Retrouvons notre bon vieux nombre de Reynolds…

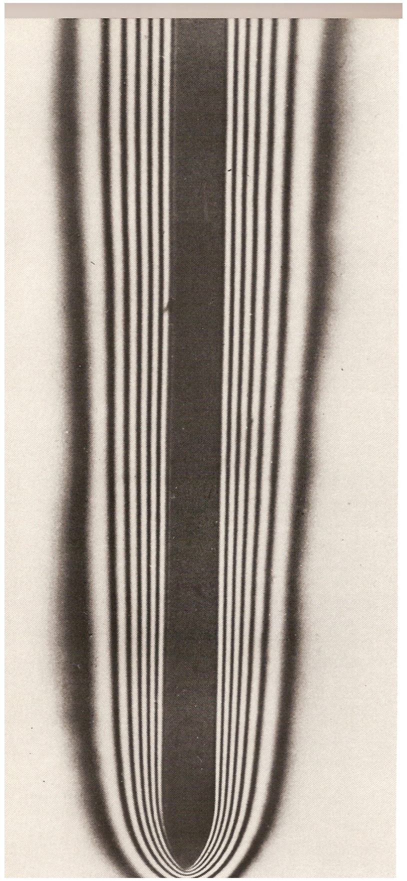
Effets visqueux
Diffusion de la quantité de mouvement

$$\boxed{\rho(\mathbf{v}\cdot\boldsymbol{\nabla})\mathbf{v} \quad \mathcal{O}(\rho U^2/L)} = -\boldsymbol{\nabla} p + \boxed{\mu\boldsymbol{\nabla}^2\mathbf{v} \quad \mathcal{O}(\mu U/L^2)}$$

Effets d'inertie
Transport de la quantité de mouvement

$$Re = \frac{\boxed{\text{Forces d'inertie}}}{\boxed{\text{Forces visqueuses}}} = \frac{\rho U^2/L}{\mu U/L^2} = \frac{\rho U L}{\mu}$$

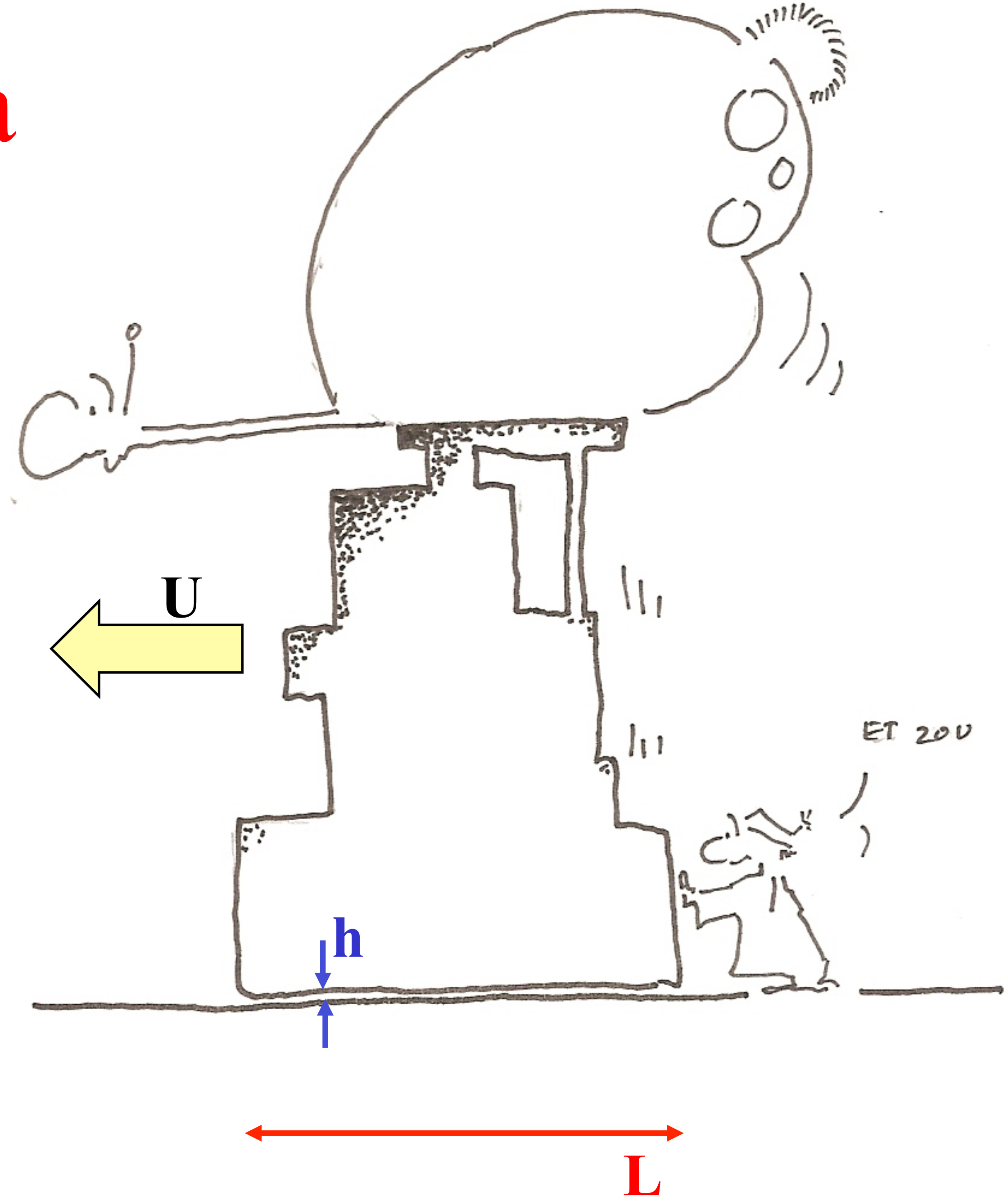
Mais que faire pour des écoulements avec deux échelles spatiales ?



