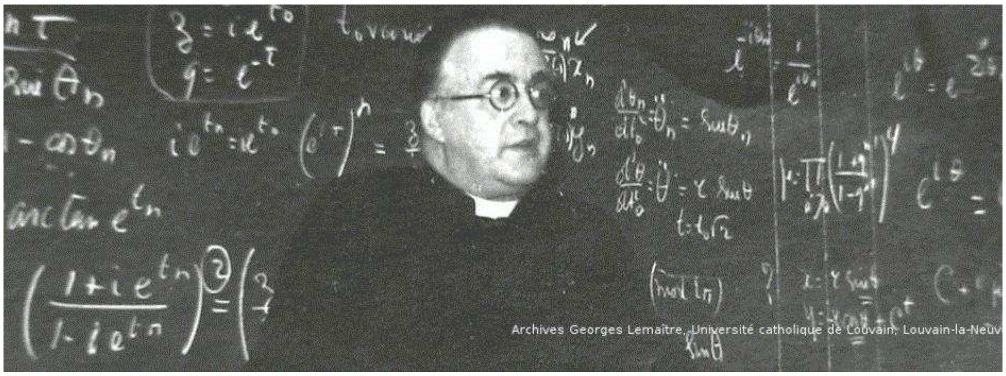
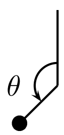


https://archives.uclouvain.be/atom/index.php/jacobi-pendule-1
https://archives.uclouvain.be/atom/index.php/lecons-de-mecanique-le-pendule-notes-de-cours

Meilleurs voeux pour
Best wishes for



$$\left(1 < \frac{x}{3} \in \mathbb{Z}, \frac{2^{32}+1}{x/3} \in \mathbb{Z}\right)$$

$$x + 100$$

où x est l'année de la publication de "L'Univers comme Espace-Temps" par A. Friedmann, Leningrad, Akademiya, contenant une théorie développée indépendammant par G. Lemaître 4 ans plus tard, "Un Univers homogène de masse constante et de rayon croissant,...", *Ann. Soc. sci. Bruxelles*, sér.A, t. **XLVII**. Le Big Bang était né!

Luc Haine nous a fait remarquer [1,3] que le tableau de la photo ci-dessus N'EST PAS rempli de formules sur les profonds mystères de l'univers, mais de développements de l'équation du mouvement du pendule simple: $d^2\theta/dt^2 =$ constante $\times \sin\theta$ est résolu par un développement de gudermanniens (quelqu'un connaît-il encore le sens de ce mot?)

where x is the year when Alexander Friedmann published "The Universe as Space-time", Leningrad, Akademiya. Lemaître published independently "An homogeneous Universe of constant mass and increasing radius,...", *Ann. Soc. sci. Bruxelles*, sér.A, t. **XLVII** 4 years later. The Big Bang was born!

Luc Haine made us notice [1,3] that the blackboard of the picture above IS NOT full of formulas about deep mysteries of the universe, but expansions of the solution of the equation of motion of the simple pendulum: $d^2\theta/dt^2 =$ constant $\times \sin\theta$ is solved by an expansion of Gudermannians (does somebody still know the meaning of this word?)

$$z = ie^{t_0}, q = -e^{-\tau}$$

$$\theta = \sum_{-\infty}^{\infty} (-1)^n \theta_n, \qquad \theta_n = 4 \arctan\, \exp(t_0 + n\tau) = -2i \log \frac{1 + i\exp(t_0 + n\tau)}{1 - i\exp(t_0 + n\tau)}, \qquad e^{(-1)^n i\theta_n} = \left(\frac{1 + z/q^n}{1 - z/q^n}\right)^2,$$

$$\frac{d\theta_n}{dt_0} = \frac{2}{\cosh(t_0 + n\tau)} = 2\sin(\theta_n/2), \quad \cos(\theta_n/2) = -\tanh(t_0 + n\tau), \quad \frac{d^2\theta_n}{dt_0^2} = \frac{-2\sinh(t_0 + n\tau)}{\cosh^2(t_0 + n\tau)} = \sin\theta_n.$$

