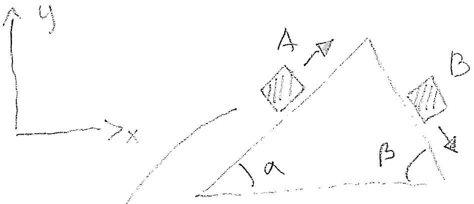
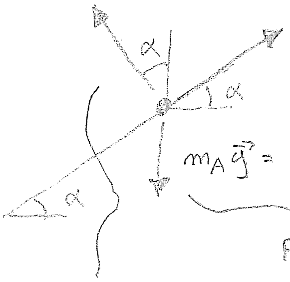
13





$\vec{T} = \begin{bmatrix} T\cos\alpha \\ T\sin\alpha \end{bmatrix}$ CORDE

$\underbrace{m_A \vec{g} = \begin{bmatrix} 0 \\ -m_A g \end{bmatrix}}_{\text{POIDS}}$

$\boxed{\sum \vec{F} = m\vec{a}}$ BLOC A

$\underbrace{\vec{N} = \begin{bmatrix} -N\sin\alpha \\ N\cos\alpha \end{bmatrix}}_{\text{REACTION DU SOL}}$

$\vec{a} = \begin{bmatrix} a\cos\alpha \\ a\sin\alpha \end{bmatrix}$

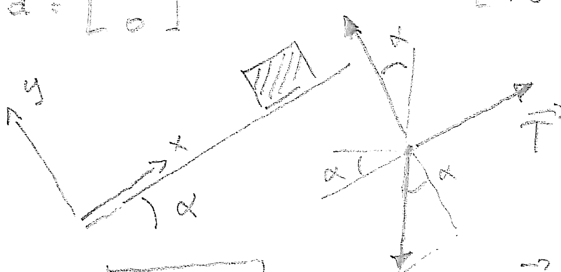
$$\begin{cases} -N\sin\alpha + T\cos\alpha = m_A\, a \cos\alpha & (1) \\ N\cos\alpha + T\sin\alpha - m_A g = m_A\, a \sin\alpha & (2) \end{cases}$$

OOOUUUPPPSSS !!
COMPLIQUÉ, COMPLIQUÉ !

PLUS MALIN !
CHOISIR LE BON SYSTEME D'AXES !

$\vec{a} = \begin{bmatrix} a \\ 0 \end{bmatrix}$

$\vec{N} = \begin{bmatrix} 0 \\ N \end{bmatrix}$



$\vec{T} = \begin{bmatrix} T \\ 0 \end{bmatrix}$

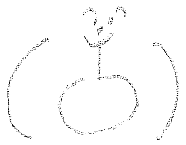
$m_A \vec{g} = \begin{bmatrix} -m_A g \sin\alpha \\ -m_A g \cos\alpha \end{bmatrix}$

BLOC A
$\sum \vec{F} = m\vec{a}$

$$\begin{cases} T - m_A g \sin\alpha = m_A a & (3) \\ N - m_A g \cos\alpha = 0 & (4) \end{cases}$$

ON RELIE DIRECTEMENT T ET a SANS CALCULER N !

$$\boxed{a = \frac{T}{m_A} - g\sin\alpha} \quad (5)$$

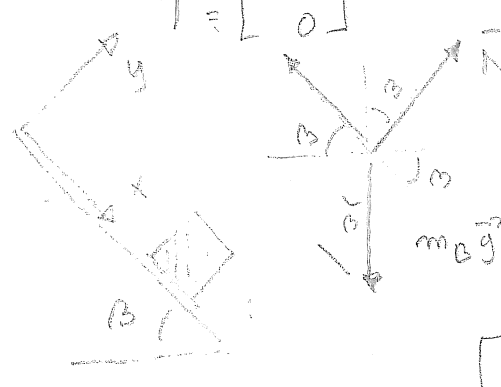
 ET 

SONT STRICTEMENT EQUIVALENTES

C'EST LA MÊME EGALITE VECTORIELLE EXPRIMEE DANS DEUX SYSTEMES D'AXES DIFFERENTS

VERIFIER !
$$\boxed{\begin{aligned} (1)\cos\alpha + (2)\sin\alpha &= (3) \\ -(1)\sin\alpha + (2)\cos\alpha &= (4) \end{aligned}}$$!