Convection naturelle le long d'une plaque verticale : écoulement laminaire permanent

Lubrification et convoyage hydraulique : butée Michell

Théorie de la lubrification



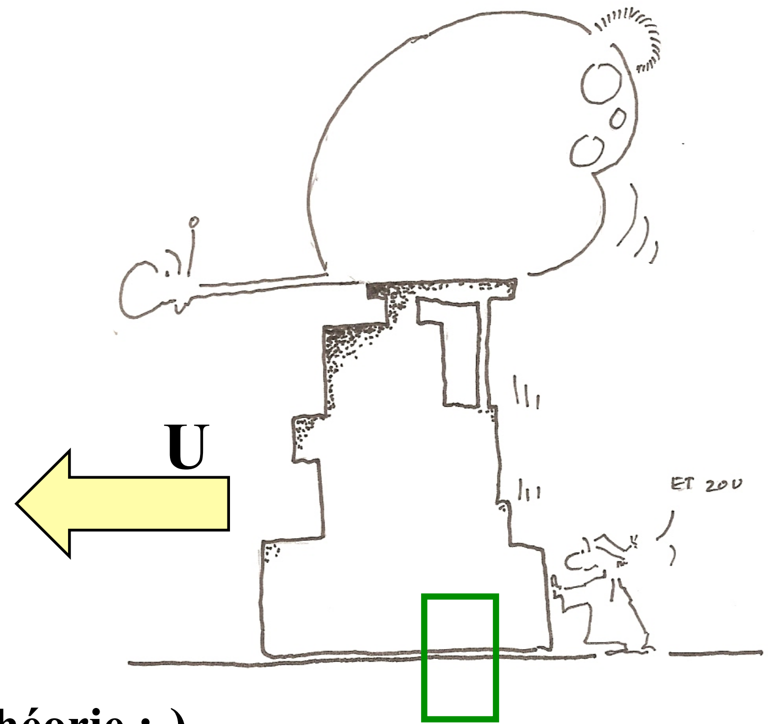
Convoyage hydraulique de charges très importantes :

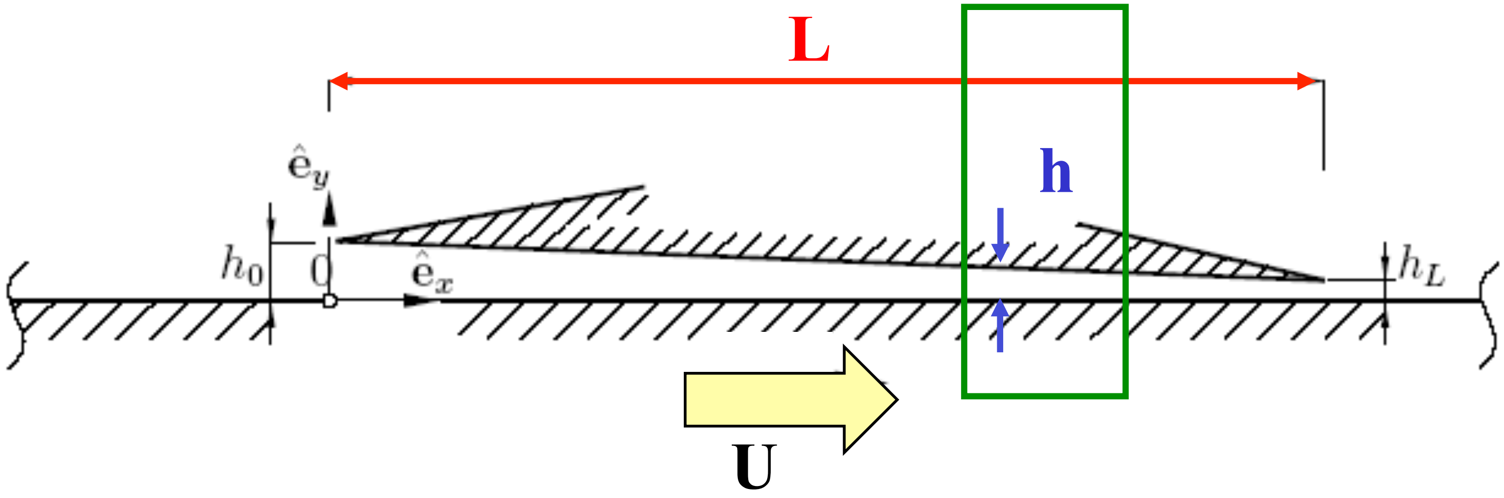
- ***turbines hydroélectriques***
- ***applications marines***
- ***butées hydrauliques***

Théorie de la lubrification

$$\boxed{h \ll L}$$

Hypothèse géométrique de base
Valable dans la zone centrale uniquement en théorie :-)



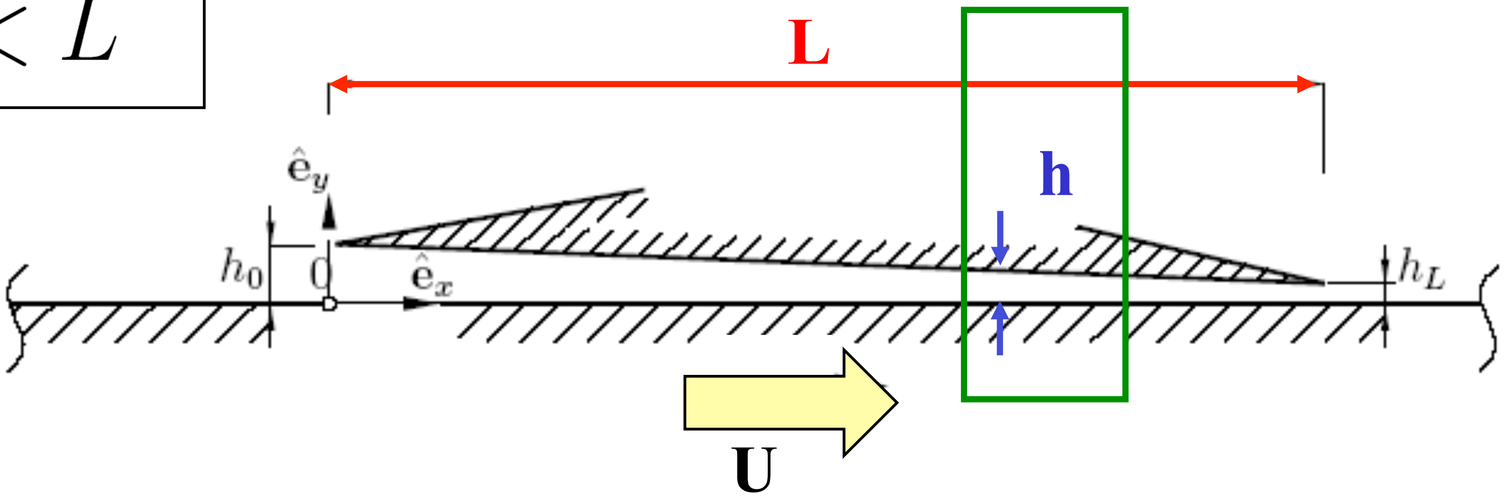


Ecoulements incompressibles plans stationnaires

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} \\
\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}
\end{aligned}
$$

Que deviennent ces équations ?

