Une piscine de fluide non-newtonien…

Voilà, un fluide vraiment difficile à modéliser !
C'est autre chose qu'un simple écoulement de fluide newtonien

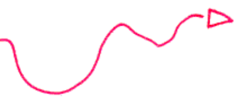
C'est l'objet du cours de rhéologie...
Les fluides : ce n'est pas que l'aéronautique !
L'océan, c'est aussi un fluide !

WHY 1D ? $\longrightarrow$ TO AVOID TECHNICALITIES !

SIMPLEST FLUID MODEL :
"THE NEWTONIAN VISCOUS MODEL"

$$
\begin{cases}
\gamma \dfrac{Dp}{Dt} - \beta \dfrac{DT}{Dt} + \underline{\nabla} \cdot \underline{v} = 0 \\[2ex]
\rho \dfrac{D\underline{v}}{Dt} = - \underline{\nabla} p + \underline{\nabla} \left(\varkappa \, \underline{\underline{S}} : \underline{\underline{d}} \right) + \underline{\nabla} \cdot \left(2\mu \, \underline{\underline{d}}^{d} \right) + \rho \underline{g} \\[2ex]
\rho c_p \dfrac{DT}{Dt} - \beta T \dfrac{Dp}{Dt} = \varkappa \left(\underline{\underline{S}} : \underline{\underline{d}} \right)^2 + 2\mu \left(\underline{\underline{d}}^{d} : \underline{\underline{d}}^{d} \right) + r + \underline{\nabla} \cdot \left(k \underline{\nabla} T \right)
\end{cases}
$$

