Make the right choice

using Markov Decision Processes

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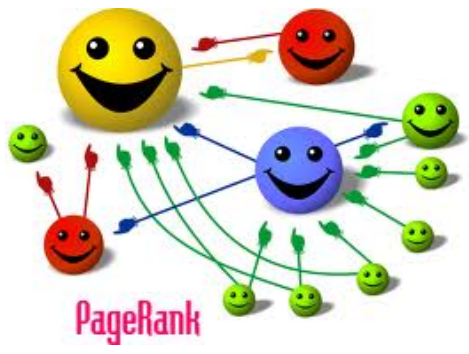
ICTEAM Tutorial Seminars

March 2013

A storage problem

A PageRank problem



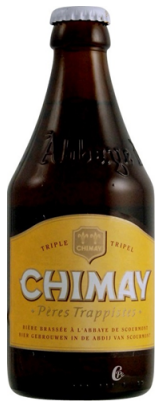
0 Number of beers left 6

0 Number of beers left 6

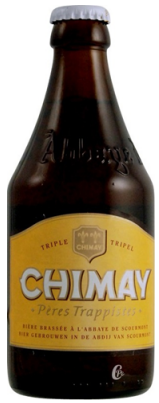
0 Number of beers left 6

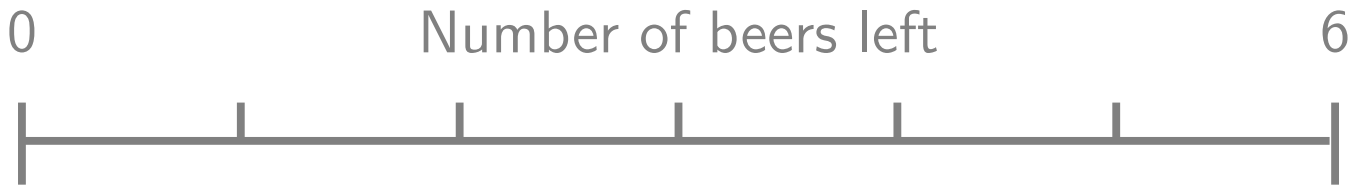
