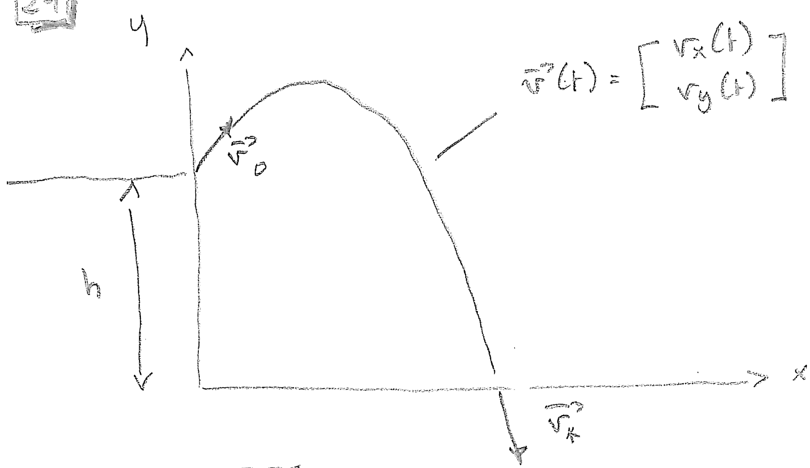


QUESTION

A QUELLE VITESSE v_* LA BALLE TOUCHE LE SOL ?

CINEMATIQUE

MRUA :-)

$$\frac{d}{dt}\left(m\vec{v}\right) = \sum \vec{F}$$

$$\frac{d}{dt}\begin{bmatrix} \cancel{m}\, v_x(t) \\ \cancel{m}\, v_y(t) \end{bmatrix} = \begin{bmatrix} 0 \\ -\cancel{m} g \end{bmatrix}$$

VITESSE INITIALE $\vec{v}(0) = \begin{bmatrix} v_{x0} \\ v_{y0} \end{bmatrix}$

$$\begin{bmatrix} v_x(t) \\ v_y(t) \end{bmatrix} = \begin{bmatrix} v_{x0} \\ v_{y0} - gt \end{bmatrix}$$

POSITION INITIALE $\vec{x}(0) = \begin{bmatrix} 0 \\ h \end{bmatrix}$

$$\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} v_{x0} t \\ h + v_{y0} t - g t^2/2 \end{bmatrix}$$

QUAND EST-CE QUE LA BALLE TOUCHE LE SOL ?

$$\boxed{\underbrace{y(t^*)}_{=0} = h + v_{y0} t^* - g \frac{t^{*2}}{2}}$$

ET

$$\boxed{\underbrace{v_y(t^*)}_{v_{y*}} = v_{y0} - g t_*}$$

$$\boxed{t_* = \frac{v_{y0} - v_{y*}}{g}}$$

$$0 = h + v_{y0} \underbrace{\left(\frac{v_{y0} - v_{y*}}{g}\right)}_{t^*} - \frac{g}{2} \underbrace{\left(\frac{v_{y0} - v_{y*}}{g}\right)^2}_{(t^*)^2}$$

$$0 = h + \frac{1}{g} v_{y0}(v_{y0} - v_{y*}) - \frac{g}{2}\frac{1}{g^2}(v_{y0} - v_{y*})^2$$

EN MULTIPLIANT TOUT PAR $2g$!

$$0 = 2hg + 2v_{y0}^2 - \cancel{2v_{y0}v_{y*}} - v_{y0}^2 - v_{y*}^2 + \cancel{2v_{y0}v_{y*}}$$

$$v_{y*}^2 = v_{y0}^2 + 2hg$$

ON CONCLUT FINALEMENT

$$v_* = \sqrt{\underbrace{v_{x*}^2}_{v_{x0}^2 \text{ MRU :-)}} + \underbrace{v_{y*}^2}_{v_{y0}^2 + 2hg}}$$

$$\boxed{v_* = \sqrt{v_0^2 + 2gh}}$$

BILAN ENERGIE

CONFIGURATION INITIALE

EN $t = 0$
ENERGIE CINETIQUE $= \frac{1}{2} m v_0^2$

CONFIGURATION FINALE

EN $t = t^*$
ENERGIE CINETIQUE $= \frac{1}{2} m v_*^2$

IL Y A UNE UNIQUE FORCE CONSTANTE $\vec{F} = m\vec{g}$!

$$\boxed{\Delta\left[\frac{1}{2} m v^2\right] = \vec{F} \cdot \Delta\vec{x}}$$

$$\begin{bmatrix} 0 \\ -mg \end{bmatrix} \cdot \begin{bmatrix} 0 \\ -h \end{bmatrix}$$

L'ALGEBRE EST BEAUCOUP PLUS SIMPLE !

$$\frac{1}{2}\cancel{m} v_*^2 - \frac{1}{2}\cancel{m} v_0^2 = \cancel{m} g h$$

$$\boxed{v_*^2 = v_0^2 + 2gh \quad \text{:-)}}$$

BILAN
D'ENERGIE

CONFIGURATION
INITIALE

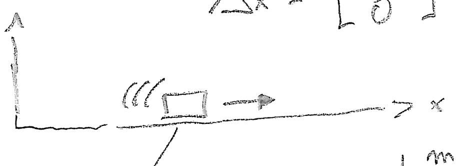
EN $t = 0$

ENERGIE
CINETIQUE $= \frac{1}{2} m v^2$

CONFIGURATION
FINALE

ENERGIE
CINETIQUE $= 0$

$\Delta\vec{x} = \begin{bmatrix} d \\ 0 \end{bmatrix}$



3 FORCES
CONSTANTES

$\vec{mg}$, $\vec{f}$, $\vec{N}$

IL N'Y A DU TRAVAIL
QUE POUR LE FROTTEMENT !