$$\vec{T} = \begin{bmatrix} -T \\ 0 \end{bmatrix} \qquad \vec{N}_B = \begin{bmatrix} 0 \\ N_B \end{bmatrix} \qquad \vec{a} = \begin{bmatrix} a \\ 0 \end{bmatrix}$$



$$m_B \vec{g} = \begin{bmatrix} m_B\, g \sin\beta \\ -m_B\, g \cos\beta \end{bmatrix}$$

BLOC B

$$\sum \vec{F} = m\vec{a}$$

$$m_B a = -T + m_B g \sin\beta$$

$$\boxed{a = g\sin\beta - \frac{T}{m_B}} \quad (6)$$

- EN COMPARANT (5) ET (6)
MEME VALEUR DE a

$$g\sin\beta - \frac{T}{m_B} = \frac{T}{m_A} - g\sin\alpha$$

$$g(\sin\beta + \sin\alpha) = T\left(\frac{1}{m_A} + \frac{1}{m_B}\right)$$

$$\boxed{T = \frac{m_B m_A}{(m_B + m_A)}\, g(\sin\beta + \sin\alpha)}$$

- EN COMPARANT (5) ET (6)
MEME VALEUR DE T

$$m_A(a + g\sin\alpha) = m_B(g\sin\beta - a)$$

$$\boxed{a = \frac{m_B g \sin\beta - m_A g \sin\alpha}{(m_A + m_B)}}$$

VALEURS NUMERIQUES

$\alpha = 30$
$\beta = 60$

$$T = \frac{30}{11} * 9{,}8 * \left(\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 36{,}5\ [N]$$

$$a = \frac{1}{11} * 9{,}8 * \left(6\frac{\sqrt{3}}{2} - 5\,\frac{1}{2}\right) = 2{,}4\ [m/s^2]$$

⚠ IL FAUT CONNAITRE LES VALEURS REMARQUABLES DES sin ET COS

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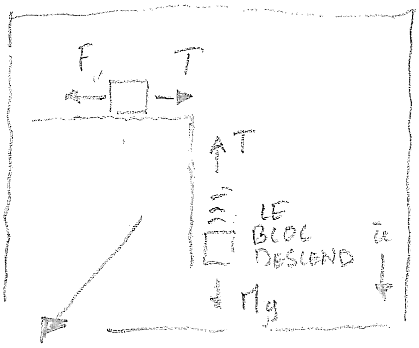
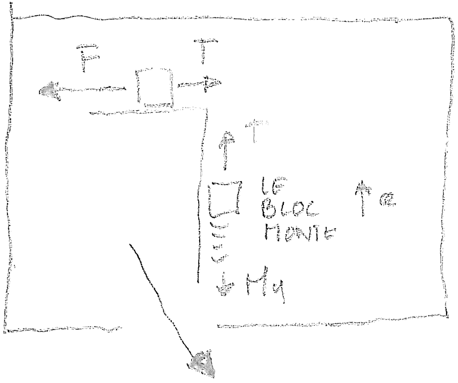




$$\begin{cases} ma = T - F \\ Ma = Mg - T \end{cases}$$

$F = 22$ [N]
$a = 1$ [m/s²]

$$\boxed{(m+M)\underbrace{a}_{1} = -\underbrace{F}_{22} + Mg}$$

$$\begin{cases} ma = F - T \\ Ma = T - Mg \end{cases}$$

$F = 44$ [N]
$a = 1{,}75$ [m/s²]

$$\boxed{(m+M)\underbrace{a}_{1{,}75} = \underbrace{F}_{44} - Mg}$$

2 EQUATIONS
2 INCONNUES m ET M

$$\begin{cases} M(1 - 9{,}8) + m = -22 \\ M(1{,}75 + 9{,}8) + m\,1{,}75 = 44 \end{cases}$$

$$\begin{cases} -8{,}8\,M + m = -22 & (1) \\ 11{,}55\,M + 1{,}75\,m = 44 & (2) \end{cases}$$

