst. Hautes Études Sci. Publ. Math. **53** (1981), 74–78. MR623535 (83b:53042)

[12] Joseph A. Wolf, *Growth of finitely generated solvable groups and curvature of Riemanniann manifolds*, J. Differential Geometry **2** (1968), 421–446. MR0248688 (40 #1939)