INCOMPRESSIBLE FLOW

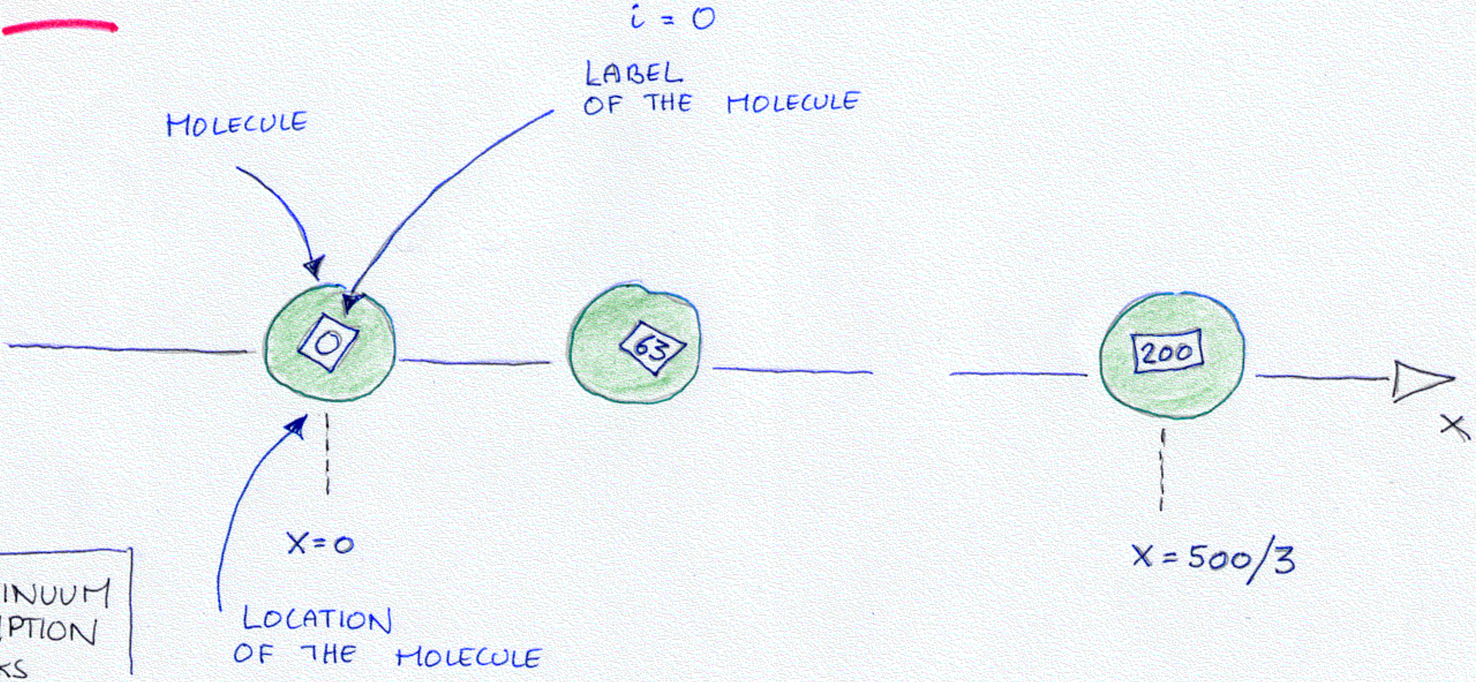


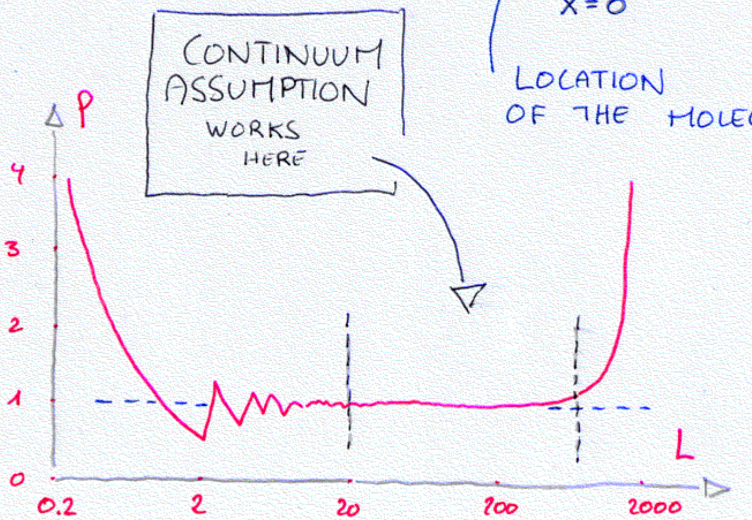
$$
\underline{\nabla} \cdot \underline{v} = 0
$$

$$
\rho \frac{D\underline{v}}{Dt} = - \underline{\nabla} p + \underline{\nabla} \cdot \left(2\mu \, \underline{\underline{d}} \right) + \rho \underline{g}
$$

$$
\rho c \frac{DT}{Dt} = 2\mu \left(\underline{\underline{d}} : \underline{\underline{d}} \right) + r + \underline{\nabla} \cdot \left(k \underline{\nabla} T \right)
$$