0 Number of beers left 6

0 Number of beers left 6

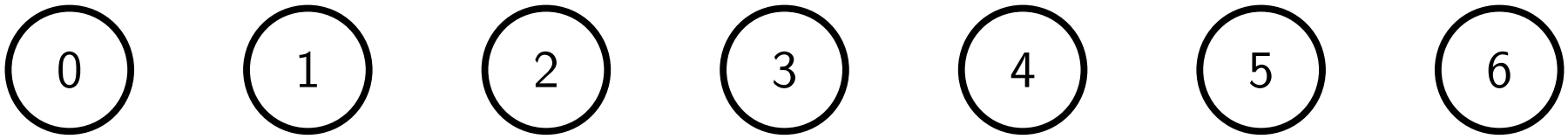


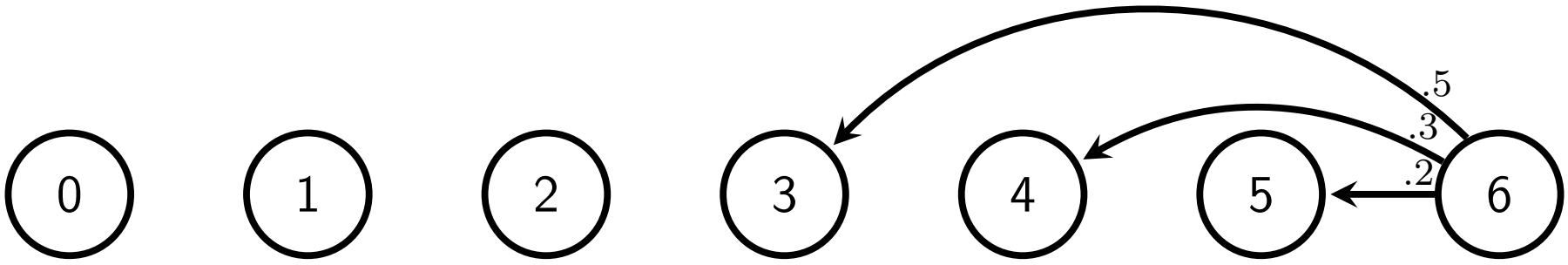
0 Number of beers left 6

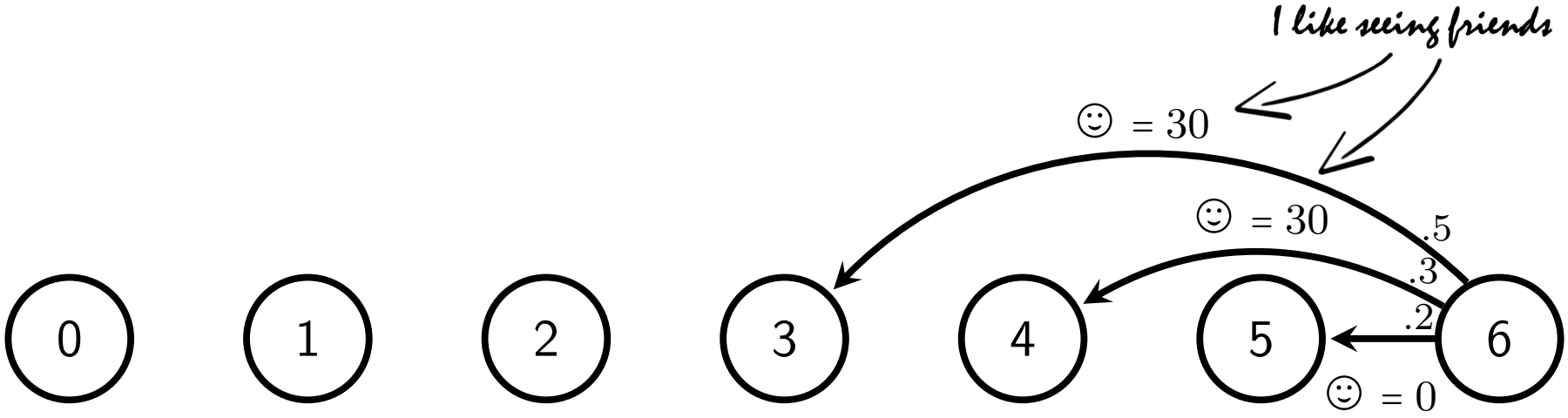


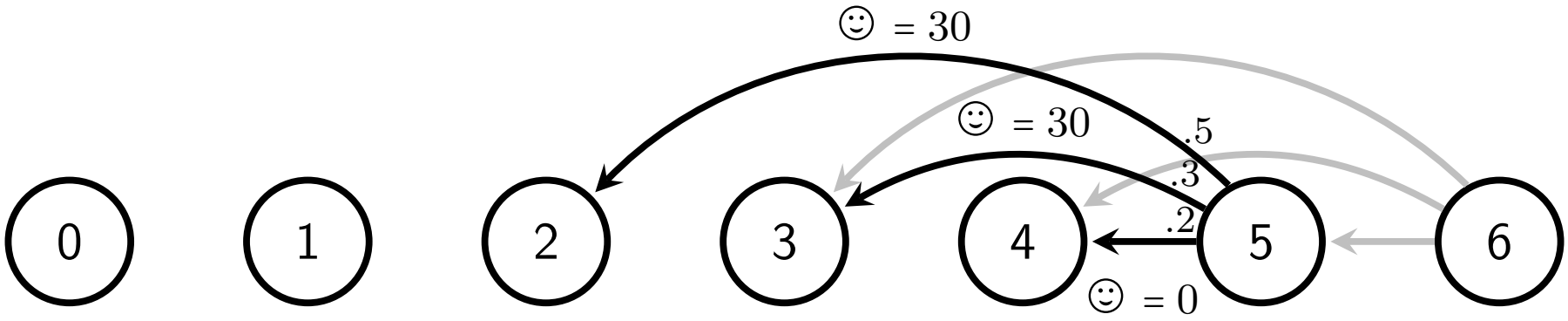

0
Number of beers left
6

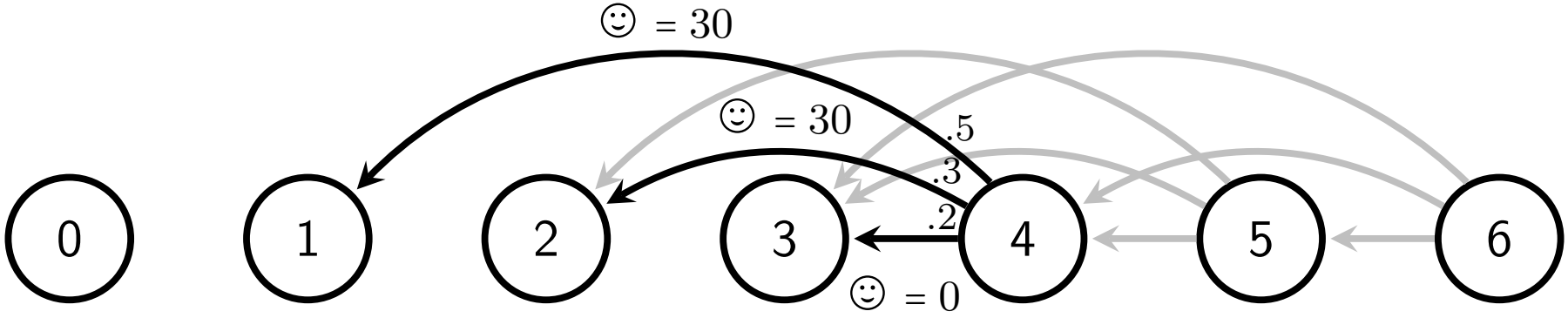

WHAT
THERE'S NO MORE BEER?
memegenerator.net

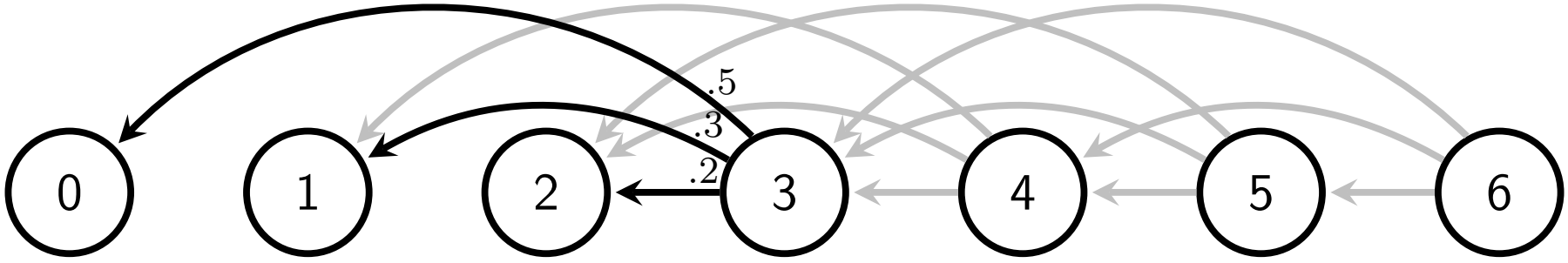

0
1
2
3
4
5
6

