

# Critical phenomena for random matrices and integrable systems

## Abstracts of talks

**Gernot Akemann (Bielefeld University): Universality in the product of two coupled random matrices with correlations**

We investigate under which conditions the singular values of the product of two coupled Gaussian random matrices with correlated entries form a determinantal point process. This process is compared with the simpler ensemble of two independent matrices and with the ensemble of a single random matrix with correlations from two sides, which are also determinantal. In all three cases their respective kernel enjoys a complex contour integral representation. In the limit of large matrix size we look at finite rank perturbations of the correlation and coupling matrices at the origin of the spectrum. We find generalisations of the Bessel-kernel of a single matrix, of the Meijer G-kernel for two matrices and of a third kernel that interpolates between these two for two coupled matrices. All three kernels are “decorated” by finite-rank perturbations. This is joint work with T. Checinski, D.-Z. Liu and E. Strahov.

**Florent Benaych-Georges (Université Paris Descartes): Extreme eigenvalues of Erdős-Rényi graphs**

In this talk, we shall present some recent works with Charles Bordenave and Antti Knowles giving estimates for the extreme eigenvalues of the centered adjacency matrices of a possibly inhomogenous Erdős-Rényi graph. In the case of the Stochastic Block Model, these works allow to assess the accuracy of the spectral clustering algorithm, whereas in the case of an homogenous Erdős-Rényi graph, they establish a transition between a compact spectrum and a diffuse one, at a threshold which coincides with the connectivity threshold (the one where a giant connected component appears). The proofs of these results bring to light two classical regimes from random matrix theory: the localized one and the delocalized one.

**Dimitris Cheliotis (University of Athens): Biorthogonal ensembles and triangular matrices**

We will discuss the singular values of certain triangular random matrices. When their elements are i.i.d. standard complex Gaussian random variables, the squares of the singular values form a biorthogonal ensemble, and with an appropriate change in the distribution of the diagonal elements, they give the biorthogonal Laguerre ensemble. For triangular Wigner matrices, we show that the empirical distribution of the appropriately rescaled squares of the singular eigenvalues converge to a distribution with support  $[0, e]$ , while the rescaled largest singular eigenvalue converges to  $\sqrt{e}$  under the additional assumption of mean zero and finite fourth moment for the law of the matrix elements.

**Maurice Duits (KTH Stockholm): TBA**

**Tamara Grava (University of Bristol, SISSA): Painlevé equations and critical behaviour of Hamiltonian PDEs**

**Igor Krasovsky (Imperial College London): Splitting of a gap in the spectrum of random matrices**

The eigenvalues in the bulk of the spectrum of typical random matrices follow the sine kernel determinantal process. The asymptotics of the probability of a large gap, i.e. an interval without eigenvalues, and of several large gaps (except for a multiplicative constant in more than one gap case) is well known. In this talk we will present the transition between the asymptotic behaviour for one gap and that for two gaps. It corresponds to the beginning of the splitting of one gap into two. The talk is based on a joint work with Benjamin Fahs.

**Arno Kuijlaars (KU Leuven): Universality for conditional measures of the sine point process**

The sine process is a random point process that is obtained as a limit from the eigenvalues of many random matrices as the size tends to infinity. This phenomenon is called universality in random matrix theory, and it also holds for many orthogonal polynomial ensembles.

In this talk I want to emphasize another connection of the sine point process with orthogonal polynomials. It comes from a surprising property called number rigidity in the sense of Ghosh and Peres. This means that for almost all configurations, the number of points in an interval  $[-R, R]$  is determined exactly by the points outside the interval. The conditional measures is the joint distribution of the points in  $[-R, R]$  given the points outside. Bufetov showed that these are orthogonal polynomial ensembles with a weight that comes from the points outside  $[-R, R]$ .

I will report on recent work with Erwin Mina-Diaz (arXiv:1703.02349) where we prove a universality result for these orthogonal polynomial ensembles that in particular implies that the correlation kernel of the orthogonal polynomial ensemble tends to the sine kernel as  $R$  tends to infinity. This answers a question posed by Alexander Bufetov.

**Gaultier Lambert (University of Zurich): On the law of large numbers for the maximum of the log-characteristic polynomial of Unitary invariant ensembles**

I will report on some of the recent results on the maximum of log-correlated process coming from random matrix theory. In particular, I will sketch a proof that the maximum of the centered log-characteristic polynomial of an  $N \times N$  unitary invariant matrix is  $\log N + o(\log N)$  with high probability. This confirms the first term in a conjecture by Fyodorov and Simm. We will try to emphasize on the connection with the orthogonal polynomials Riemann-Hilbert problem and the second moment originating from the theory of branching random walks. This is joint work with Elliot Paquette.

