Introduction to scientific computing: homework 3

Here, we'd like that you verify the different properties of standard iterative methods like Jacobi or Gauss-Seidel. The smoother that you have already implemented is clearly a Jacobi iteration.

1. Compute the initial Fourier spectrum of your image. For that, you have to use an external library/program. You can do that in matlab too (`fft2`).

2. Apply some Jacobi iterations to the image and verify the impact of the process on the amplitude of the Fourier modes. More precisely, verify the relation that was demonstrated during the lecture 3 i.e. that the eigenvalues of the amplification matrix were:

$$\lambda^{k,l} = \frac{1}{2}\left(\cos\frac{k\pi}{N} + \cos\frac{l\pi}{N}\right) \quad \text{if} \quad \omega = 1,$$

$$\lambda^{k,l} = \frac{1}{2} + \frac{1}{4}\left(\cos\frac{k\pi}{N} + \cos\frac{l\pi}{N}\right) \quad \text{if} \quad \omega = \frac{1}{2}.$$

3. Write a program that implements a Multi-Grid iteration using full weighting restriction and linear interpolation for prolongation. In principle, this program should be able to deal with $\mu = 1, 2$ and with an arbitrary number of grids. Here, you should make the process converge. Tell us too to what this process is supposed to converge. Compare the rate of convergence of the multigrid scheme to the one of a simple one level Jacobi smoother.

Note : Howework 3 is due tuesday march 16th.