$-1{,}75\,(1) + (2) \rightarrow 1{,}75 * 8{,}8\,M + 11{,}55\,M = 1{,}75 * 22 + 44$

$$\boxed{\begin{aligned} M &= 3{,}06 \text{ kg} \\ m &= 4{,}94 \text{ kg} \end{aligned}}$$

$m = -22 + 8{,}8 * 3{,}06$
EN REPRENANT L'EQUATION (1)

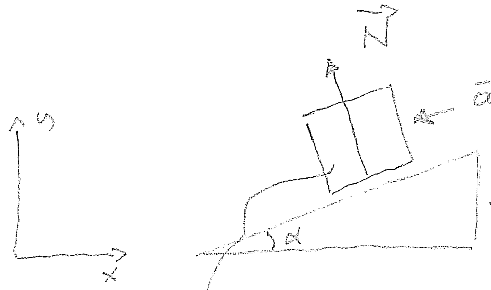
NOTE

LE BLOC LE PLUS MASSIF EST m ET NON M :-)

LES NOTATIONS PEUVENT ETRE CONTRE-INTUITIVES !

BE CAREFUL !

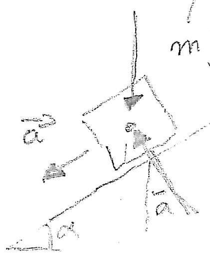
15



$\vec{\alpha}$ ACCELERATION DU BLOC PAR RAPPORT AU COIN

$\vec{A} = \begin{bmatrix} A \\ 0 \end{bmatrix}$ ACCELERATION DU COIN PAR RAPPORT AU SOL

BLOC

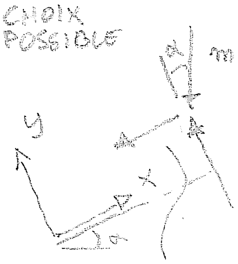


$m\vec{g} = \begin{bmatrix} 0 \\ -mg \end{bmatrix}$

$\vec{N} = \begin{bmatrix} -N\sin\alpha \\ N\cos\alpha \end{bmatrix}$

ACCELERATION BLOC % COIN $\vec{\alpha} = \begin{bmatrix} -a\cos\alpha \\ -a\sin\alpha \end{bmatrix}$

AUTRE CHOIX POSSIBLE



$m\vec{g} = \begin{bmatrix} -mg\sin\alpha \\ -mg\cos\alpha \end{bmatrix}$

$\vec{N} = \begin{bmatrix} 0 \\ N \end{bmatrix}$

$\vec{\alpha} = \begin{bmatrix} -a \\ 0 \end{bmatrix}$

$\vec{A} = \begin{bmatrix} A\cos\alpha \\ A\sin\alpha \end{bmatrix}$

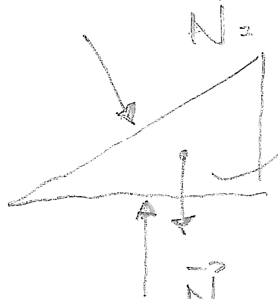
$\sum \vec{F} = m\vec{\alpha}$

REPERE INERTIEL → SOL !

$$\begin{cases} -N\sin\alpha = m(A - a\cos\alpha) \\ N\cos\alpha - mg = m(\ \ -a\sin\alpha) \end{cases}$$

IL FAUT METTRE L'ACCELERATION DU COIN !