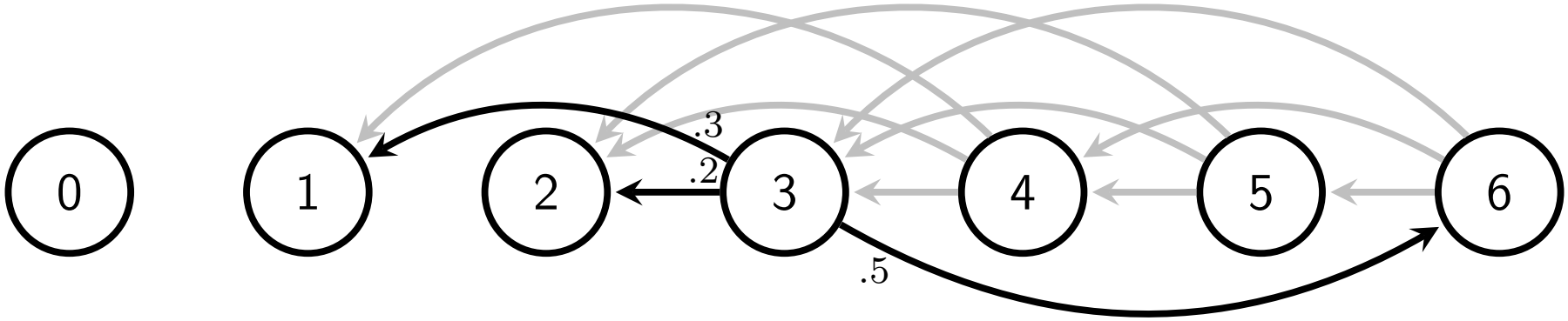

0
1
2
3
4
5
6
.5
.3
.2

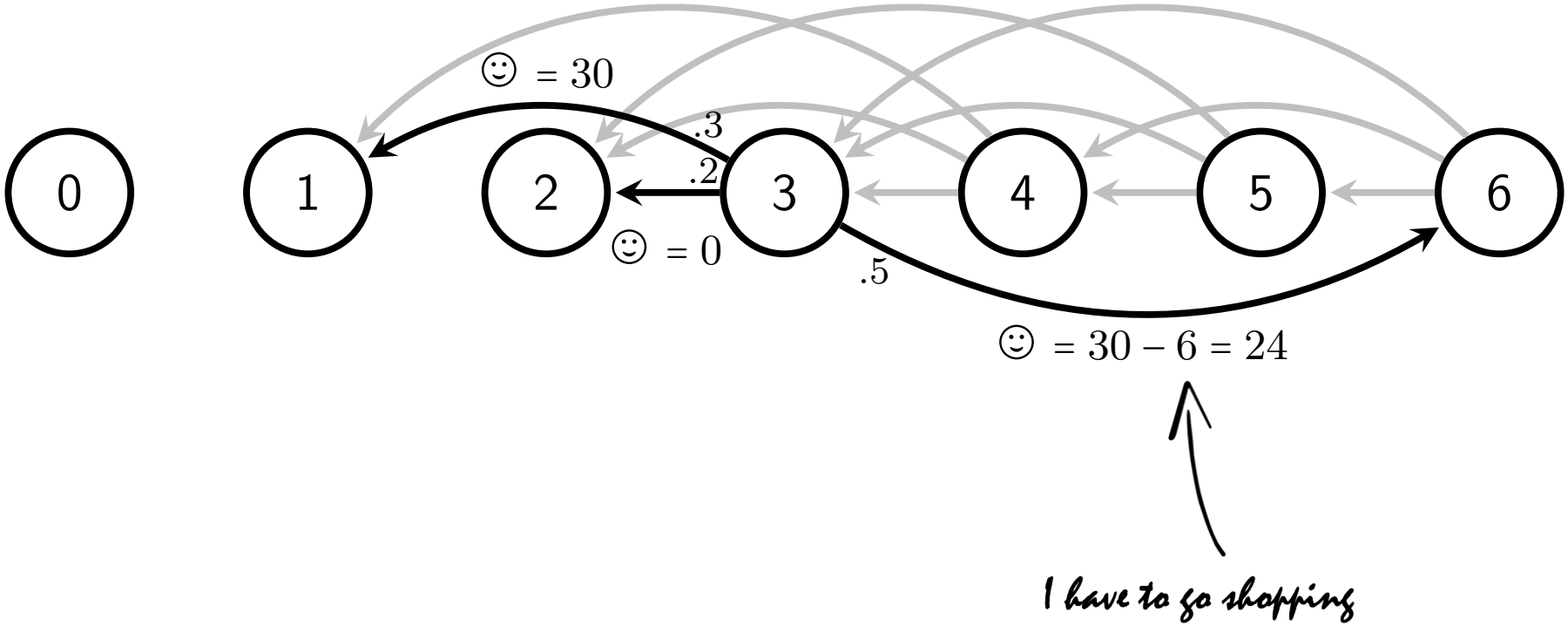

I like seeing friends
☺ = 30
☺ = 30
.5
.3
.2
0
1
2
3
4
5
6
☺ = 0

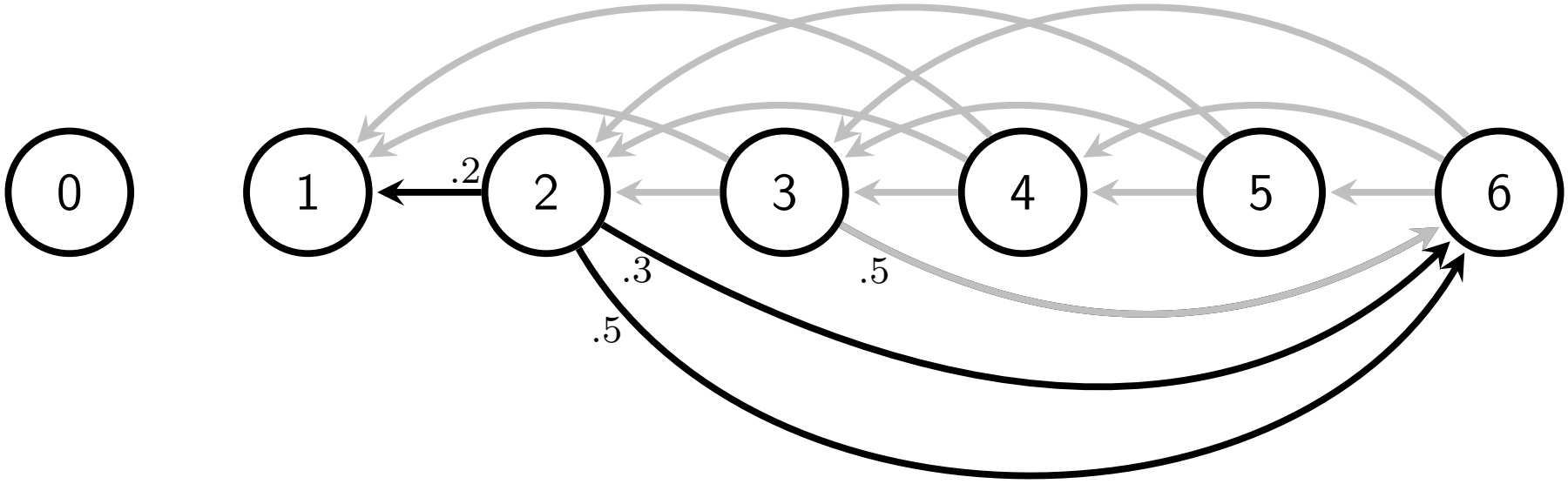

☺ = 30
☺ = 30
.5
.3
.2
0
1
2
3
4
5
6
☺ = 0

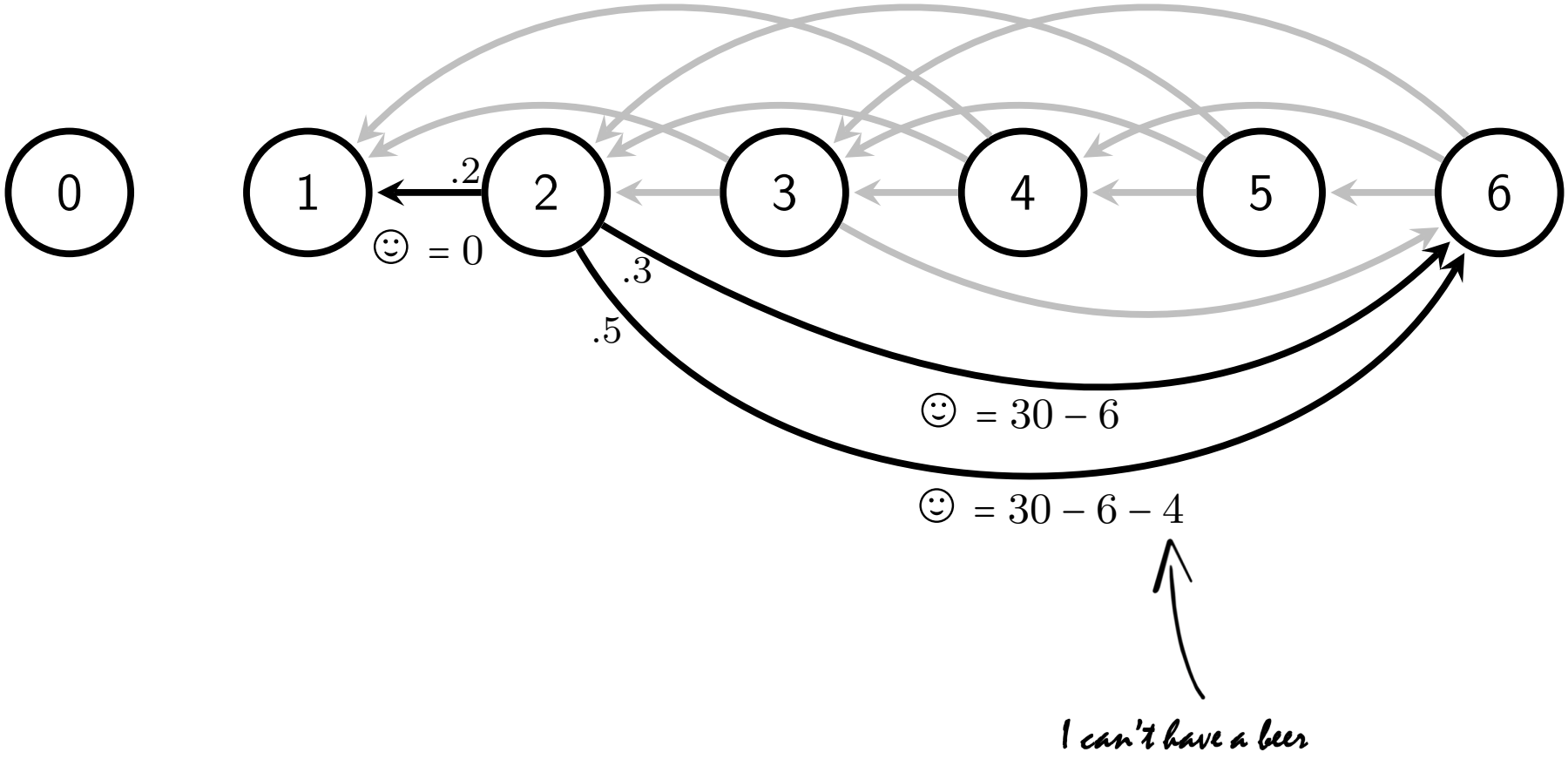

☺ = 30
☺ = 30
.5
.3
.2
0
1
2
3
4
5
6
☺ = 0

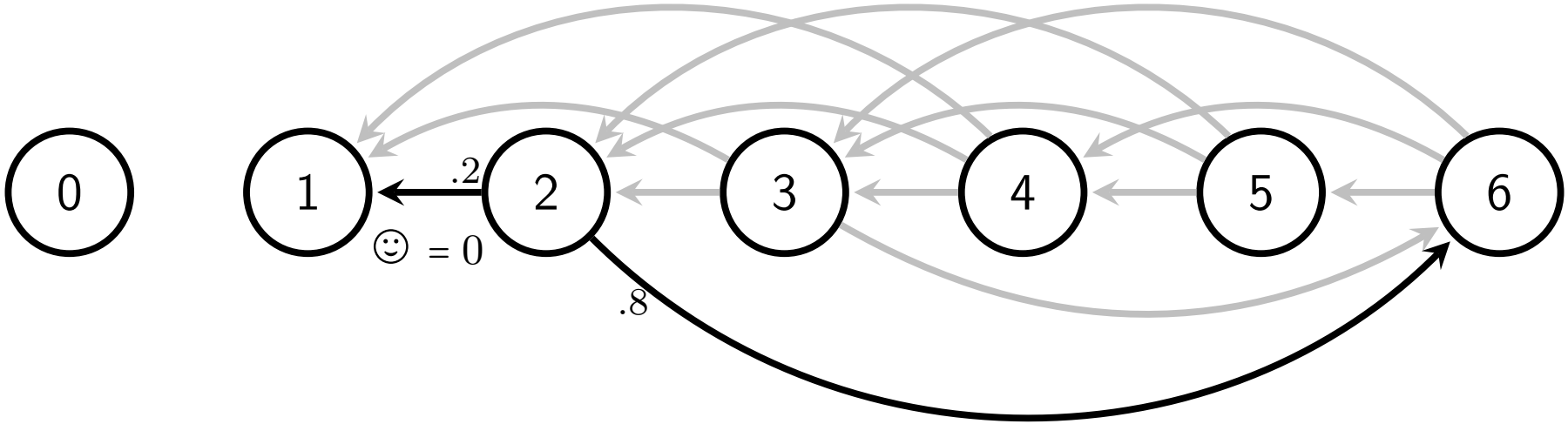

.5
.3
.2
0
1
2
3
4
5
6


0
1
2
3
4
5
6
.3
.2
.5