$W = -d \mu m g$

$\underbrace{\begin{bmatrix} -\mu m g \\ 0 \end{bmatrix}}_{\text{FORCE}} \cdot \underbrace{\begin{bmatrix} d \\ 0 \end{bmatrix}}_{\text{DEPLACEMENT}}$

LE
TRAVAIL
A UN
SIGNE !

LE TRAVAIL
DE LA FORCE
DE GRAVITE EST NUL !

$\underbrace{\begin{bmatrix} 0 \\ -mg \end{bmatrix} \cdot \begin{bmatrix} d \\ 0 \end{bmatrix}}_{= 0 \,!}$

IDEM
POUR LA
REACTION !

$$\boxed{\underbrace{\Delta\left(\frac{1}{2} m v^2\right)}_{0 - \frac{1}{2} m v^2} = \underbrace{\sum \vec{F} \cdot \Delta\vec{x}}_{-d \mu m g}}$$

$\frac{1}{2} \cancel{m} v^2 = d \mu \cancel{m} g$

$$\boxed{d = \frac{v^2}{2 \mu g}}$$

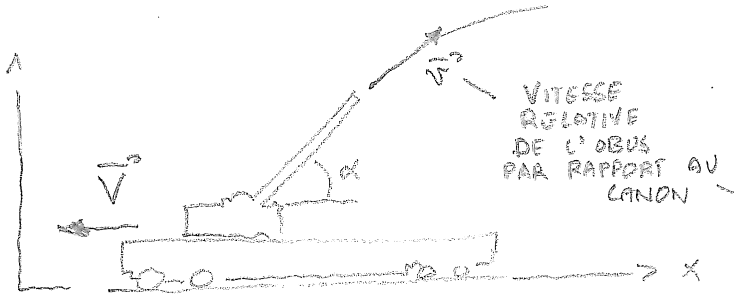
$[m^2/s^2]$

$[m/s^2]$

CHECK
DIMENSIONS !

VALEURS
NUMERIQUES

$d = \underbrace{\frac{4 \times 4}{2 \times 0{,}6 \times 9{,}81}}_{1{,}36 \text{ m}}$



LA VITESSE ABSOLUE REELLE EST DONC $\vec{v} + \vec{V}$!! POUR UN REPERE INERTIEL

CONFIGURATION AVANT

$$\boxed{\text{QUANTITE DE MOUVEMENT HORIZONTALE} = 0}$$

CONFIGURATION APRES LE TIR

$$\boxed{\text{QUANTITE DE MOUVEMENT HORIZONTALE} = m(v\cos\alpha - V) - MV}$$

PAS DE FORCES HORIZONTALES

ON SUPPOSE QUE LE WAGON ROULE SANS AUCUN FROTTEMENT SUR LES RAILS

$$\boxed{\Delta(m v_x) = 0 \quad !}$$

$$m(v\cos\alpha - V) - MV = 0$$

$$\boxed{V = \frac{m v \cos\alpha}{(M+m)}}$$

$$\boxed{\operatorname{tgt}\Theta = \frac{v_y}{v_x} = \frac{v\sin\alpha}{v\cos\alpha - V}}$$

$$= \frac{v\sin\alpha}{v\cos\alpha\left(1 - \frac{m}{(M+m)}\right)}$$

$$\boxed{\operatorname{tgt}\Theta = \frac{(M+m)}{M}\operatorname{tgt}\alpha}$$

ON OBSERVE BIEN QUE $\Theta \neq \alpha$!

$\boxed{27}$

JUSTE APRES LE CHOC		À L'ARRET
ENERGIE CINETIQUE $\frac{1}{2}(m_1+m_2)\,v_c^2$	$\longrightarrow$	ENERGIE CINETIQUE 0 !