COIN



$\vec{N} = \begin{bmatrix} N\sin\alpha \\ -N\cos\alpha \end{bmatrix}$

$M\vec{g} = \begin{bmatrix} 0 \\ -Mg \end{bmatrix}$

$\vec{N}_{sol} = \begin{bmatrix} 0 \\ N_{sol} \end{bmatrix}$

$$\begin{cases} N\sin\alpha = MA \\ -N\cos\alpha - Mg + N_{sol} = 0 \end{cases}$$

(PAS VRAIMENT UTILE SAUF POUR CONNAITRE N_{sol} NON DEMANDE ICI !)

3 EQUATIONS
A 3 INCONNUES
N, A et a

$$\begin{cases} -N\sin\alpha = m(A - a\cos\alpha) & (1) \\ N\cos\alpha - mg = -ma\sin\alpha & (2) \\ N\sin\alpha = MA & (3) \end{cases}$$

$(1)+(3) \longrightarrow \quad (M+m)A = ma\cos\alpha$ } EGALITE HORIZ POUR ENSEMBLE BLOC + COIN !

$\cos(\alpha)(1) + \sin(\alpha)(2) \longrightarrow \quad \underbrace{-mg\sin\alpha = mA\cos\alpha - ma}$

EGALITE LE LONG DE LA DIRECTION OBLIQUE POUR LE COIN !

$$-mg\sin\alpha = mA\cos\alpha - \frac{(M+m)}{\cos\alpha}A$$

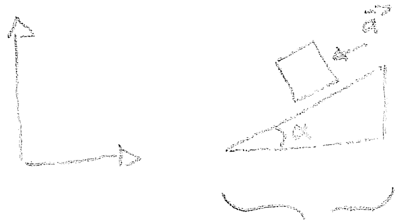
$$mg\sin\alpha\cos\alpha = \underbrace{-mA\cos^2\alpha + mA}_{mA(\underbrace{1-\cos^2\alpha}_{\sin^2\alpha})} + MA$$

$$\boxed{A = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}}$$

$$\boxed{N = \frac{Mmg\cos\alpha}{M + m\sin^2\alpha}}$$ CAR $N\sin\alpha = MA$:-)

$$\boxed{a = \frac{(M+m)g\sin\alpha}{M + m\sin^2\alpha}}$$ CAR $ma\cos\alpha = (M+m)A$:-)

15 PIECE OF CAKE ! :-) THE FAST WAY



$\vec{a}$ ACCELERATION DU BLOC PAR RAPPORT AU COIN

$$\vec{a} = \begin{bmatrix} -a\cos\alpha \\ -a\sin\alpha \end{bmatrix}$$

$$\vec{A} = \begin{bmatrix} A \\ 0 \end{bmatrix}$$

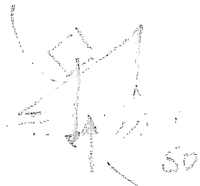
ACCELERATION DU COIN PAR RAPPORT AU SOL

ENSEMBLE BLOC + COIN

$$m\vec{a} = \sum \vec{F}$$ COMPOSANTE HORIZONTALE

$$\boxed{(m+M)A - ma\cos\alpha = \underbrace{0}_{}}$$

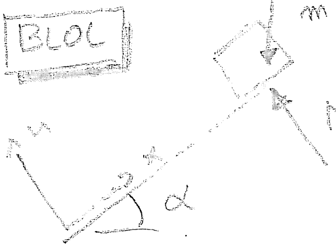
PAS DE FORCE HORIZ !



BLOC

$$mg = \begin{bmatrix} -mg\sin\alpha \\ -mg\cos\alpha \end{bmatrix}$$

$$N = \begin{bmatrix} 0 \\ N \end{bmatrix}$$



$$m(\vec{A} + \vec{a}) = \sum \vec{F}$$ COMPOSANTE HORIZONTALE

$$\boxed{m(\underbrace{A\cos\alpha - a}_{}) = -mg\sin\alpha}$$

ACCELERATION PAR RAPPORT AU SOL !