STUPID EXAMPLE



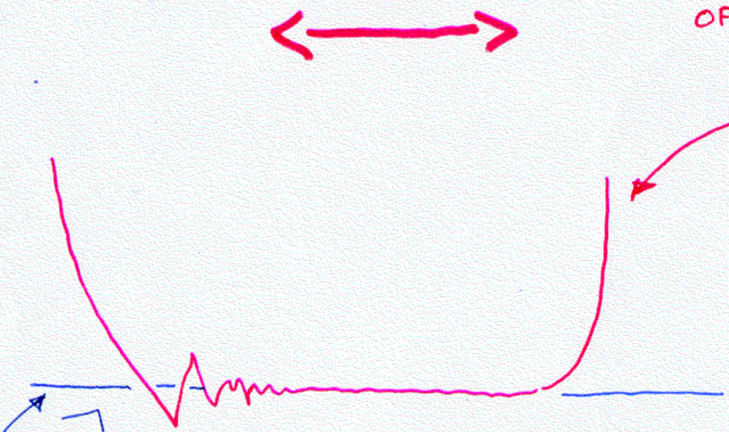


ASSUME : $X = \dfrac{i}{1 + \dfrac{i}{1000}}$

SOLIDS
LIQUIDS
GASES

SEPARATION OF LENGTH SCALES

10^5	LINEAR	1D
10^{15}	VOLUME	3D



AVERAGED MASS OF THE CONTINUUM

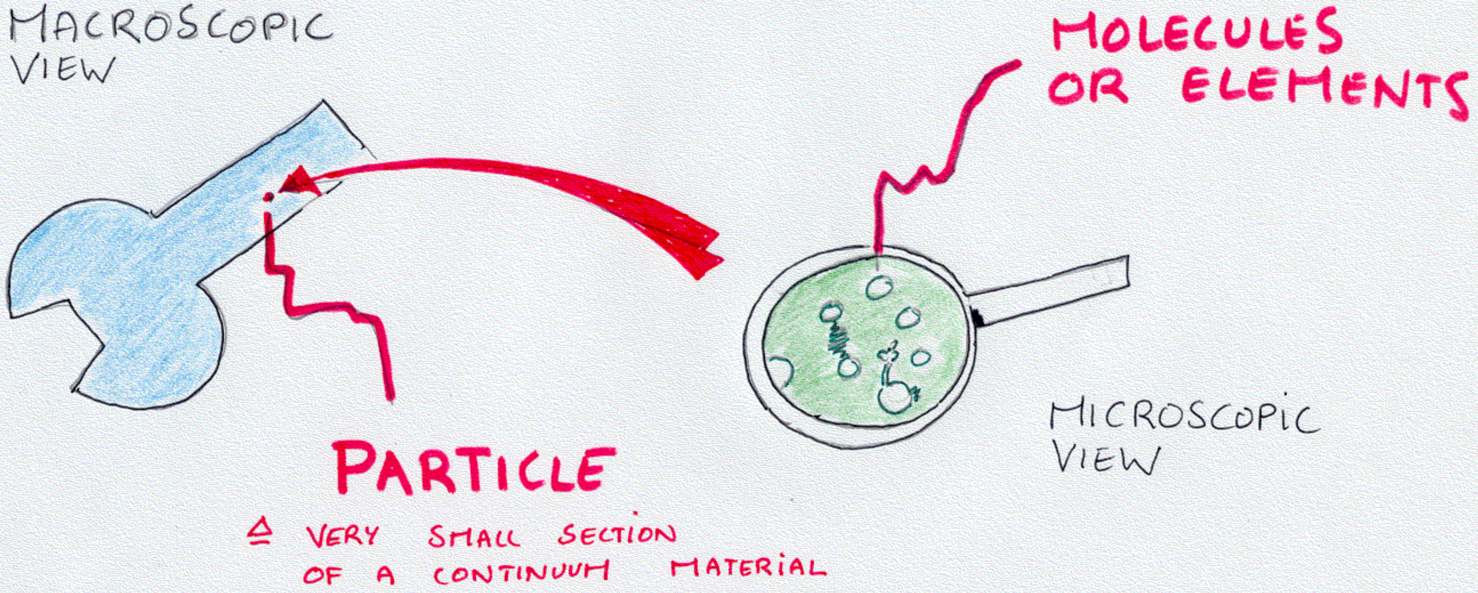
$\rho(L, x, t)$

$\cong$ CONTINUUM ASSUMPTION

$\rho(x, t)$ DENSITY OF THE CONTINUUM

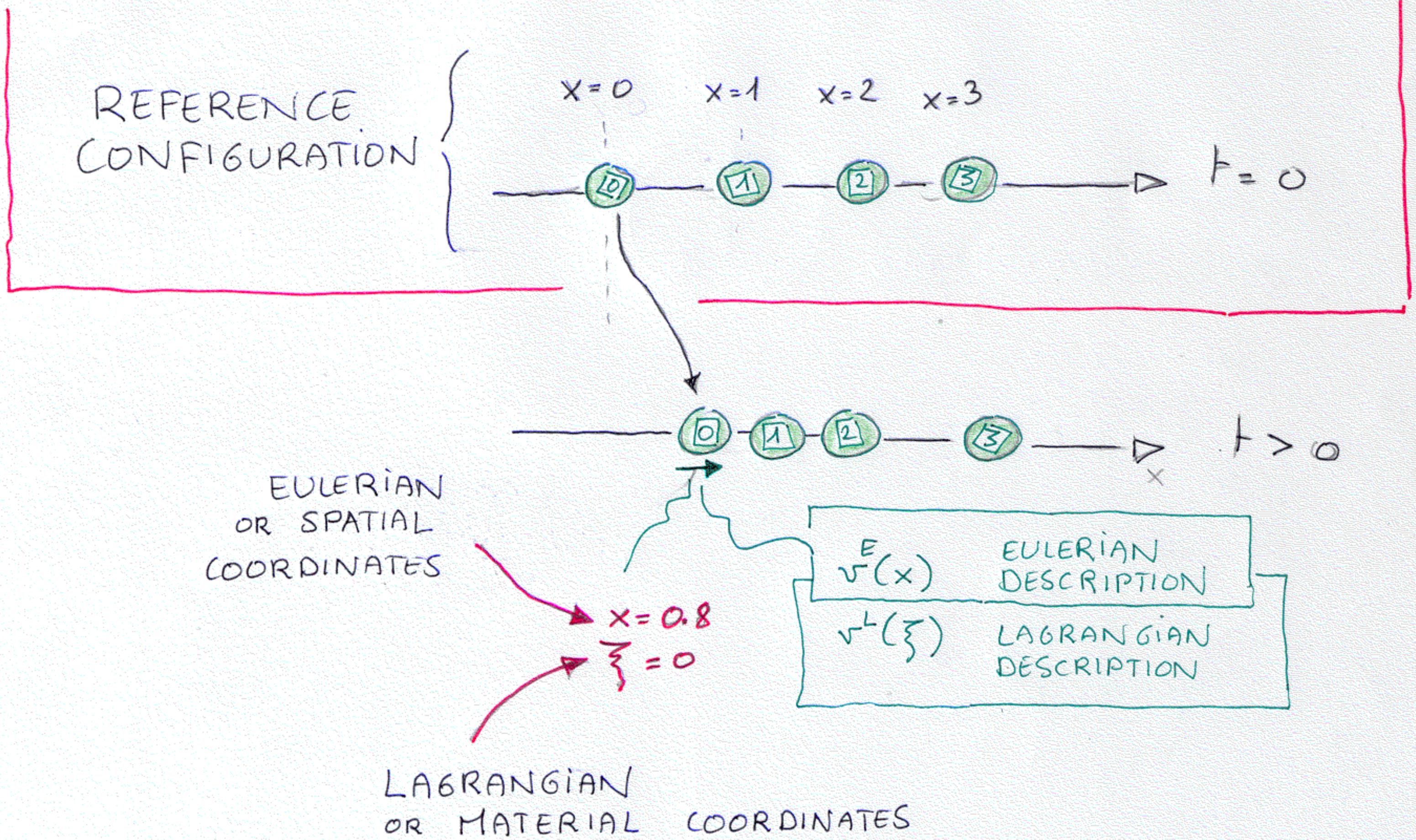
FORM OF UNCERTAINTY PRINCIPLE

ACCURACY OF THE CONTINUUM HYPOTHESIS IS DIRECTY DEPENDENT UPON THE SEPARATION OF THE LENGTH SCALES OF MOLECULAR FLUCTUATIONS AND THE LENGTH SCALES OF MACROSCOPIC VARIATIONS



