Séminaire itinérant de catégories
Louvain-la-Neuve, le 19 mai 2012
Programme

9: **45** *Acceuil*

10: **00** M. M. Clementino, « Filter monads, injectivity and weak factorization systems »
10: **55** D. Coumans, « Canonical extension for categories and Makkai's topos of types »
11: **50** A. Burroni, « Les T-polygraphes »

12: **40** *Déjeuner*

14: **20** R. Duncan, « Interacting Observables in Categorical Quantum Mechanics »
15: **15** J.-P. Laffineur, « Autour d'un théorème de Guitart et Lair sur l'esquissabilité des théories du premier ordre »
16: **05** *Pause café*

16: **20** D. Rodelo, « Techniques variétales pour les catégories de Goursat »

17: **10** *Fin*

Maria Manuel Clementino *Filter monads, injectivity and weak factorization systems*
Using Escardó-Flagg approach to injectivity via Kock-Zoberlein (or lax idempotent) monads in T_0 topological spaces [2], and Hofmann's recent study of injectivity for spaces [3], we characterize continuous maps which are injective with respect to special classes of embeddings as algebras of filter monads on slices of Top. The use of fibrewise way-below relations shows that these continuous maps can be studied as fibrewise continuous lattices, fibrewise continuous Scott domains or fibrewise sober spaces. Moreover, it is shown that, together with the corresponding embeddings, injective continuous maps form a weak factorization system in the category of topological (T_0-) spaces and continuous maps. The techniques used lead to categorical open problems related to the lifting-preservation of Kock-Zoberlein monads and to the study of special weak factorization systems. (The first part is based on joint work with Francesca Cagliari and Sandra Mantovani [1], while the second part is based on work in progress with Ignacio López Franco.)

[1] F. Cagliari, M.M. Clementino, S. Mantovani, *Fibrewise injectivity and Kock-Zoberlein monads*, J. Pure Appl. Algebra (available online), doi:10.1016/j.jpaa.2012.03.019

[2] M. Escardó, R. Flagg, *Semantic domains, injective spaces and monads*, Electr. Notes in Theor. Comp. Science **20**, electronic paper 15 (1999).

[3] D. Hofmann, *A four for the price of one duality principle for distributive topological spaces*, preprint, arXiv:math.GN/1102.2605.

Dion Coumans *Canonical extension for categories and Makkai's topos of types*
In the 1950s Jonsson and Tarksi introduced the notion of canonical extension for Boolean algebras with operators. In this setting, canonical extension provides an algebraic description of Stone's topological duality. By now, the theory of canonical extensions has been developed further and it has proven be a powerful tool in the algebraic study of propositional logics. After a brief introduction in this theory, we'll define a notion of canonical extension for coherent categories, the categorical analogues of distributive lattices. This notion extends the existing notion of canonical extension for distributive lattices (viewed as coherent categories). Furthermore, it may be characterized by a universal property. This construction opens the way to applications of the theory of canonical extension in the study of first order logics. Our construction of canonical extension for coherent categories has led to an alternative description of the topos of types, introduced by Makkai in 1981. This allows us to give new and transparent proofs of some properties of the action of the topos of types construction on morphisms. Furthermore, we apply this description to relate, for a coherent category, its topos of types to its category of models (in Set).

Albert Burroni *Les T-polygraphes*

Les T-polygraphes généralisent les T-multigraphes où T est une monade. (Le cas qui nous intéresse est celui de la monade dans Graph dont les algèbres sont les catégories.) Nous définissons une catégorie laxe non biaisée des T-spans bilatères dont les objets monades sont des T-polycatégories.

Ross Duncan *Interacting Observables in Categorical Quantum Mechanics*

One of the most shocking features of quantum mechanics is the possibility that quantum observables may be incompatible: when one property is well-defined, another cannot have a definite value assigned to it. This is most clearly seen in the case of complementary observables, such as the spin along the X and Z axes. Perfect knowledge of the X spin implies complete ignorance of the Z spin. Historically complementarity has been viewed as a negative property, a failure of some classical behaviour; however, from a different point of view a positive attitude is possible.

Non-degenerate quantum measurements are equivalent to coalgebras whose action is to copy and delete the eigenstates associated with the observable; in this way we may view each observable as embedding a classical data type in the quantum world. When two complementary observables are considered together, they jointly form a structure closely related to a Hopf algebra. These interacting algebras give rise to much of the equational structure exploited in quantum computing. Further, all of the axiomatisation can be performed in an abstract categorical setting, and a beautiful, highly legible graphical notation results.

I will introduce the basic ideas of categorical quantum mechanics, and the the various algebra structures and demonstrate some applications in concerning quantum circuits and measurement-based quantum computing.

Jean-Pierre Laffineur *Autour d'un théorème de Guitart et Lair sur l'esquissabilité des théories du premier ordre*

Nous présentons le plan de la démonstration de l'esquissabilité dans les ensembles des théories du premier ordre, puis celui de la démonstration de l'équivalence entre théories $L\infty, \infty$ et théories esquissables dans les ensembles. Nous disons un mot de la différence entre catégories κ-accessibles au sens de Makkai-Paré et catégories θ-modelables au sens de Lair, puis nous parlons du processus d'atomisation de Skolem.

Diana Rodelo *Techniques variétales pour les catégories de Goursat*

We prove that varietal techniques based on the existence of operations of a certain arity can be extended to Goursat (i.e. 3-permutable) categories with finite coproducts using a method developed by D. Bourn and Z. Janelidze [1]. In particular, we give a categorical version of the characterisation theorems for n-permutable varieties due to J. Hagemann and A. Mitschke [2], when $n = 3$. (Joint work with Tim Van der Linden)

[1] D. Bourn and Z. Janelidze, *Approximate Mal'tsev operations*, Theory and Appl. of Categ. **21** (2008), no. 8, 152–171.

[2] J. Hagemann and A. Mitschke, *On n-permutable congruences*, Algebra Universalis **3** (1973), 8–12.