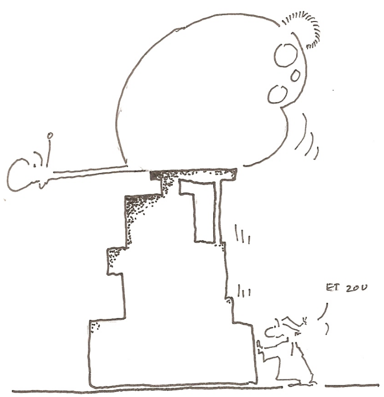
$$\boxed{h \ll L}$$



$$h \ll L$$

$$\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\rho\, u \frac{\partial u}{\partial x} + \rho\, v \frac{\partial u}{\partial y} &= -\frac{\partial p}{\partial x} + \mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} \\
\rho\, u \frac{\partial v}{\partial x} + \rho\, v \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu \frac{\partial^2 v}{\partial x^2} + \mu \frac{\partial^2 v}{\partial y^2}
\end{aligned}$$

Longueur horizontale caractéristique : L
Longueur verticale caractérisitique : h
Vitesse horizontale caractéristique : U



Comment choisir une vitesse verticale caractéristique ?

$$\underbrace{\boxed{\frac{\partial u}{\partial x}}}_{\mathcal{O}(U/L)} + \underbrace{\boxed{\frac{\partial v}{\partial y}}}_{\mathcal{O}(V/h)} = 0$$

Il ne faut pas définir de vitesse caractéristique verticale !

$$\boxed{V = \frac{Uh}{L} \ll U}$$

Quand peut-on négliger les termes d'inertie ?

$$\underbrace{\boxed{\rho\, u\, \frac{\partial u}{\partial x}}}_{}^{\mathcal{O}(\rho U^2/L)} + \overbrace{\boxed{\rho\, v\, \frac{\partial u}{\partial y}}}^{\mathcal{O}(\rho U^2/L)}_{\mathcal{O}(\rho V U/h)} = -\frac{\partial p}{\partial x} + \boxed{\mu \cancel{\frac{\partial^2 u}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 u}{\partial y^2}}$$

$$\mathcal{O}(\mu U/L^2) \ll \mathcal{O}(\mu U/h^2)$$

Hypothèse de lubrification :
Écoulements rampants

$$\frac{\boxed{\text{Forces d'inertie}}}{\boxed{\text{Forces visqueuses}}} = \frac{\rho U^2/L}{\mu U/h^2} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Et l’autre équation ?

$$\overset{\mathcal{O}(\rho U^2 h/L^2)}{\boxed{\rho\, u\, \frac{\partial v}{\partial x}}} + \overset{\mathcal{O}(\rho U^2 h/L^2)}{\boxed{\rho\, v\, \frac{\partial v}{\partial y}}} = -\frac{\partial p}{\partial y} + \boxed{\cancel{\mu \frac{\partial^2 v}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 v}{\partial y^2}}$$

$$\mathcal{O}(\mu U h/L^3) \ll \mathcal{O}(\mu U/Lh)$$

On obtient la
<u>même condition</u>…

$$\frac{\boxed{\text{Forces d'inertie}}}{\boxed{\text{Forces visqueuses}}} = \frac{\rho U^2 h/L^2}{\mu U/Lh} = \underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\boxed{\cancel{\rho\, u \frac{\partial v}{\partial x}}} + \boxed{\cancel{\rho\, v \frac{\partial v}{\partial y}}} = -\frac{\partial p}{\partial y} + \boxed{\cancel{\mu \frac{\partial^2 v}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 v}{\partial y^2}}$$

$$\mathcal{O}(\mu U/Lh)$$

Et la pression ?

$$p(x,y) - p_0 = \boxed{p(x,0) - p_0} + \boxed{y \cancel{\frac{\partial p}{\partial y}}\bigg|_{y=0}}$$

$$\mathcal{O}(\mu UL/h^2) \gg \mathcal{O}(\mu UL/L^2)$$

$$\mathcal{O}(\mu U/h^2)$$

$$\boxed{\cancel{\rho\, u \frac{\partial u}{\partial x}}} + \boxed{\cancel{\rho\, v \frac{\partial u}{\partial y}}} = -\frac{\partial p}{\partial x} + \boxed{\cancel{\mu \frac{\partial^2 u}{\partial x^2}}} + \boxed{\mu \frac{\partial^2 u}{\partial y^2}}$$

Equations de Reynolds (1889)

Théorie de la lubrification

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{dp}{dx} + \mu \frac{\partial^2 u}{\partial y^2}$$

Film fluide mince

$$h \ll L$$

Hypothèse de lubrification :
Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

Est-ce que l'hypothèse de lubrification est réaliste ?