TRAVAIL DU FROTTEMENT

$$W = -\mu_c (m_1+m_2)\, g\, d$$

BILAN D'ENERGIE

$$\frac{1}{2} v_c^2 = \mu_c\, g\, d$$

$$\boxed{v_c = \sqrt{2\,\mu_c\, g\, d}}$$

$\mu_c = 0{,}6$, $g = 9{,}81$, $d = 4$

$= 6{,}86$ m/s

DIMENSION CHECK $\sqrt{\left[\frac{m}{s^2}\right][m]}$ OK :-)

BILAN DE QUANTITE DE MOUVEMENT HORIZONTALE

AVANT LE CHOC		APRES LE CHOC
QUANTITE DE MVT $m_1 v_1$	$\longrightarrow$	QUANTITE DE MVT $(m_1+m_2)\,v_c$

$$\boxed{v_1 = \frac{(m_1+m_2)\,v_c}{m_1}}$$

VALEURS NUMERIQUE

$$v_c = \underbrace{\frac{2500}{1500}\ 6{,}86}_{11{,}43\ m/s}$$

$\boxed{28}$

AVANT LE CHOC

QUANTITE DE MVT

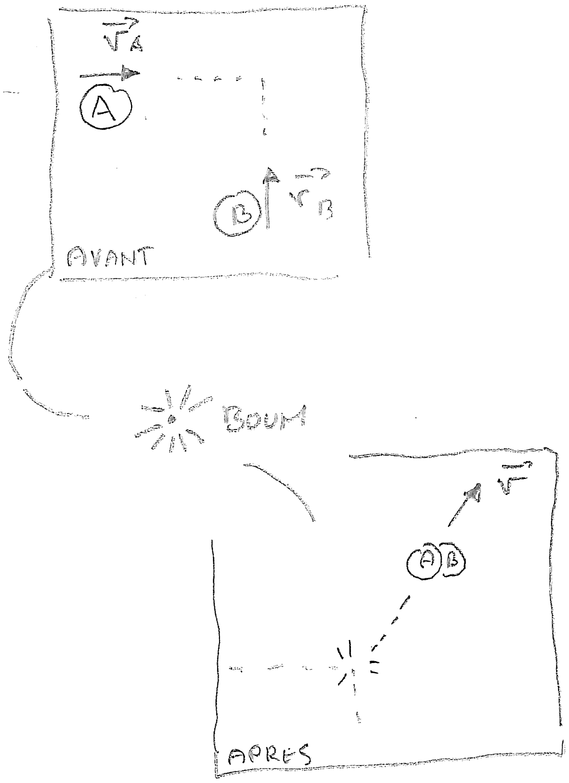
$$m_A \begin{bmatrix} v_A \\ 0 \end{bmatrix} + m_B \begin{bmatrix} 0 \\ v_B \end{bmatrix} \longrightarrow$$

APRES LE CHOC

$$m_A + m_B \underbrace{\begin{bmatrix} v_x \\ v_y \end{bmatrix}}_{\vec{v}}$$

$$\begin{cases} m_A v_A = (m_A + m_B) v_x \\ m_B v_B = (m_A + m_B) v_y \end{cases}$$

$$\vec{v} = \begin{bmatrix} \frac{90}{200} \; 8 \\ \\ \frac{110}{200} \; 7{,}5 \end{bmatrix} = \begin{bmatrix} 3{,}6 \\ \\ 4{,}13 \end{bmatrix}$$



PERTE ENERGIE CINETIQUE

$$= \underbrace{\underbrace{\left[\frac{1}{2} m_A v_A^2 + \frac{1}{2} m_B v_B^2\right]}_{5974 \text{ J}} - \underbrace{\left[\frac{1}{2}(m_A + m_B)(v_x^2 + v_y^2)\right]}_{3003 \text{ J}}}_{2971 \text{ J}}$$

29

220 [Km/h]

$$\boxed{220 \; \frac{1000}{3600} \; [m/s] = 61,1 \; [m/s]}$$

A SAVOIR FAIRE !

AVANT

QUANTITE DE MOUVEMENT = 0

APRES

QUANTITE DE MOUVEMENT = $m v$

0,046   61,1

$$\boxed{\Delta [m v] = \underbrace{F \Delta t}_{\text{IMPULSION}}}$$

$$F = \frac{m v}{\Delta t}$$

$m = 46 \; 10^{-3}$, $v = 61,1$, $\Delta t = 5 \; 10^{-4}$

F = 5620 N

30

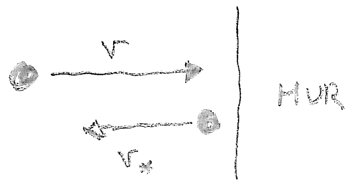
AVANT

ENERGIE CINETIQUE = $\frac{1}{2} m v^2$

APRES

$$\underbrace{0,81 \; \frac{1}{2} m v^2}_{\frac{1}{2} m v_*^2} = \text{ENERGIE}$$

$$v_*^2 = 0,81 \, v^2$$
$$v_* = 0,9 \, v$$



$$\boxed{\Delta [m v] = \underbrace{F \Delta T}_{\text{IMPULSION}}}$$

ATTENTION LA VITESSE CHANGE D'ORIENTATION !

$$\underbrace{m \begin{bmatrix} -0,9 v \\ 0 \end{bmatrix}}_{\text{APRES !}} - \underbrace{m \begin{bmatrix} v \\ 0 \end{bmatrix}}_{\text{AVANT !}}$$

IMPULSION = $1,9 \; m v$

$m = 60 \; 10^{-3}$, $v = 30$

= 3,42 [kg m/s]