DESCRIBING THE MOTION USING AVERAGED CONTINUOUS QUANTITIES

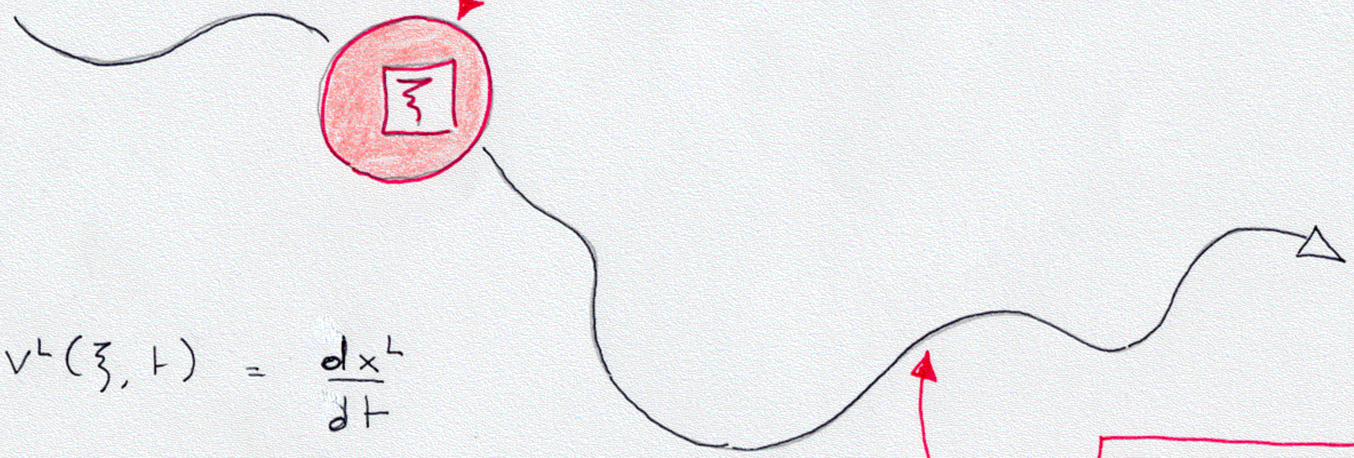
LIKE HOUSE NUMBERING ALONG A STREET



(IT IS IN FACT A "CONTINUOUS" NUMBERING!)

LAGRANGIAN DESCRIPTION

PARTICLE ($\neq$ MOLECULE !)



PARTICLE PATH $\underline{x}^L(\xi, t)$

$$v^L(\xi, t) = \frac{dx^L}{dt}$$

$$a^L(\xi, t) = \frac{d^2x^L}{dt^2}$$

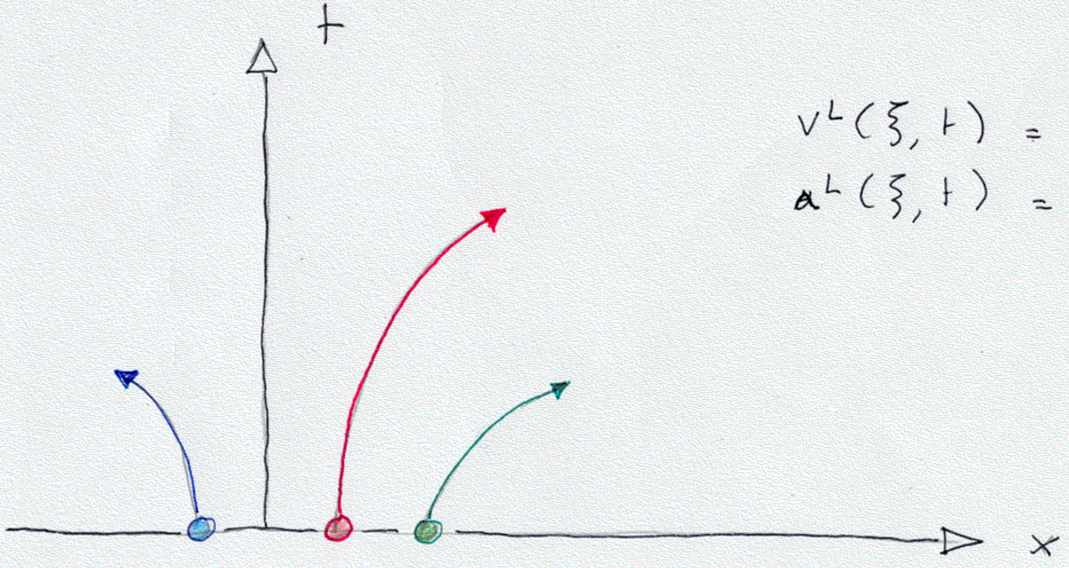
LAGRANGIAN DESCRIPTION OF THE VELOCITY AND THE ACCELERATION

$\frac{d}{dt} = \frac{\partial}{\partial t}$ BECAUSE ξ DOES NOT DEPEND OF t !