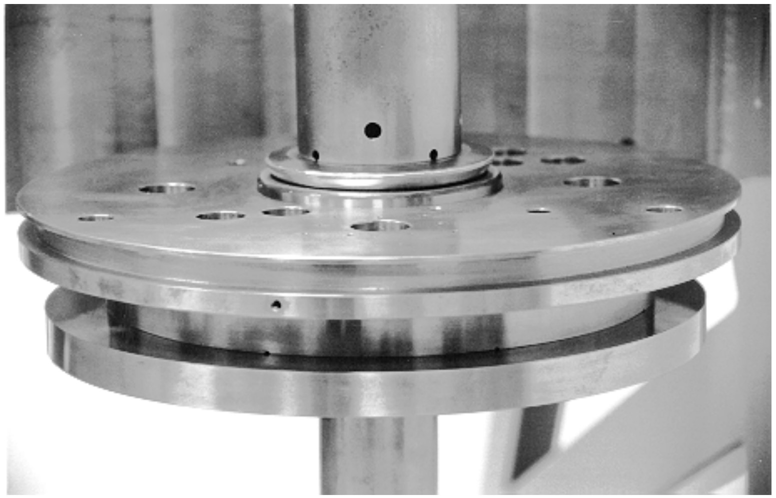
$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

0.0375

$$\begin{aligned} L &= 10\ cm \\ h &= 0.5\ mm \\ U &= 1\ m/s \end{aligned}$$

$$\begin{aligned} \rho &= 900\ kg/m^3 \\ \mu &= 60\ 10^{-3}\ Ns/m^2 \end{aligned}$$

Huile SAE50 à 60 degrés



Huile SAE 50
C’est quoi ?

Transport maritime

Marine LCX

Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

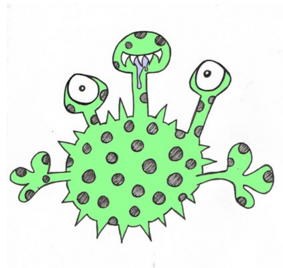
Grades offerts :
SAE 50

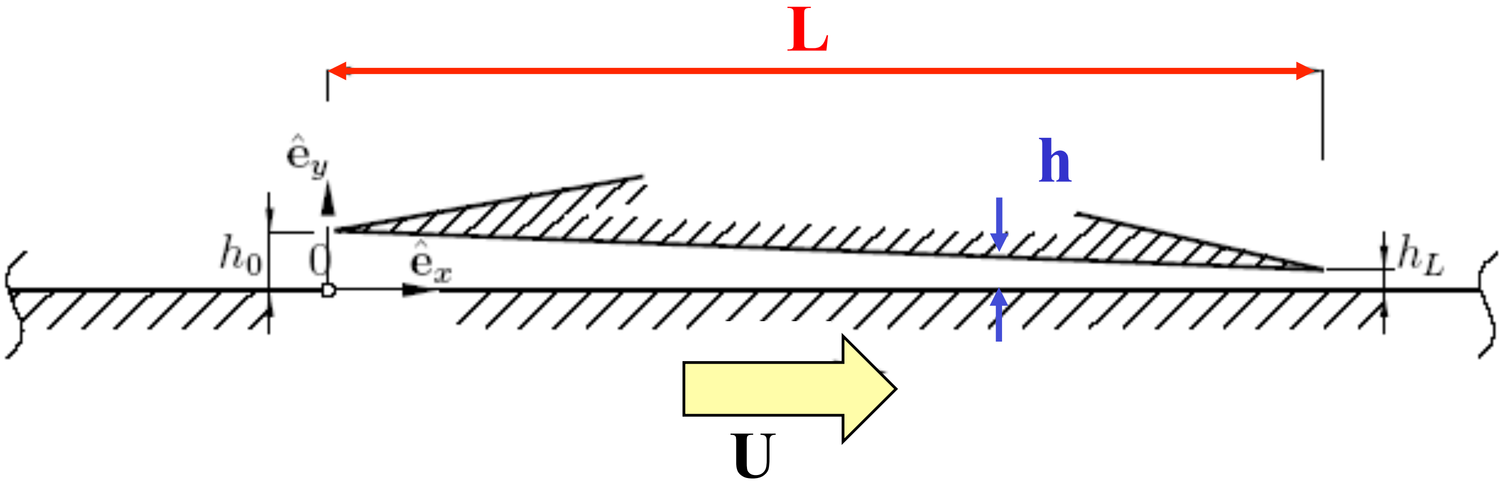
Fiche technique
Fiche signalétique

$$\begin{cases} \dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0 \\ -\dfrac{dp}{dx} + \mu \dfrac{\partial^2 u}{\partial y^2} = 0 \end{cases}$$

-i- calcul de u(x,y)

$$u(x,y) = -\frac{dp}{dx}\frac{h^2}{2\mu}\frac{y}{h}\left(1-\frac{y}{h}\right) + U\left(1-\frac{y}{h}\right)$$





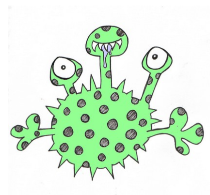
-ii- calcul de p(x)

$$\left\{\begin{array}{rcl} \boxed{-\dfrac{dp}{dx} + \mu\dfrac{\partial^2 u}{\partial y^2} = 0} \\ \boxed{\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} = 0} \end{array}\right.$$

$$0 = \int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} dy$$

$$0 = \frac{d}{dx}\overbrace{\int_0^h u(x,y)dy}^{Q(x)} + \cancel{\left[v(x,y) \right]_0^h}$$

En utilisant l'expression de u(x,y)



Equation classique de Reynolds (1889)

$$\boxed{0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2}\right)}$$

$$0 = \frac{d}{dx}\left(-\frac{dp}{dx}\frac{h^3}{12\mu} + \frac{Uh}{2}\right)$$

$$\frac{d}{dx}\left(h^3(x)\,\frac{dp}{dx}(x)\right) = 6\mu U \boxed{\frac{dh}{dx}}$$

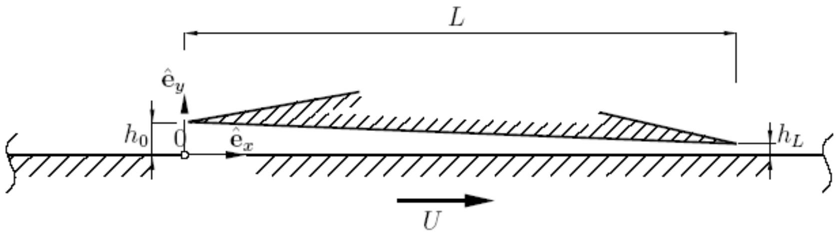
$$-\frac{d}{dh}\left(h^3\frac{dp}{dh}(h)\right) = \frac{6\mu U L}{h_0 - h_L}$$

$$-h^3\frac{dp}{dh}(h) = \frac{6\mu U L}{h_0 - h_L}(h + A)$$

$$-\frac{dp}{dh}(h) = \frac{6\mu U L}{h_0 - h_L}\left(\frac{1}{h^2} + \frac{A}{h^3}\right)$$

$$p(h) = \frac{6\mu U L}{h_0 - h_L}\left(B + \frac{1}{h} + \frac{A}{2h^2}\right)$$