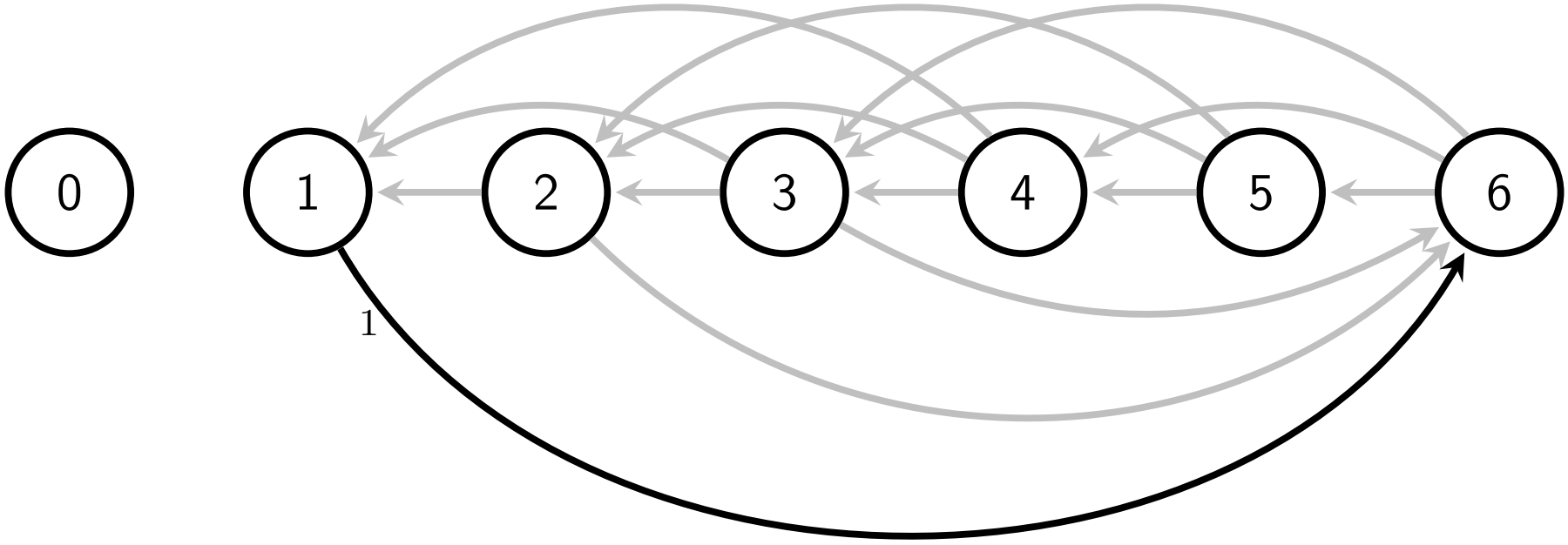

☺ = 30
.3
.2
0
1
2
3
4
5
6
☺ = 0
.5
☺ = 30 – 6 = 24
I have to go shopping

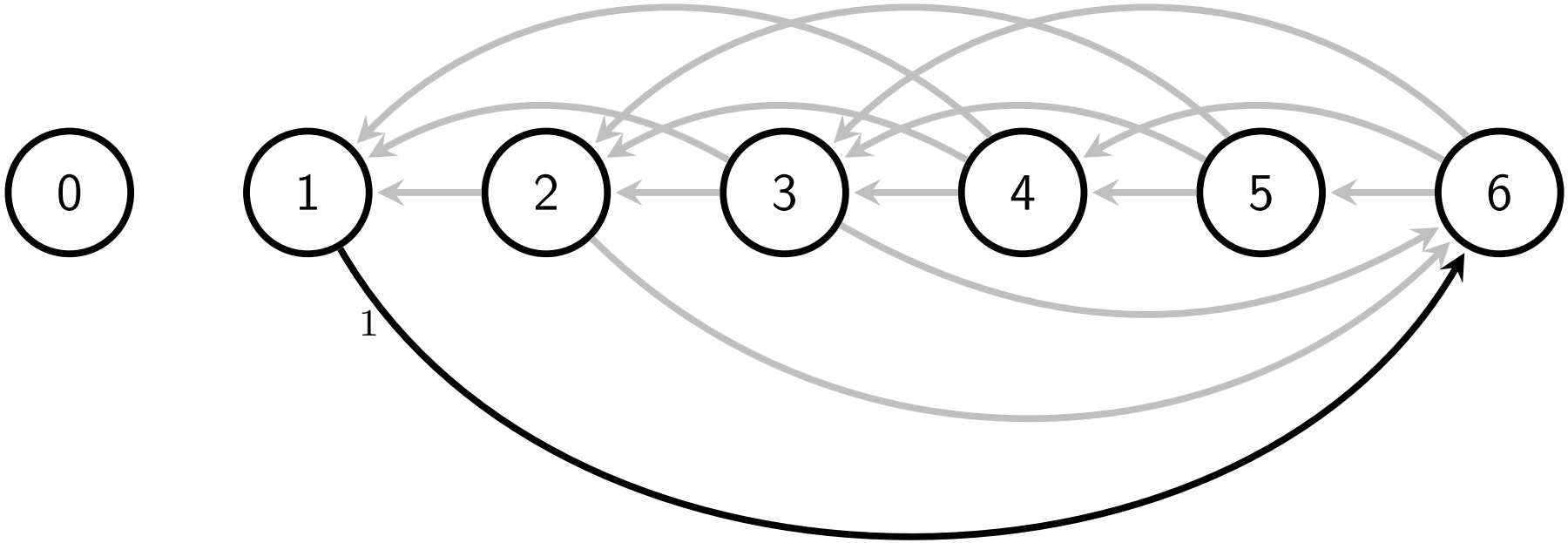

0
1
2
3
4
5
6
.2
.3
.5
.5


0
1
2
3
4
5
6
.2
☺ = 0
.3
.5
☺ = 30 – 6
☺ = 30 – 6 – 4
I can't have a beer



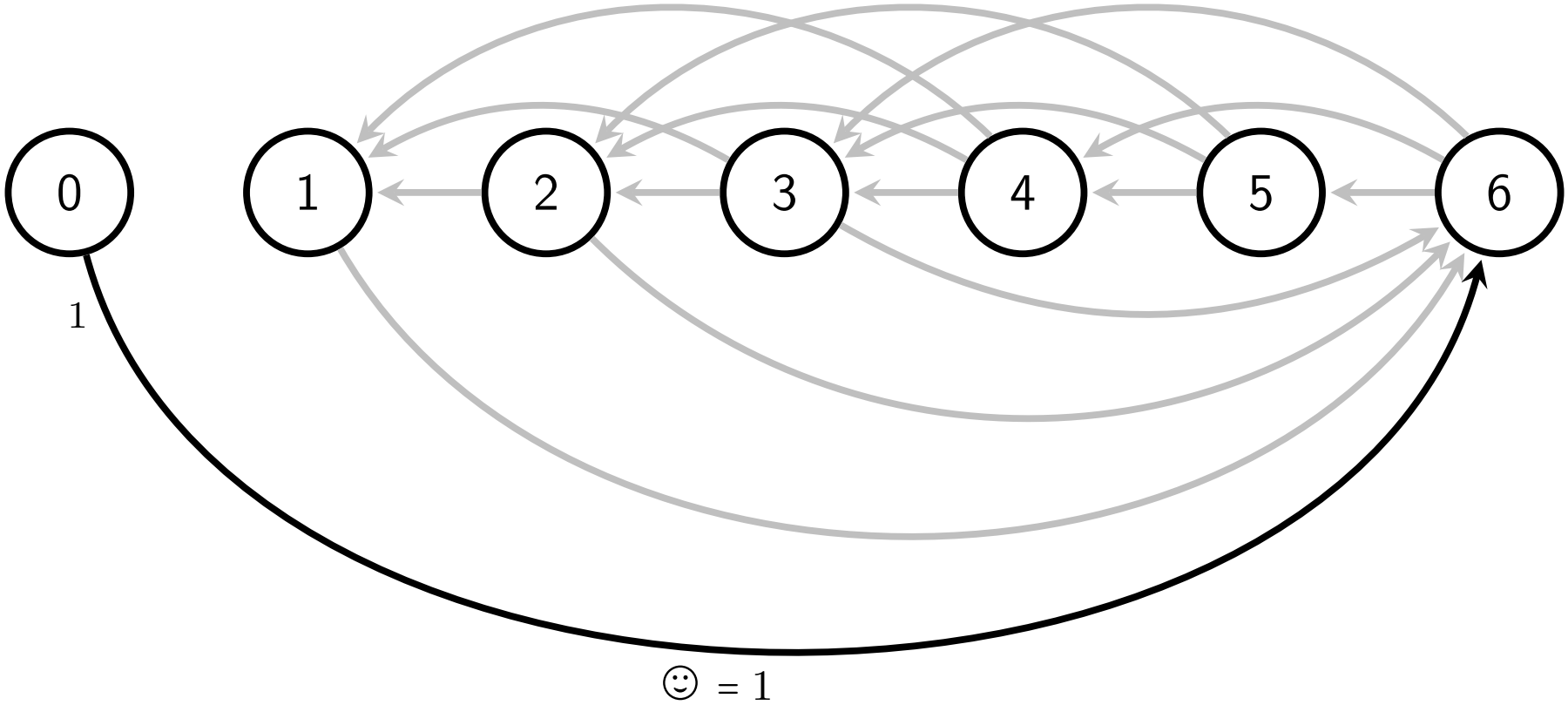
$$\begin{aligned}
☺ &= \tfrac{.3}{.8} \times (30 - 6) \\
&\quad + \tfrac{.5}{.8} \times (30 - 6 - 4) \\
&= 21.5
\end{aligned}$$