EXAMPLE 2 :

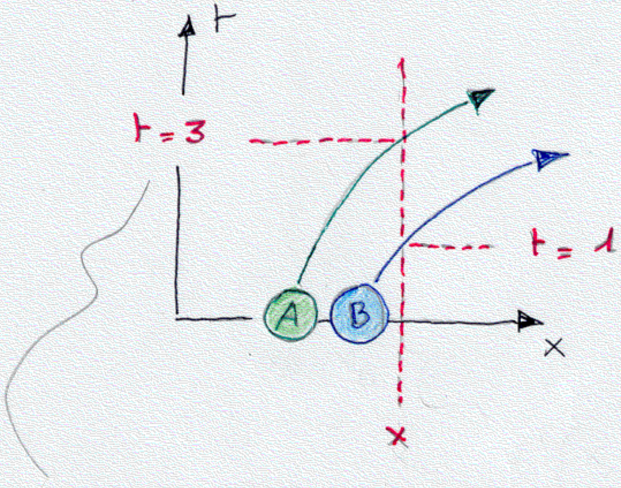
$$\underbrace{x = x^L(\xi, t)} = \underbrace{\xi + \xi t^2}$$

ASSUME THAT PARTICLES
OF CONTINUUM MOVE ACCORDINGLY TO SUCH A RULE



$$v^L(\xi, t) = 2\xi t$$
$$a^L(\xi, t) = 2\xi$$

EULERIAN DESCRIPTION



$$\overbrace{v^E(x, t=1)}^{\text{EULERIAN DESCRIPTION OF THE VELOCITY}} = \underbrace{v^L(\xi = B, t=1)}_{\text{LAGRANGIAN DESCRIPTION OF THE VELOCITY}}$$

$$v^E(x, t=3) = v^L(\xi = A, t=3)$$

THE USUAL DESCRIPTION TO USE IS THE EULERIAN DESCRIPTION

BUT, HOW TO FIND v_E ?

QUANTITY OF IMPORTANCE

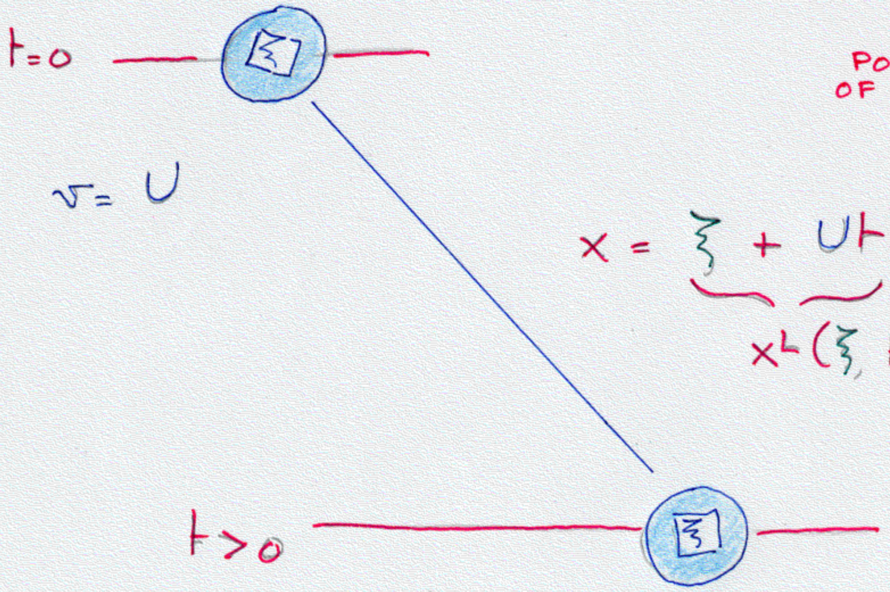
$$\underbrace{\xi}_{\text{LABEL OF PARTICLE WHICH IS AT POSITION } x \text{ AT TIME } t}{}^{E}(x,t)$$

INVERSE FUNCTION *

OF $\underbrace{x}_{\text{POSITION OF PARTICLE } \xi \text{ AT TIME } t}{}^{L}(\xi,t)$

$$\boxed{\begin{aligned} \xi^E\left(x^L(\xi,t),t\right) &= \xi \\ x^L\left(\xi^E(x,t),t\right) &= x \end{aligned}}$$

* SUBSCRITS E AND L ARE IN FACT USELESS AS $\xi^L = \xi$ AND $x^E = x$!
(BUT .... FOR SOME NOTATIONAL SYMETRY, THE AUTHOR LIKES THEM)



$$x = \underbrace{\xi + Ut}_{x^L(\xi,t)} \longrightarrow \xi = \underbrace{x - Ut}_{\xi^E(x,t)}$$

* FOR A GIVEN TIME OF COURSE !