$$\begin{cases} \sin\theta = \dfrac{1}{2i}\left[\prod_{-\infty}^{\infty}\left(\dfrac{z+q^n}{z-q^n}\right)^2 - \prod_{-\infty}^{\infty}\left(\dfrac{z-q^n}{z+q^n}\right)^2\right] = \sum_{m=-\infty}^{\infty} \underbrace{\dfrac{A_m}{(z-q^m)^2} + \dfrac{B_m}{z-q^m} - \dfrac{A_m}{(z+q^m)^2} + \dfrac{B_m}{z+q^m}}_{\frac{4A_m z q^m + 2B_m z(z^2-q^{2m}) \; = 2A_m z[2q^m + (B_m/A_m)(z^2-q^{2m})] \; = -8i\mu z^2 q^{2m} i(-1)^m \sinh(t_0+m\tau)}{(-e^{2t_0} - e^{-2m\tau})^2 \; = -4z^2 q^{2m} \cosh^2(t_0+m\tau)}} \\ = \mu \dfrac{d^2\theta}{dt_0^2}. \qquad \left(A_m = \frac{4q^{2m}}{2i} \prod_{n\neq m}\left(\frac{q^m+q^n}{q^m-q^n}\right)^2 = -2i\mu q^{2m}, \quad \frac{B_m}{A_m} = q^{-m} + 2\sum_{n\neq m}\left(\frac{1}{q^m+q^n} - \frac{1}{q^m-q^n}\right) = q^{-m}\right) \end{cases}$$

à rapprocher d' une formule "elegantissimas censeri debet" de Jacobi [2, p.108]

Lemaître consacrait un temps anormalement long de son enseignement à ces questions. Einstein considérait Lemaître comme un mathématicien virtuose dénué de tout sens physique.

Personne n'aurait osé mettre en doute le sens physique de Feynman, qui consacra pourtant beaucoup de temps à une démonstration géométrique élémentaire(!) des orbites elliptiques à partir de la loi en $1/r^2$ [4].

to compare to Jacobi's "formulas elegantissimas censeri debet" [2, p.108].

Lemaître devoted too much time to the teaching of these subjects. No wonder that Einstein considered him to be a mathematical virtuoso without any physical insight.

Nobody would have dared to question the physical insight of Feynman, who however spent quite an amount of time on an elementary(!) geometric proof of the elliptic orbits from the $1/r^2$ law [4].

[1] L. Haine, On a generalization of Jacobi's Elegantissima, *14th Int. Sympos. Orthogonal Polynomials, Special Functions and Applications*, Univ. of Kent, 2017; *Group Analysis of Differential Equations and Integrable Systems*, Larnaca 2018.
[2] C.G.J. Jacobi, *Fundamenta nova theoriæ functionum ellipticarum*, 1829, https://archive.org/details/fundamentanovat00jacogoog
[3] Fonds FG LEM - Archives de Georges Lemaître https://archives.uclouvain.be/atom/index.php/archives-de-georges-lemaitre
[4]Feynman Lost Lecture of gravitational path of planets https://archive.org/download/12073960FeynmanSLostLecture

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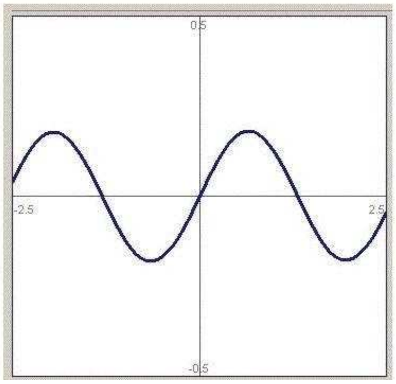
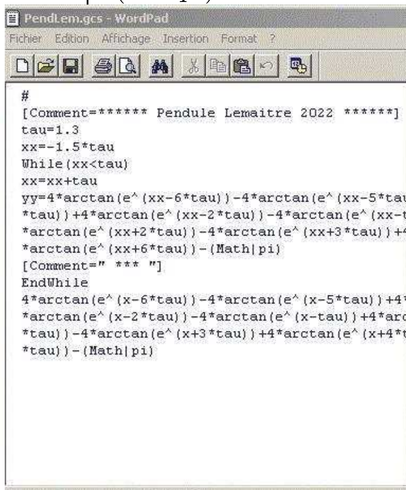
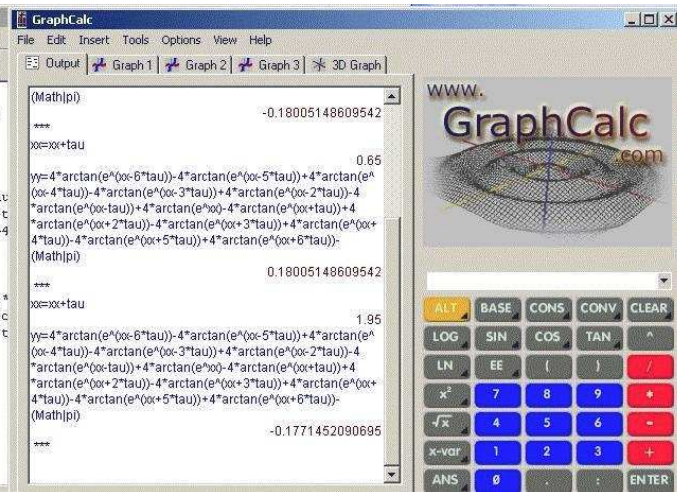
https://sourceforge.net/projects/pgwinsuite/files/GraphCalc%20Portable/