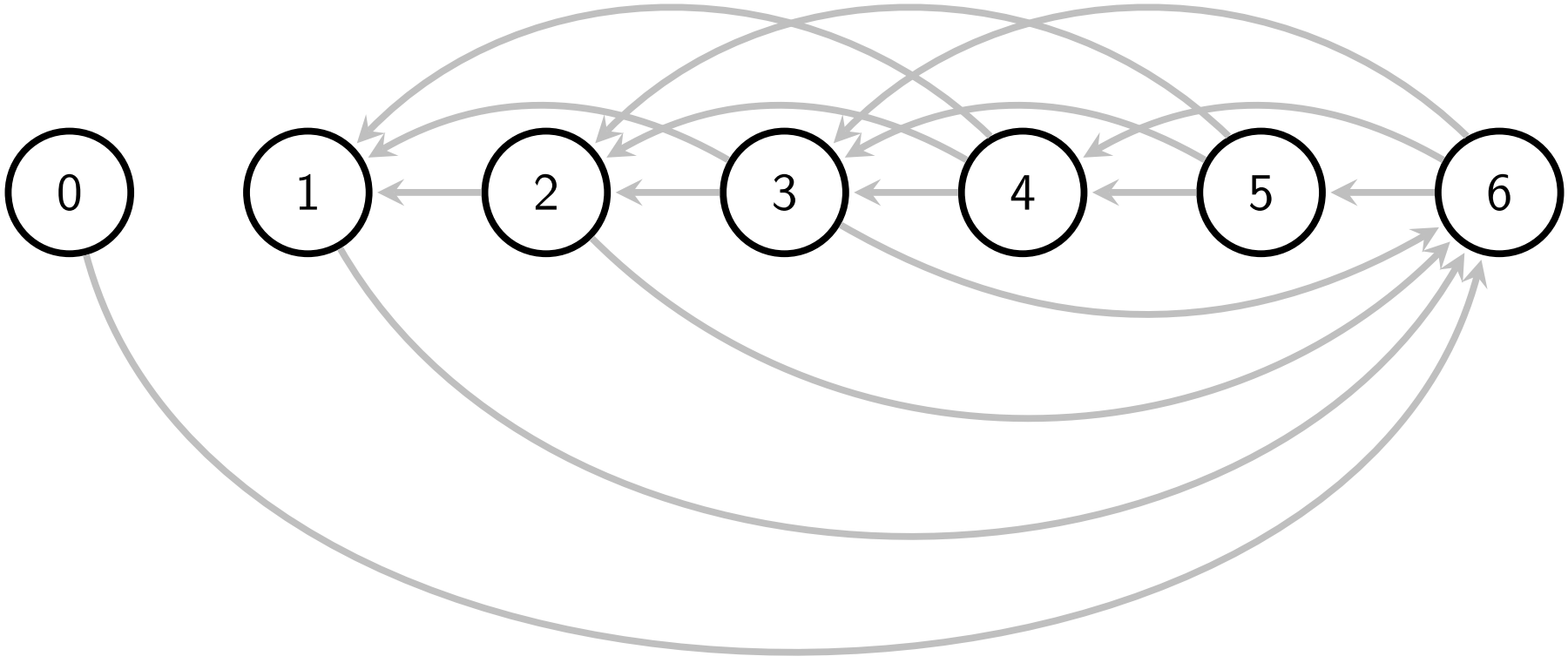

0
1
2
3
4
5
6
1

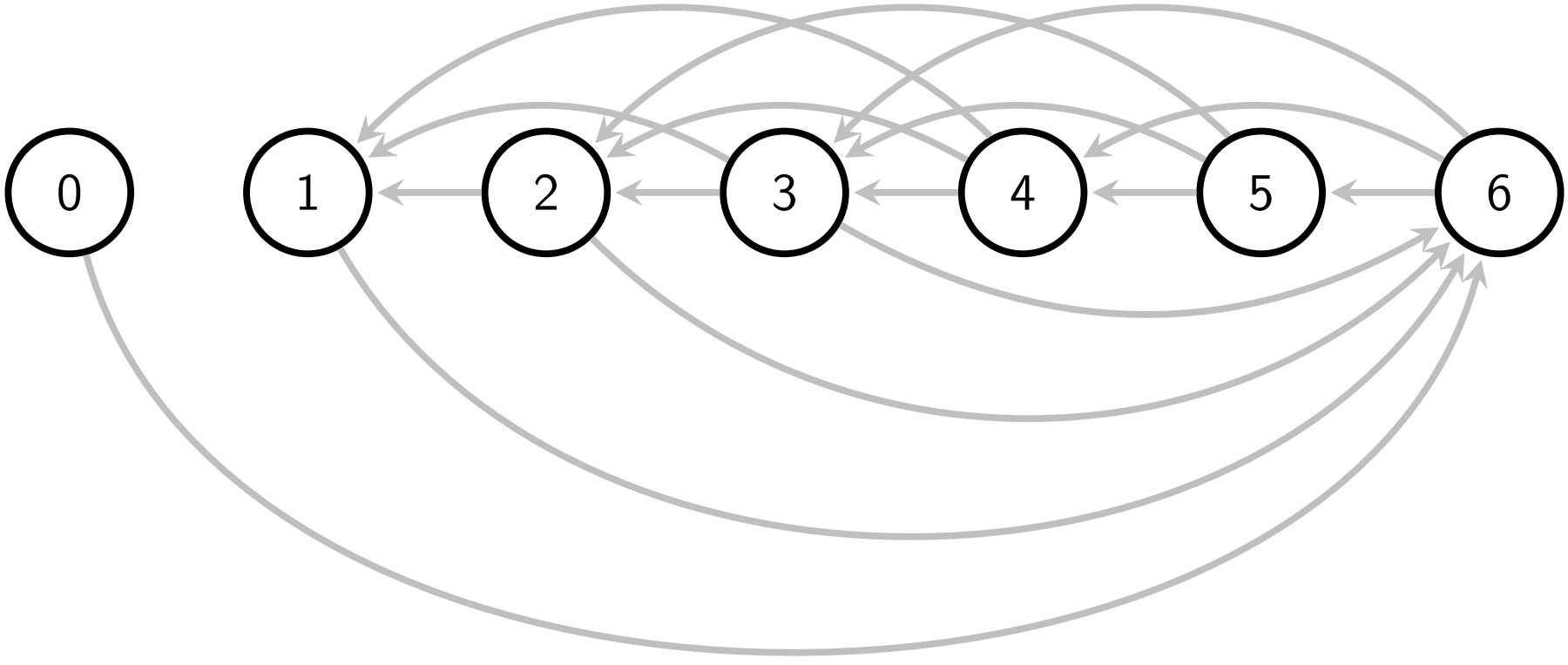


$$\begin{aligned}
\text{☺} &= .2 \times (\quad\; - 6) \\
&+ .3 \times (30 - 6 - 4) \\
&+ .5 \times (30 - 6 - 4 - 10) \\
&= 9.8
\end{aligned}$$