**Mylène Maida (Université de Lille 1): Concentration of measure for Coulomb gases**

A Coulomb gas is the canonical Gibbs measure associated with a system of particles in electrostatic interaction. As the number of particles grows to infinity, the empirical measure of a Coulomb gas converges weakly towards an equilibrium measure, characterized by a variational principle. We obtain sub-gaussian concentration inequalities around this equilibrium measure, in the weak and Wasserstein topologies. This yields for instance a concentration inequality at the correct rate for the Ginibre ensemble. The proof relies on new functional inequalities, which are counterparts of Talagrand’s transport inequality in the Coulomb interaction setting. Joint work with Djalil Chafaï and Adrien Hardy.

**Nicholas Simm (University of Warwick): Regularized counting statistics and Gaussian multiplicative chaos**

We define a statistic which counts the number of eigenvalues of a random matrix falling in a given interval, regularized at some appropriate scale. Varying the interval, we obtain a random process which, for large dimensional matrices, is asymptotically Gaussian and has a logarithmic covariance structure. I will discuss results about constructing the exponential of this process in terms of the theory of Gaussian multiplicative chaos, obtaining convergence in the full sub-critical phase. As a by-product of the analysis, we prove a smoothed analogue of a conjecture due to Fyodorov, Hiary and Keating about integrated moments of characteristic polynomials. This is joint work with Gaultier Lambert and Dmitry Ostrovsky.

**Christophe Texier (Université Paris-Sud): Truncated linear statistics associated with the eigenvalues of random matrices**

Given a certain invariant random matrix ensemble characterised by the joint probability distribution of eigenvalues  $P(\lambda_1, \dots, \lambda_N)$ , the study of linear statistics of the eigenvalues  $L = \sum_{i=1}^N f(\lambda_i)$ , where  $f(\lambda)$  is a known function, has played an important role in many applications of random matrix theory. I will discuss the distribution of truncated linear statistics of the form  $\tilde{L} = \sum_{i=1}^{N_1} f(\lambda_i)$ , when the sum runs over a fraction of the eigenvalues ( $N_1 < N$ ). By using the Coulomb gas technique, the large deviation function controlling the distribution of such sums in the limit of large  $N$ , with  $0 < N_1/N < 1$  fixed, will be analysed. Two situations will be considered leading to two different universal scenarii:

1) the case where the truncated linear statistics is restricted to the largest (or smallest) eigenvalues. We have shown that the constraint that  $\tilde{L} = \sum_{i=1}^{N_1} f(\lambda_i)$  is fixed drives an infinite order phase transition in the underlying Coulomb gas. This transition corresponds to a change in the density of the gas, from a density defined on two disjoint intervals to a single interval. In this latter case the density presents a logarithmic divergence inside the bulk.

2) the second situation is the case without further restriction on the ordering of the eigenvalues contributing to the truncated linear statistics (this can be viewed as a new ensemble which is related, but not equivalent, to the “thinned ensembles” introduced by Bohigas and Pato). In this case, a region opens in the phase diagram of the Coulomb gas, where the large deviation function is mostly controlled by entropy (in particular this induces a change in the scaling of the relative fluctuations of the truncated linear statistics, from the usual  $1/N$  for  $N_1 = N$ , to  $1/\sqrt{N}$  when  $N_1 < N$ ). Our analysis relies on the mapping on a problem of  $N_1$  fictitious non-interacting fermions in  $N$  energy levels, which can exhibit both positive and negative effective (absolute) temperatures.

- Aurélien Grabsch, Satya N. Majumdar and Christophe Texier, Truncated linear statistics associated with the top eigenvalues of random matrices, *J. Stat. Phys.* **167**(2), 234–259 (2017).
- Aurélien Grabsch, Satya N. Majumdar and Christophe Texier, Truncated linear statistics of the eigenvalues of random matrices II. Partial sums of proper time delays in chaotic quantum dots, *J. Stat. Phys.* **167**(6), 1452–1488 (2017).

**Pierre van Moerbeke (Université catholique de Louvain, Brandeis University):  
Tilings of nonconvex domains and a new universality class**

We show that tilings of nonconvex domains lead asymptotically to new universality classes. In particular, such new statistics is seen for lozenge tilings of hexagons with cuts, by taking the limit when the size of both, the hexagon and the cuts, tend to infinity, while keeping certain geometric data fixed. (joint work with Mark Adler and Kurt Johansson)

**Christian Webb (Aalto University): Multiplicative chaos in random matrix theory and number theory**

I will give an overview of recent developments on the connection between global statistics of large random matrices, number theory, and random geometry. More precisely I will discuss how on the global scale, the asymptotic behavior of characteristic polynomials of e.g. CUE matrices can be described in terms of random fractal measures known as Gaussian multiplicative chaos measures and how similar objects also arise when studying the statistical behavior of the Riemann zeta function on the critical line.