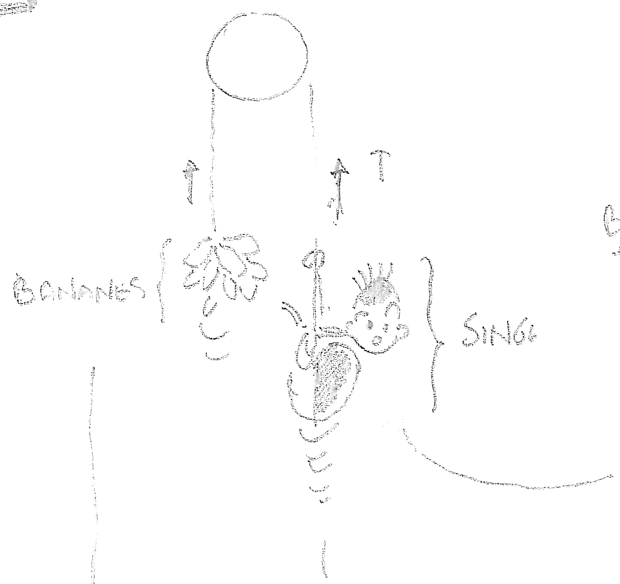
$$mA\cos\alpha - \frac{(M+m)A}{\cos\alpha} = -mg\sin\alpha$$

$$-mA\cos^2\alpha + mA + MA = mg\sin\alpha\cos\alpha$$

$$mA(\underbrace{1-\cos^2\alpha}_{\sin^2\alpha}) + MA = mg\sin\alpha\cos\alpha$$

$$\boxed{A = \frac{mg\sin\alpha\cos\alpha}{M + m\sin^2\alpha}}$$

$\boxed{16}$



BANANES + CORDE

T — FORCE EXERCÉE SUR LA CORDE

T — LE SINGE TIRE LA CORDE } FORCE MOTRICE DE TRACTION

mg

LES BANANES MONTENT SI $T - \underset{\text{POIDS DES BANANES}}{12g} > 0$

$\Downarrow$

$\boxed{T > 12g}$

DYNAMIQUE DU SINGE

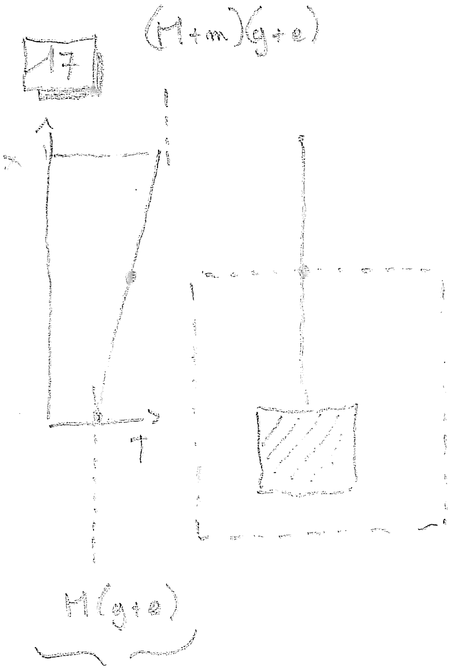
$T - 10g = 10\,a$

a : ACCÉLÉRATION DU SINGE PAR RAPPORT À LA CORDE

CONCLUSION

IL FAUT $a > \underbrace{\dfrac{12g - 10g}{10}}_{1{,}96\ m/s^2}$

$\boxed{17}$



CORPS = ½ CORDE + BLOC

ON APPLIQUE NEWTON !

$\vec{T}$

$\Sigma \vec{F} = (m/2 + M)\,\vec{a}$

$m/2 + M$

$(m/2 + M)\,\vec{g}$

SI ON TIENT COMPTE DU POIDS DE LA CORDE, EN HAUT, LA TENSION REPREND LE POIDS DU BLOC ET DE LA CORDE !

$$T = \underbrace{(m/2 + M)}_{0{,}215}\ \underbrace{(g + e)}_{13{,}8} = 2{,}97\ N$$