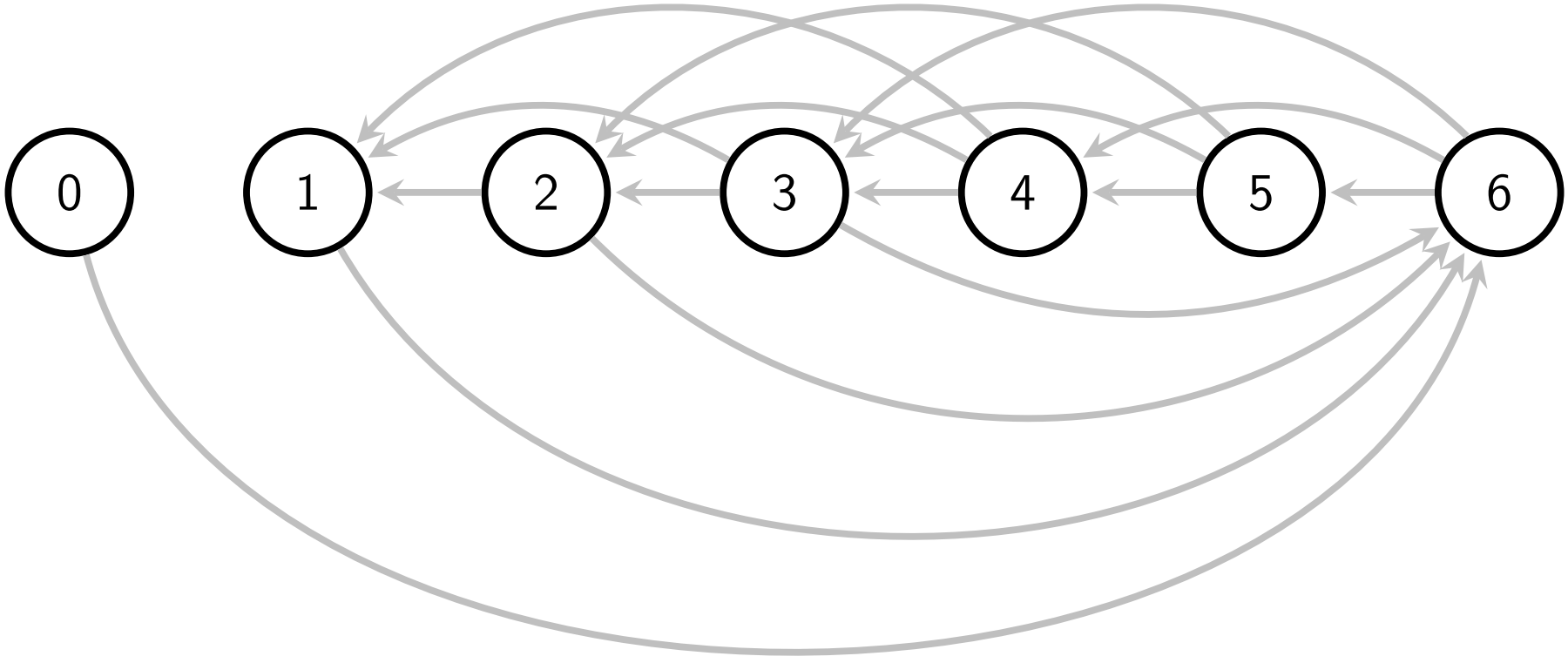
My friend can't have a beer


0
1
2
3
4
5
6
1
☺ = 1


0
1
2
3
4
5
6



Transition probabilities and happiness are stored in matrices



Transition probabilities and happiness are stored in matrices

Stationnary probabilities and the average happiness are easy to compute

But I was not very smart, was I?

There are other strategies...

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

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$$n = 0 \quad : \quad \bar{\text{☺}} = 20.5$$

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

$$n = 0 \quad : \quad \bar{\text{☺}} = 20.5$$

$$n = 1 \quad : \quad \bar{\text{☺}} = 21.3$$

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

$$n = 0 \quad : \quad \bar{\text{☺}} = 20.5$$

$$n = 1 \quad : \quad \bar{\text{☺}} = 21.3$$

$$n = 2 \quad : \quad \bar{\text{☺}} = 21.2$$

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

$$n = 0 \quad : \quad \bar{\text{☺}} = 20.5$$

$$n = 1 \quad : \quad \bar{\text{☺}} = 21.3$$

$$n = 2 \quad : \quad \bar{\text{☺}} = 21.2$$

$$n = 3 \quad : \quad \bar{\text{☺}} = 20.1$$

$$\ldots$$

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

$n = 0$: $\bar{\text{☺}} = 20.5$

$n = 1$: $\bar{\text{☺}} = 21.3$

$n = 2$: $\bar{\text{☺}} = 21.2$

$n = 3$: $\bar{\text{☺}} = 20.1$

$\ldots$

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

$n = 0$: $\bar{\text{☺}} = 20.5$

$n = 1$: $\bar{\text{☺}} = 21.3$

$n = 2$: $\bar{\text{☺}} = 21.2$

$n = 3$: $\bar{\text{☺}} = 20.1$

$\ldots$

- Less veggies, more beer!

There are other strategies...

- Fill in the fridge whenever there are less than n beers left

$$n = 0 \quad : \quad \bar{\text{☺}} = 20.5$$

$$n = 1 \quad : \quad \bar{\text{☺}} = 21.3$$

$$n = 2 \quad : \quad \bar{\text{☺}} = 21.2$$

$$n = 3 \quad : \quad \bar{\text{☺}} = 20.1$$

$$\ldots$$

- Less veggies, more beer!
- ...

Markov Decision Processes

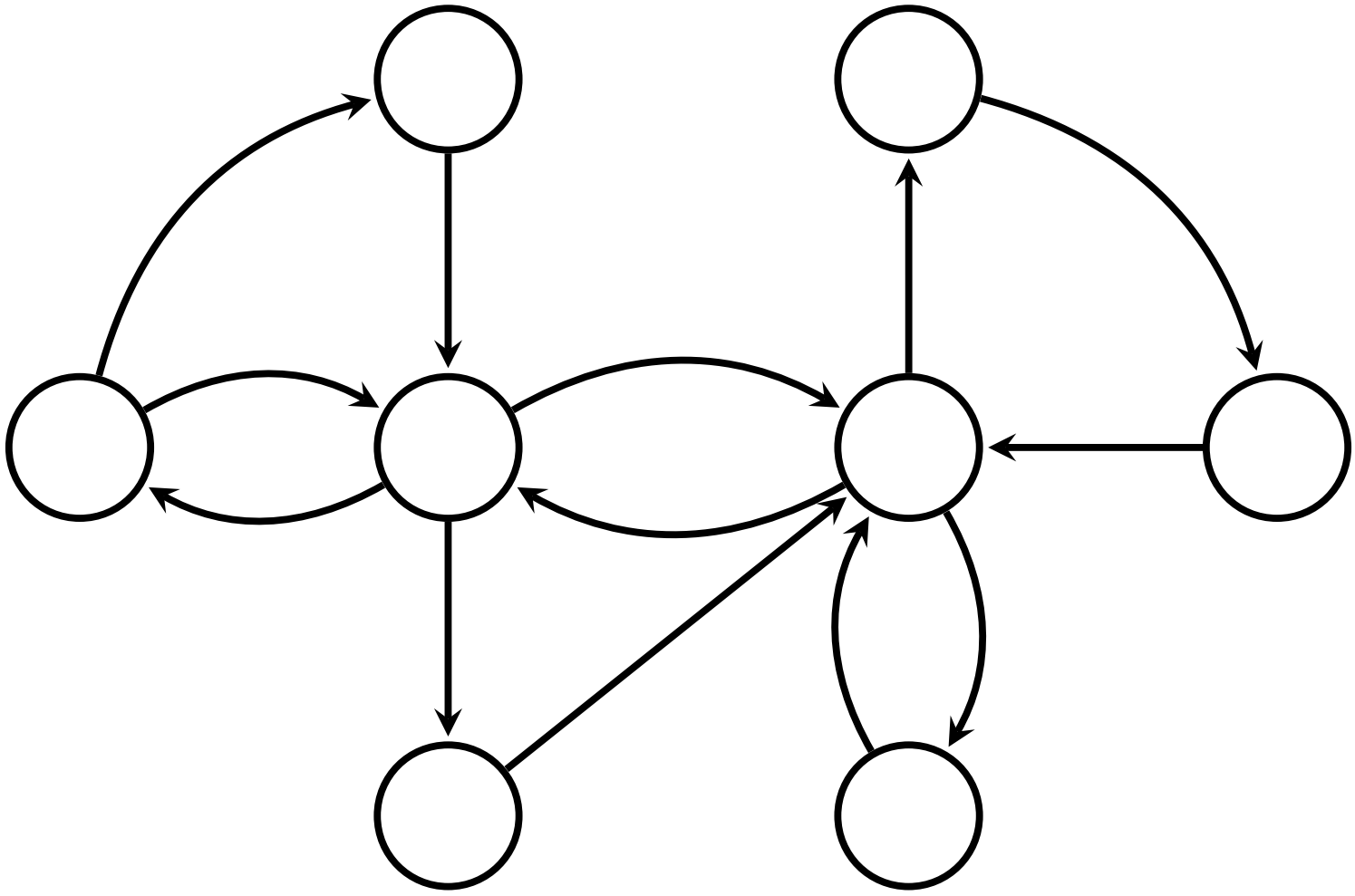
Model:	Represent the decision problem with states and actions
Goal:	Find the strategy with highest reward
Resolution:	Efficient algorithms exist (Policy Iteration, Dynamic Programming, Linear Programming, ...)