EXAMPLE 2 REVISITED ....

$$x = \underbrace{\xi + \xi t^2}_{x^L(\xi, t)}$$

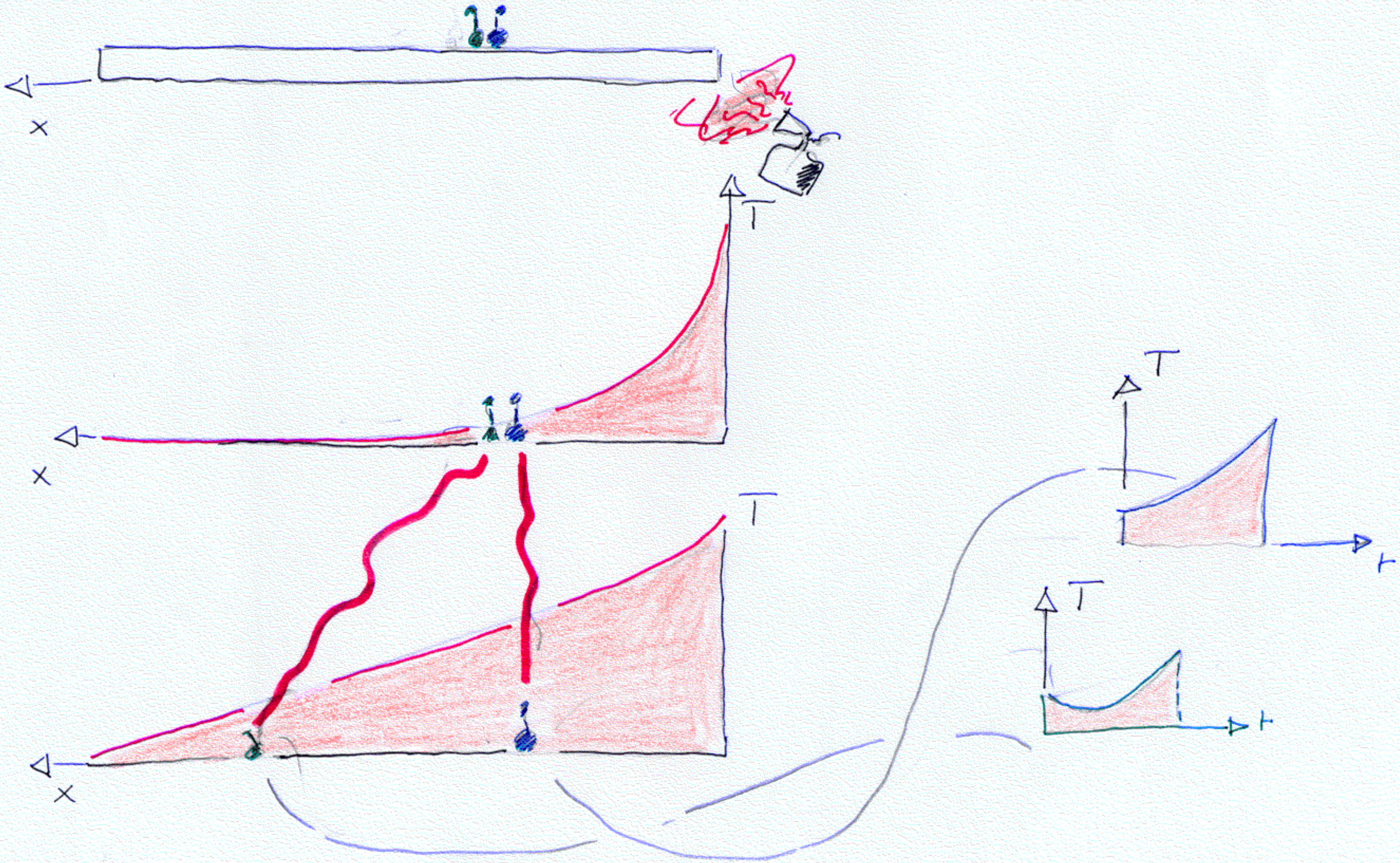
$$\xi(1+t^2) = x$$

$$\xi = \underbrace{\frac{x}{1+t^2}}_{\xi^E(x,t)}$$

$$v^E(x,t) = \underbrace{v^L(\xi^E(x,t), t)}_{= 2\xi t} = 2\frac{x}{1+t^2}\,t = \frac{2xt}{1+t^2}$$

$$a^E(x,t) = \frac{2x}{1+t^2}$$

DIFFERENCE BETWEEN WHAT A STATIONARY OBSERVER AND A MOVING PARTICLE EXPERIENCE !