$$\tau = 1.3 : q = -0.27253, \mu = \prod_{1}^{\infty} \frac{(1+q^n)^4}{(1-q^n)^4} = 0.17054, \tau/\sqrt{\mu} = 3.147935, \qquad \pi - 0.17951 \leqslant \theta \leqslant \pi + 0.17951$$







46) $\vartheta = \text{Arc}\sin k = \frac{4\sqrt{q}}{1+q} - \frac{4\sqrt{q^3}}{3(1+q^3)} + \frac{4\sqrt{q^5}}{5(1+q^5)} - \frac{4\sqrt{q^7}}{7(1+q^7)} + \dots$

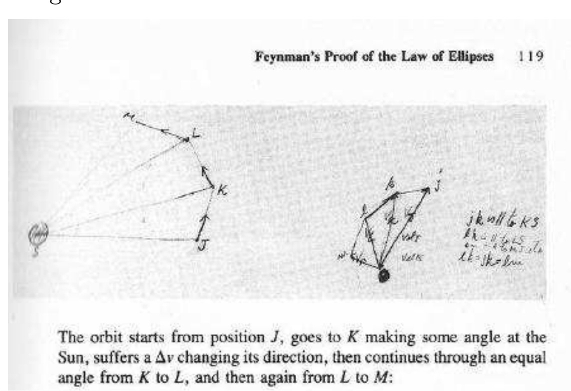
quae in hanc facile transformatur:

47) $\frac{\vartheta}{4} = \text{Arc}\,\text{tg}\sqrt{q} - \text{Arc}\,\text{tg}\sqrt{q^3} + \text{Arc}\,\text{tg}\sqrt{q^5} - \text{Arc}\,\text{tg}\sqrt{q^7} + \dots,$

quae inter formulas elegantissimas censeri debet.

It is now our problem to demonstrate-and it is the purpose of this lecture mainly to demonstrate-that therefore the orbit is an ellipse. It is not difficult, when one knows the calculus, and to write the differential equations and to solve them, to show that it's an ellipse. I believe in the lectures here-or at least in the book- [you] calculated the orbit by numerical methods and saw that it looked like an ellipse. That's not exactly the same thing as proving that it is exactly an ellipse. The Mathematics Department ordinarily is left the job of proving that it's an ellipse, so that they have something to do over there with their differential equations. [Laughter] I prefer to give you a demonstration that it's an ellipse in a completely strange, unique, [and] different way than you are used to. I am going to give what I will call an elementary demonstration. [But] "elementary" does not mean easy to understand. "Elementary" means that very little is required to know ahead of time in order to understand it, except to have an infinite amount of intelligence. It is not necessary to have knowledge but to have intelligence, in order to understand an elementary demonstration. There may be a large number of steps that are very hard to follow, but each step does not require already knowing calculus, already knowing Fourier transforms, and so on. So by an elementary demonstration I mean one that goes back as far as one can with regard to how much has to be learned. …
"The Motion of Planets Around the Sun" 161

So here is the problem, here's what we have discovered: that if we draw a circle and take an off-center point, then take an angle in the orbit-any angle you want in the orbit-and draw the corresponding angle inside this constructed circle and draw a line from the eccentric point, then this line will be the direction of the tangent. Because the velocity is evidently the direction of motion at the moment and is in the direction of the tangent to the curve. So our problem is to find the curve such that if we draw a point from an eccentric center, the direction of the tangent of that curve will always be parallel to that when the angle of the curve is given by the angle in the center of that circle.



The orbit starts from position J, goes to K making some angle at the Sun, suffers a Δv changing its direction, then continues through an equal angle from K to L, and then again from L to M:

… But we proved that that was one of the properties of an ellipse-the reflection property. Therefore, the solution to the problem is an ellipse-or the other way around, really, is what I proved: that the ellipse is a possible solution to the problem.