A storage problem

A PageRank problem

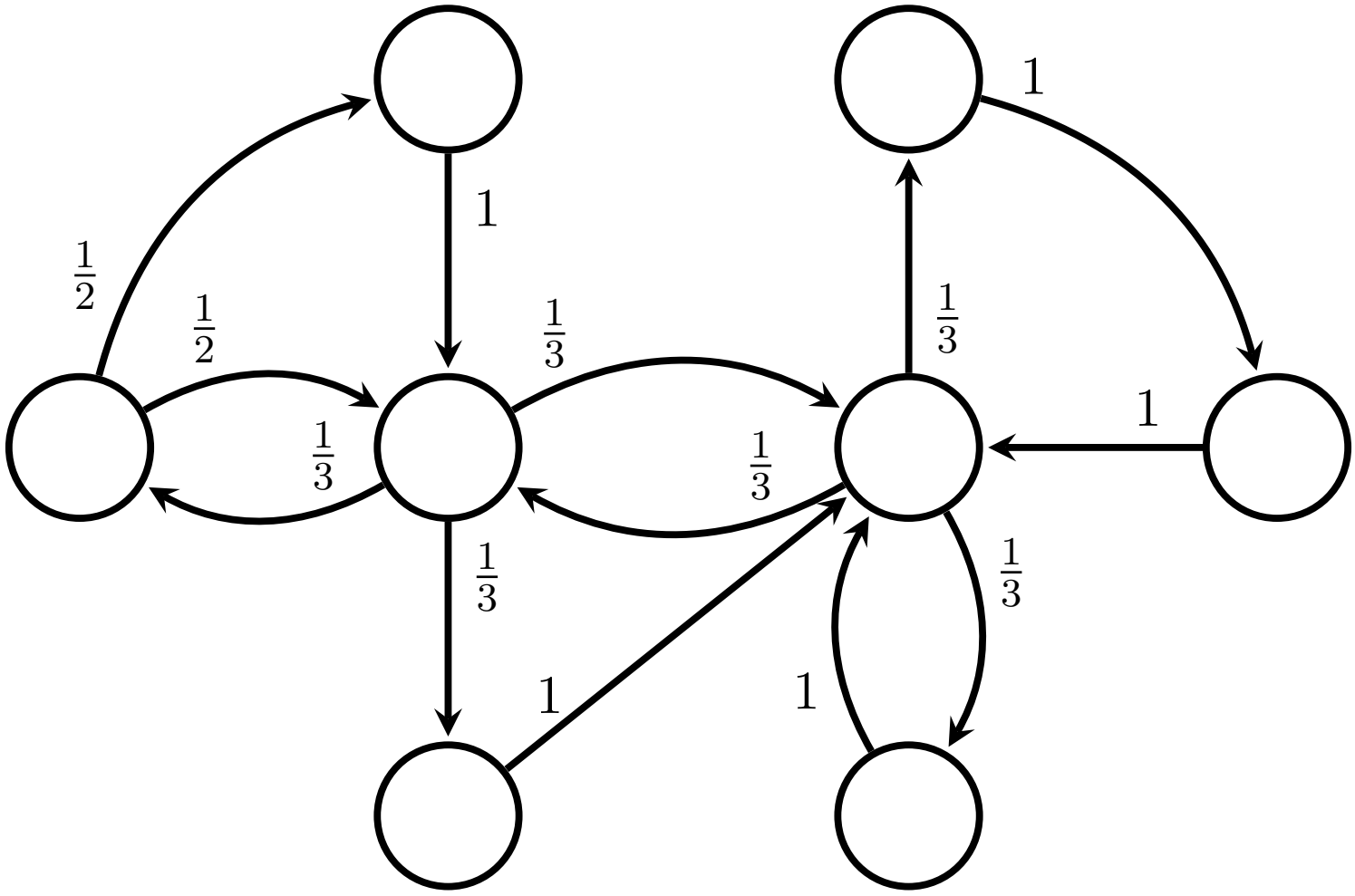
PageRank is the average portion of time spent in a node

during an infinite random walk



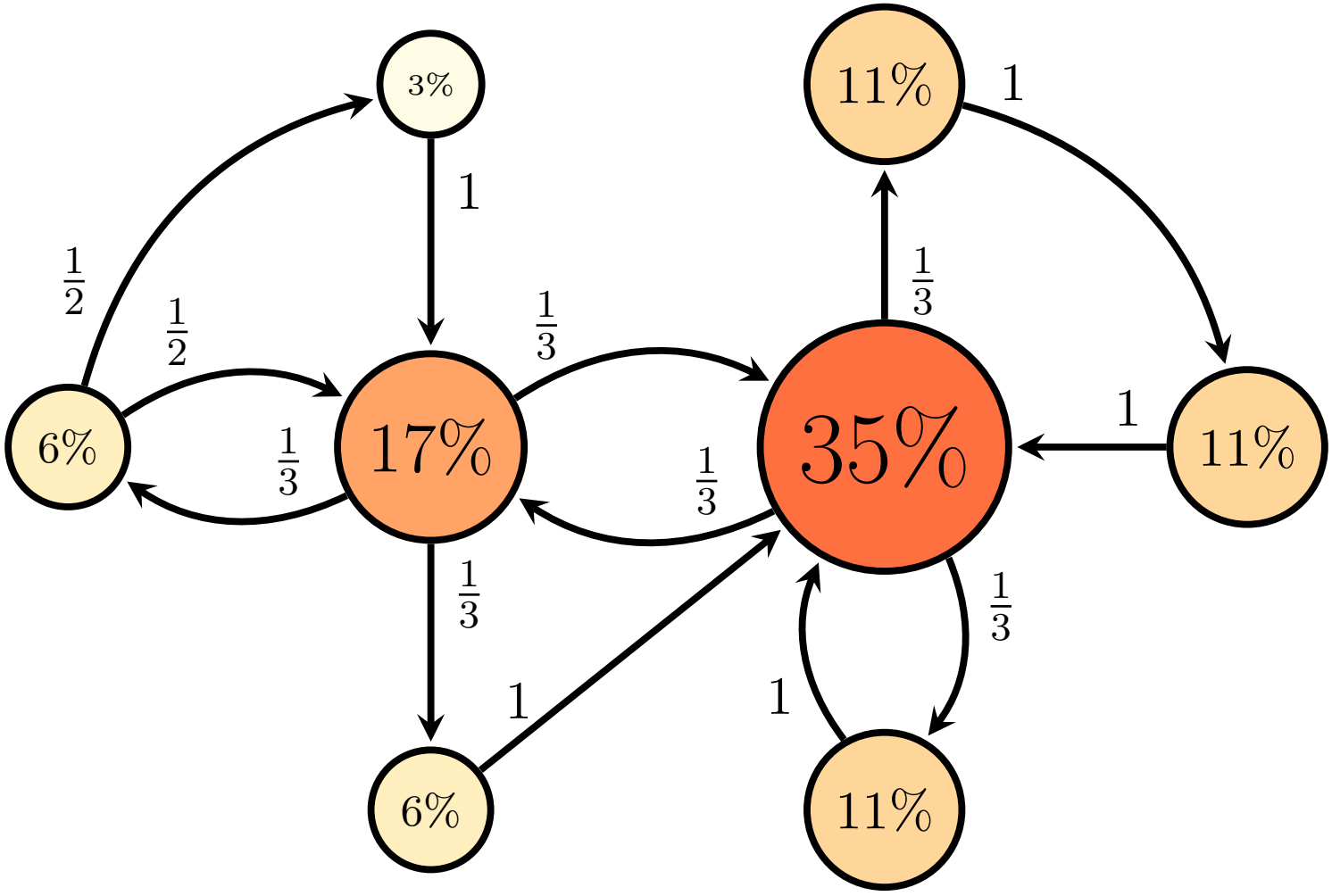
PageRank is the average portion of time spent in a node