MATERIAL DERIVATIVE

$$f = f^L(\xi, t) = f^E(\overbrace{x^L(\xi,t)}^{x}, t)$$

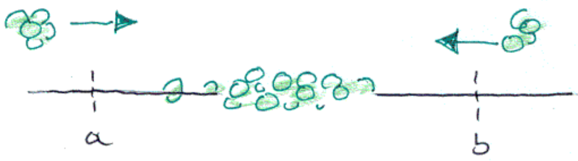
$$\frac{df}{dt} = \frac{\partial f^L}{\partial t} = \frac{\partial f^E}{\partial t} + \frac{\partial f^E}{\partial x}\underbrace{\frac{\partial x^L}{\partial t}}_{v^L = v^E}$$

$$= \underbrace{\frac{\partial f^E}{\partial t} + v^E \frac{\partial f^E}{\partial x}}_{\frac{Df^E}{Dt}(x,t)}$$

RATE OF CHANGE OF f AT FIXED POSITION

RATE OF CHANGE OF f THAT A PARTICLE CAN "SEE" BECAUSE IT MOVES THROUGH A SPATIALLY NON-UNIFORM FIELD OF f

MASS CONSERVATION



RATE OF MASS INCREASE IN THE INTERVAL

= RATE OF MASS INFLUX ACROSS THE ENDS

$\forall$ INTERVALS !

$$\frac{\partial}{\partial t}\int_a^b \rho \, dx = \rho_a v_a - \rho_b v_b$$

$$\int_a^b \frac{\partial \rho}{\partial t}\, dx + [\rho v]_a^b = 0$$

$$\int_a^b \left(\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) \right) dx = 0$$

$\forall a < b$

IF F CONTINUOUS !!

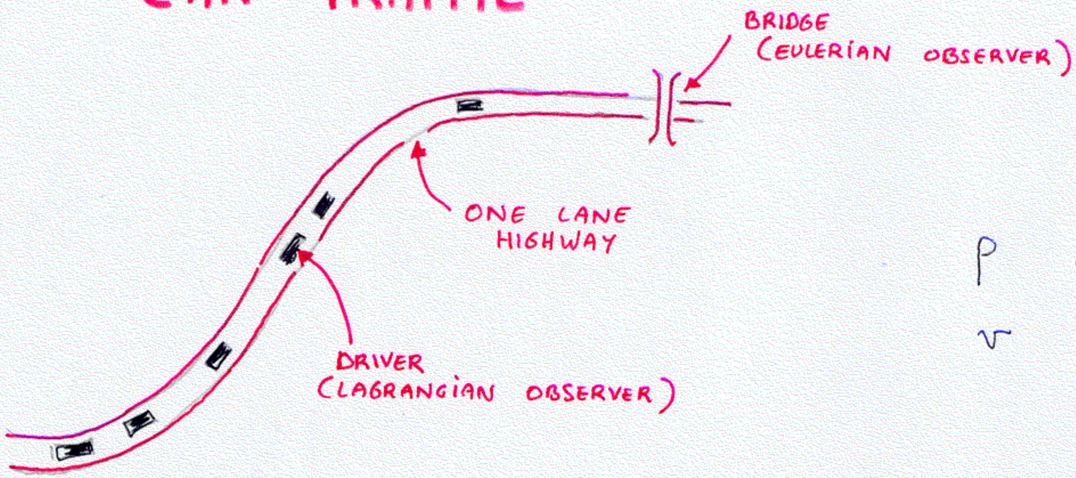
$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0}$$

CONTINUITY EQUATION

ALSO

$$\frac{D\rho}{Dt} + \rho \frac{\partial v}{\partial x} = 0$$

CAR TRAFFIC



ρ DENSITY OF CARS [CARS / Km]

v VELOCITY OF CARS [Km / h]