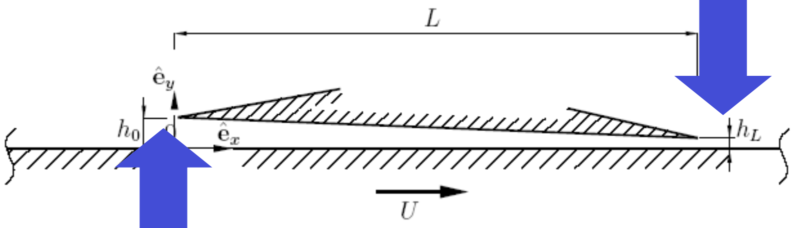
Palier plat



$$\frac{x}{L} = \frac{h_0 - h(x)}{h_0 - h_L}$$

$$\frac{dh}{dx} = -\frac{h_0 - h_L}{L}$$

$$p(h_L) = p_0$$



$$p(h_0) = p_0$$

Deux conditions aux limites

Deux constantes

$$p(h) = \frac{6\mu U L}{h_0 - h_L}\left(\boxed{B} + \frac{1}{h} + \frac{\boxed{A}}{2h^2}\right)$$

$$\downarrow$$

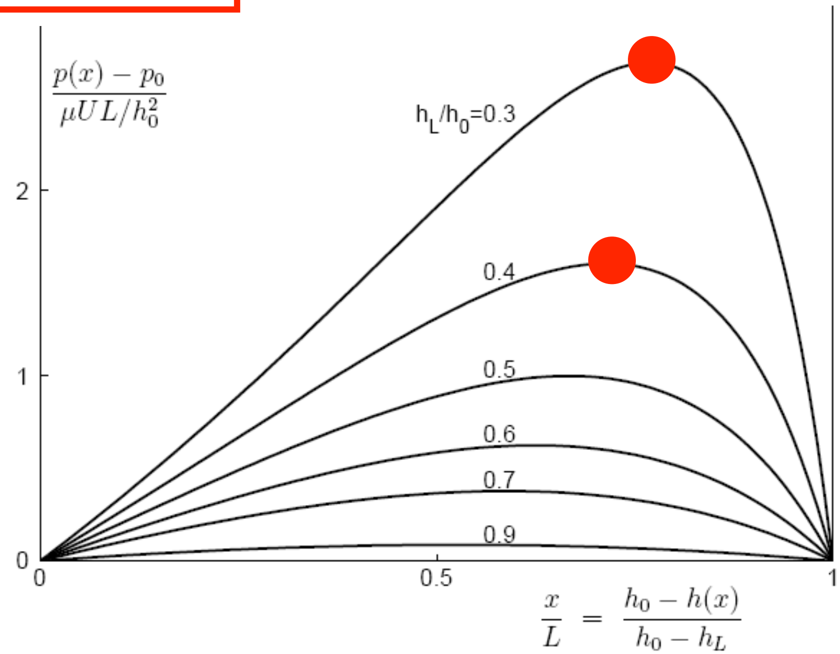
$$p(h) - p_0 = \frac{6\mu U L (h_0 - h)(h - h_L)}{(h_0^2 - h_L^2)h^2}$$

$$0 = \frac{dp}{dh}(h)$$

$$0 = \frac{6\mu U L}{(h_0^2 - h_L^2)}\left(-\frac{(h_0 + h_L)}{h^2} + \frac{2h_0 h_L}{h^3}\right)$$

$$\boxed{h = \frac{2\,h_0\,h_L}{(h_0 + h_L)}}$$

Où la pression est-elle maximale ?

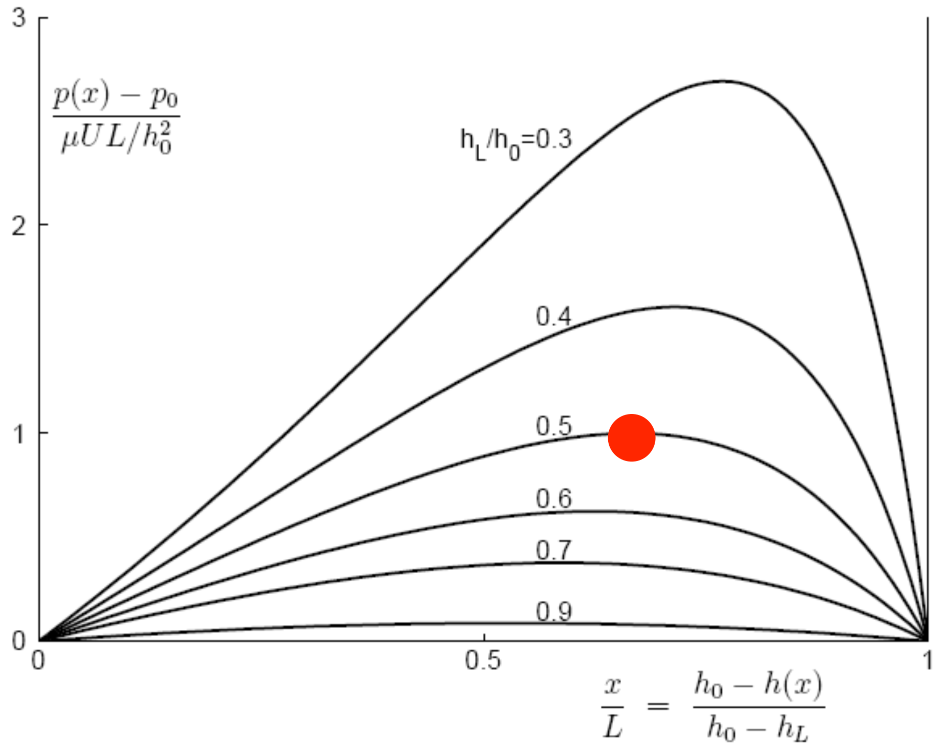


Cette pression peut être énorme !