during an infinite random walk



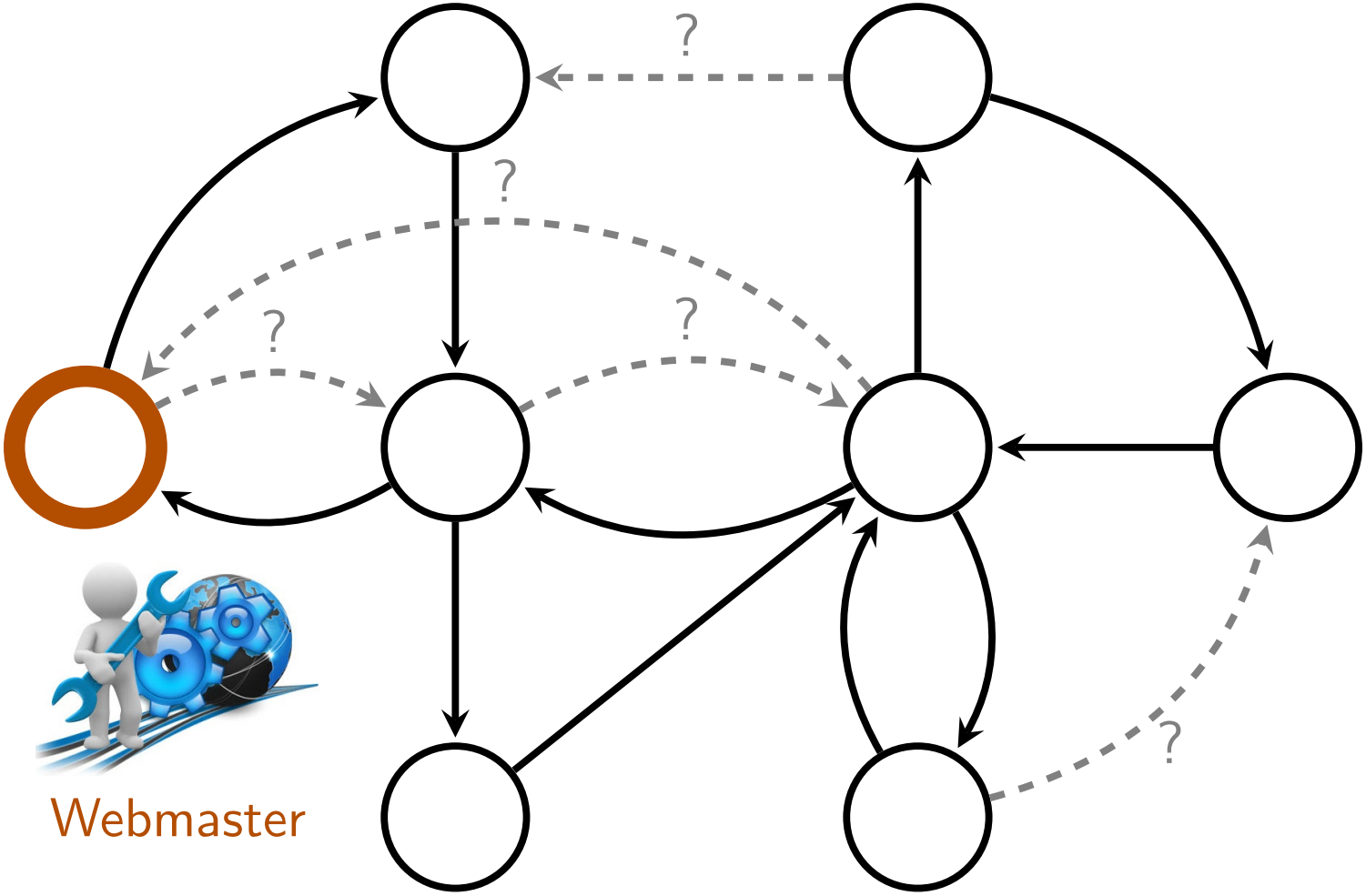
PageRank is the average portion of time spent in a node

during an infinite random walk



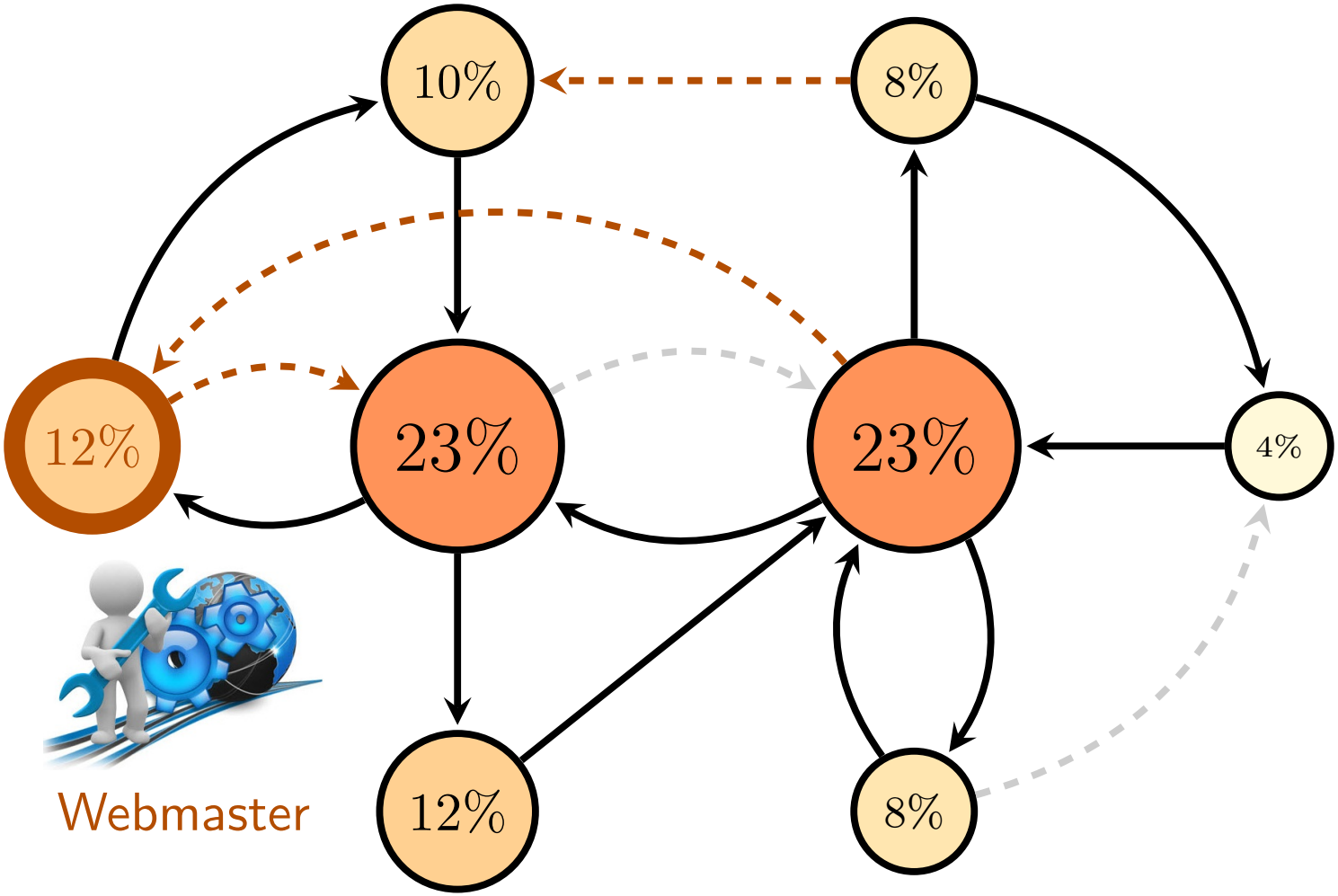
How can I improve my PageRank?

If I can tune the web a little bit...



Improvents in the ranking can be important

I have become the third most important page. Hurray!



Take home messages

Many decision problems can be modeled as Markov decision processes

These processes can be solved efficiently

Maybe they can help for one of your own problems?

Thanks for your attention!