CONTINUITY EQUATION

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho v) = 0}$$

$+$

STATE OR CONSTITUTIVE EQUATION

$$\boxed{v = \hat{v}(\rho)}$$

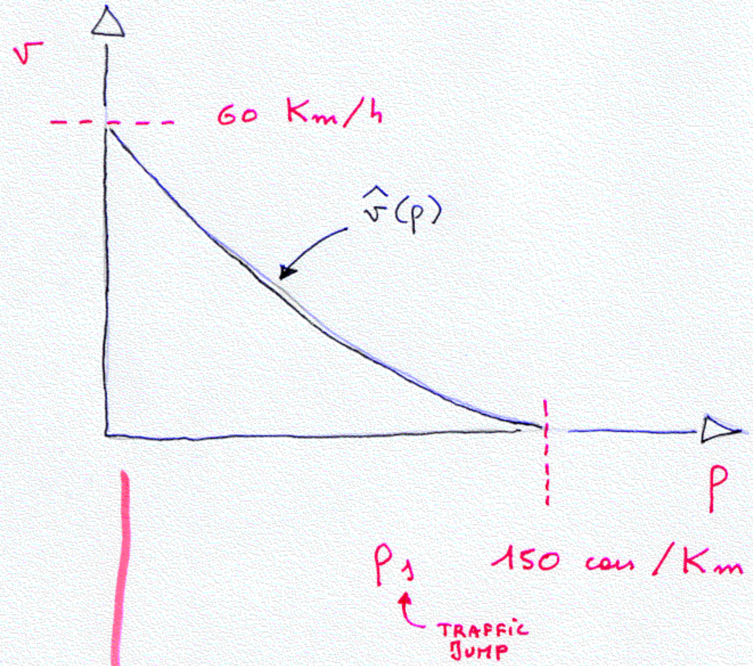
= REASONABLE MODEL OF THE DRIVER BEHAVIOUR

LIGHT TRAFFIC $\longrightarrow$ SPEED LIMIT !

HEAVY TRAFFIC $\longrightarrow$ SLOW DOWN !

CONSTITUTIVE MODEL IS NEEDED TO CLOSE THE PROBLEM

STATE EQUATION FOR CAR TRAFFIC

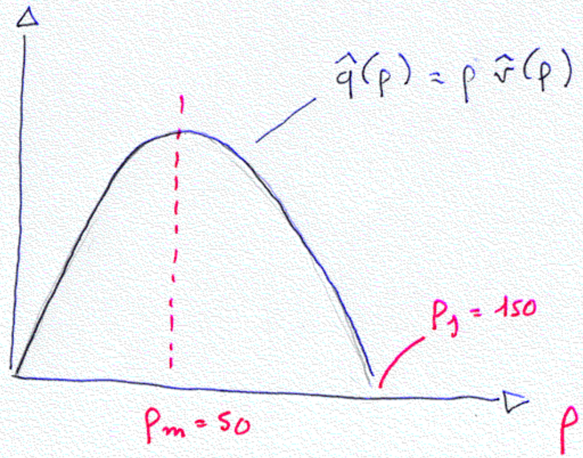


IN AGREEMENT WITH EXPERIMENTAL EVIDENCES

[LINCOLN TUNNEL NEW YORK ...]

FLUX OF CARS

$q \triangleq \rho v$



SOLVING THE CAR TRAFFIC PROBLEM

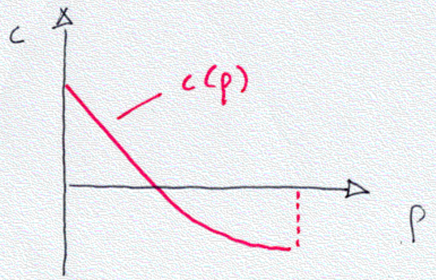
$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\underbrace{\rho v}_{= \hat{q}(\rho)}) = 0$$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\hat{q}(\rho)) = 0$$

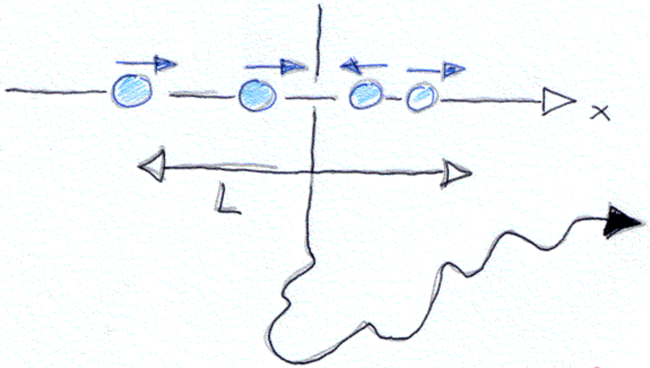
$$\boxed{\frac{\partial \rho}{\partial t} + \underbrace{\hat{q}'(\rho)}_{\triangleq\, c(\rho)} \frac{\partial \rho}{\partial x} = 0}$$

VELOCITY AT WHICH FLUCTUATIONS IN TRAFIC DENSITY PROGRESS.

SINGLE NON-LINEAR HYPERBOLIC DIFFERENTIAL EQUATION



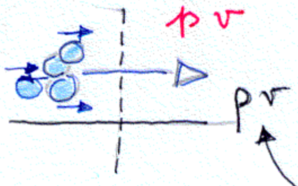
MOMENTUM DENSITY



$$\overbrace{p(x,t)}^{\text{MOMENTUM DENSITY}} \triangleq \underbrace{\rho(x,t)}_{\text{MASS DENSITY}} \, v(x,t)$$

THIS IS IN FACT THE DEFINITION OF THE CONTINUOUS VELOCITY, THE MOMENTUM AND THE MASS DENSITY BEING DEFINED BY THE AVERAGING PROCESS !

MOMENTUM FLUX



MASS FLUX