$$
\begin{aligned}
L &= 10\ cm \\
h_0 &= 0.1\ mm \\
h_L &= 0.05\ mm \\
U &= 10\ m/s
\end{aligned}
$$

$$
\begin{aligned}
\rho &= 900\ kg/m^3 \\
\mu &= 0.1\ Ns/m^2
\end{aligned}
$$

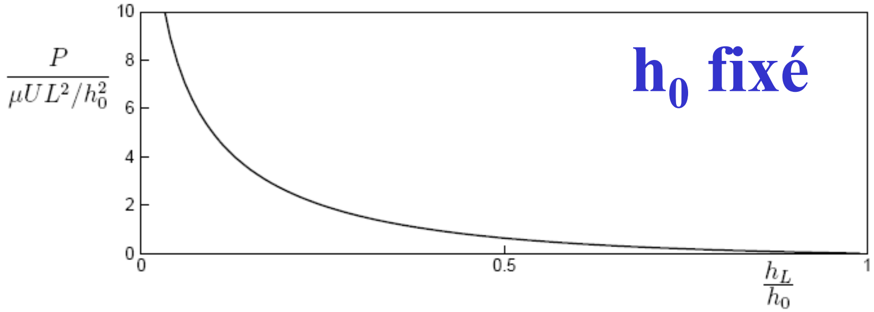
Huile SAE50 à 50 degrés

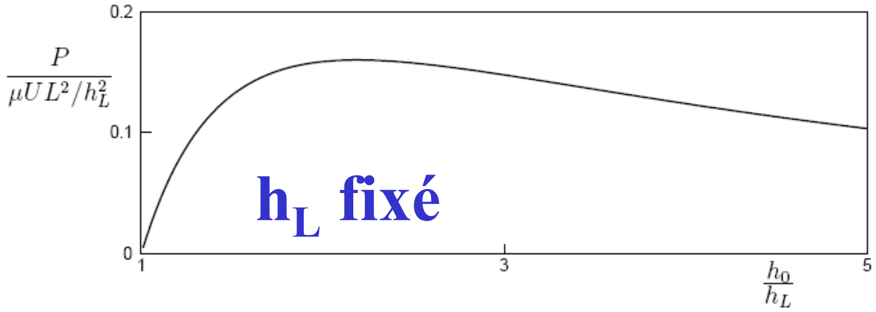


$$p_{\max} - p_0 = \boxed{\frac{3}{2}\frac{\mu\, U\, L}{h_0\, h_L}\,\frac{(h_0 - h_L)}{(h_0 + h_L)}}$$

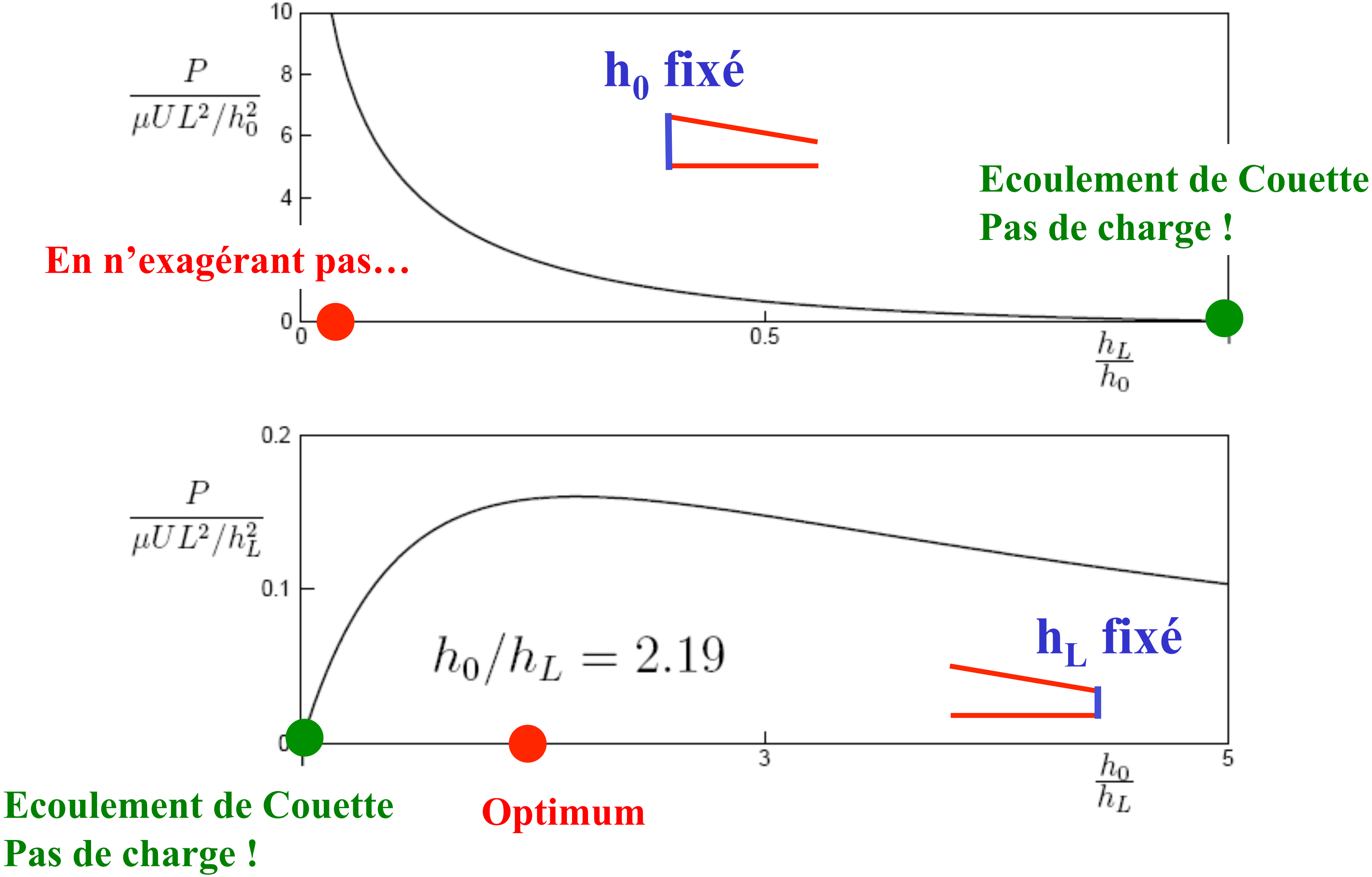
10^7 Pascal

Charge utile



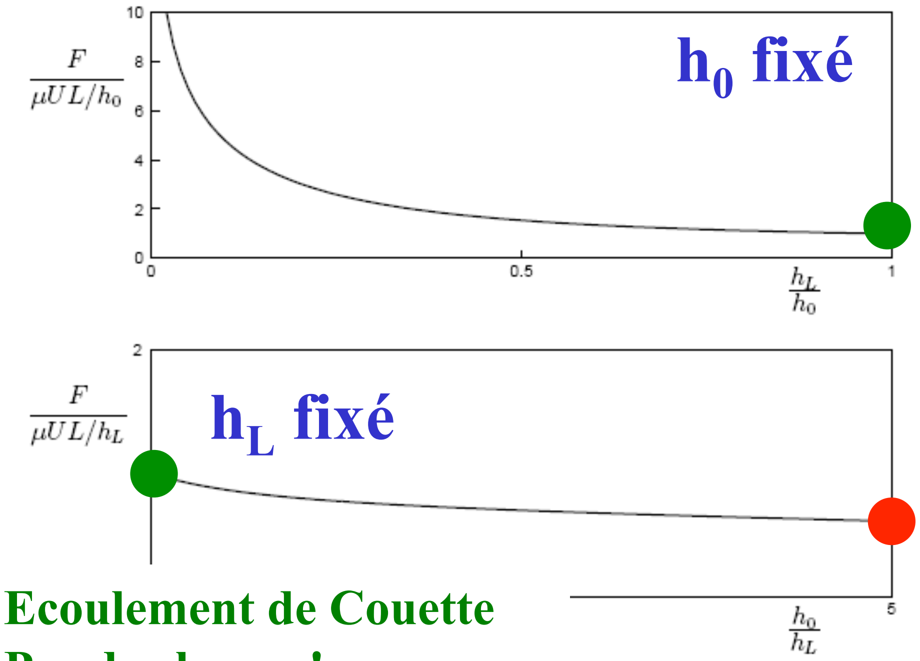


$$
\begin{aligned}
P &= \int_0^L (p(x) - p_0)\; dx \\
&= -\frac{L}{(h_0 - h_L)} \int_{h_0}^{h_L} (p(h) - p_0)\; dh \\
&= -\frac{L}{(h_0 - h_L)} \frac{6\,\mu\, U\, L}{(h_0^2 - h_L^2)} \int_{h_0}^{h_L} \left[(h_0 + h_L)\frac{1}{h} - \frac{h_0\, h_L}{h^2} - 1 \right] dh \\
&= -6\,\mu\, U\, L^2 \left[\frac{1}{(h_0 - h_L)^2} \log\left(\frac{h_L}{h_0}\right) + \frac{2}{(h_0^2 - h_L^2)} \right]
\end{aligned}
$$



Rapport optimal…

Force exercée par le fluide sur la partie mobile



Ecoulement de Couette
Pas de charge !

$$F = -\int_0^L \mu \left.\frac{\partial u}{\partial y}\right|_{y=0} dx$$

$$= \frac{\mu U L}{(h_0 - h_L)} \int_{h_0}^{h_L} \left[\frac{6}{h^2}\,\frac{h_0\,h_L}{(h_0 + h_L)} - \frac{4}{h}\right] dh$$

$$= -\mu U L \left[\frac{6}{(h_0 + h_L)} + \frac{4}{(h_0 - h_L)} \log\left(\frac{h_L}{h_0}\right)\right]$$

La force diminue de façon monotone lorsque le rapport augmente…

La puissance consommée est dissipée…

$$F\,U = -\frac{\mu\,U^2\,L}{h_0}\left[\frac{6}{(1+h_L/h_0)} + \frac{4}{(1-h_L/h_0)}\log\left(\frac{h_L}{h_0}\right)\right]$$

Embêtant…

S’assurer que l’huile est bien refroidie car la viscosité (et donc la charge utile) décroît rapidement avec la température…

…en chaleur !

A propos de la viscosité de notre huile SAE 50

Transport maritime

Marine LCX

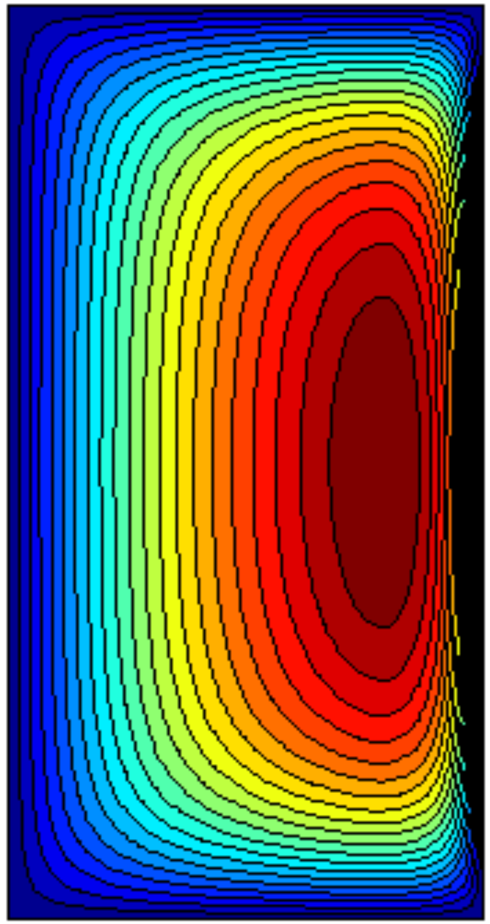
Une huile formulée spécialement pour la lubrification des gros moteurs diesel marins à crosse. Elle lubrifie les cylindres grâce à un indice de basicité très élevé de 70 et un grade SAE* 50.

Grades offerts :
SAE 50

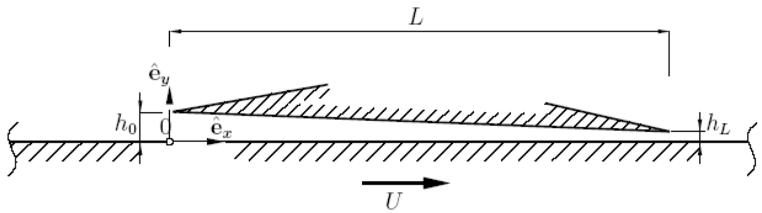
Fiche techniqu
Fiche signalét

$$
\begin{aligned}
T &= 20^0C & \mu &= 1.100\ Ns/m^2 \\
T &= 40^0C & \mu &= 0.210\ Ns/m^2 \\
T &= 50^0C & \mu &= 0.100\ Ns/m^2 \\
T &= 60^0C & \mu &= 0.060\ Ns/m^2 \\
T &= 80^0C & \mu &= 0.025\ Ns/m^2 \\
T &= 100^0C & \mu &= 0.013\ Ns/m^2
\end{aligned}
$$

Analyse « tridimensionnelle » du palier plat



pression sous un palier dont la largeur vaut le double de la longueur



Lubrification 2D ½

$$
\begin{aligned}
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} &= 0 \\
\frac{\partial p}{\partial x} &= \mu \frac{\partial^2 u}{\partial z^2} \\
\frac{\partial p}{\partial y} &= \mu \frac{\partial^2 v}{\partial z^2} \\
\frac{\partial p}{\partial z} &= 0
\end{aligned}
$$

Théorie de la lubrification

Film fluide mince

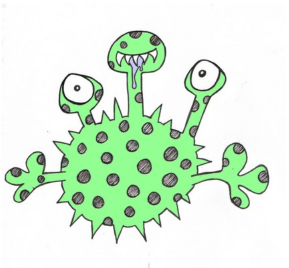
$$h \ll L$$

Hypothèse de lubrification :
Écoulements rampants

$$\underbrace{\frac{\rho U L}{\mu}}_{Re_L} \frac{h^2}{L^2} \ll 1$$

$$\left\{\begin{aligned} \boxed{\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z} &= 0} \\ -\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial z^2} &= 0 \\ -\frac{\partial p}{\partial y}+\mu\frac{\partial^2 v}{\partial z^2} &= 0 \end{aligned}\right.$$

-i- calcul de u(x,y,z) et de v(x,y,z)



$$u(x,y,z) = -\frac{\partial p}{\partial x}\frac{h^2}{2\mu}\frac{z}{h}\left(1-\frac{z}{h}\right)+U\left(1-\frac{z}{h}\right)$$

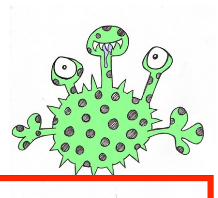
$$v(x,y,z) = -\frac{\partial p}{\partial y}\frac{h^2}{2\mu}\frac{z}{h}\left(1-\frac{z}{h}\right)$$

$$
\begin{cases}
-\dfrac{\partial p}{\partial x} + \mu \dfrac{\partial^2 u}{\partial z^2} = 0 \\[2ex]
-\dfrac{\partial p}{\partial y} + \mu \dfrac{\partial^2 v}{\partial z^2} = 0 \\[2ex]
\dfrac{\partial u}{\partial x} + \dfrac{\partial v}{\partial y} + \dfrac{\partial w}{\partial z} = 0
\end{cases}
$$

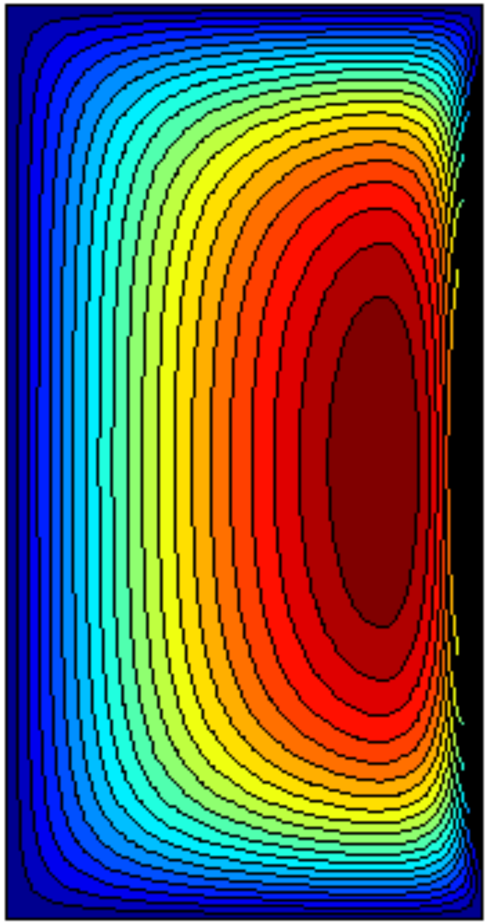
-ii- calcul de p(x,y)

$$
\int_0^h \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\, dz = 0
$$

$$
\frac{\partial}{\partial x}\int_0^h u(x,y,z)dz + \frac{\partial}{\partial y}\int_0^h v(x,y,z)dz + \Big[\cancel{w(x,y,z)}\Big]_0^h = 0
$$



$$
\boxed{\frac{\partial}{\partial x}\left(h^3\frac{\partial p}{\partial x}\right) + \frac{\partial}{\partial y}\left(h^3\frac{\partial p}{\partial y}\right) = 6\mu U\frac{dh}{dx}}
$$



-iiii- calcul numérique par différences finies de p(x,y)

$$h^3\left(\frac{\partial^2 p}{\partial x^2}+\frac{\partial^2 p}{\partial y^2}\right)+3h^2\left(\frac{h_L-h_0}{L}\right)\frac{\partial p}{\partial x} = 6\mu U\left(\frac{h_L-h_0}{L}\right)$$