

A colorful description of '1/9'.

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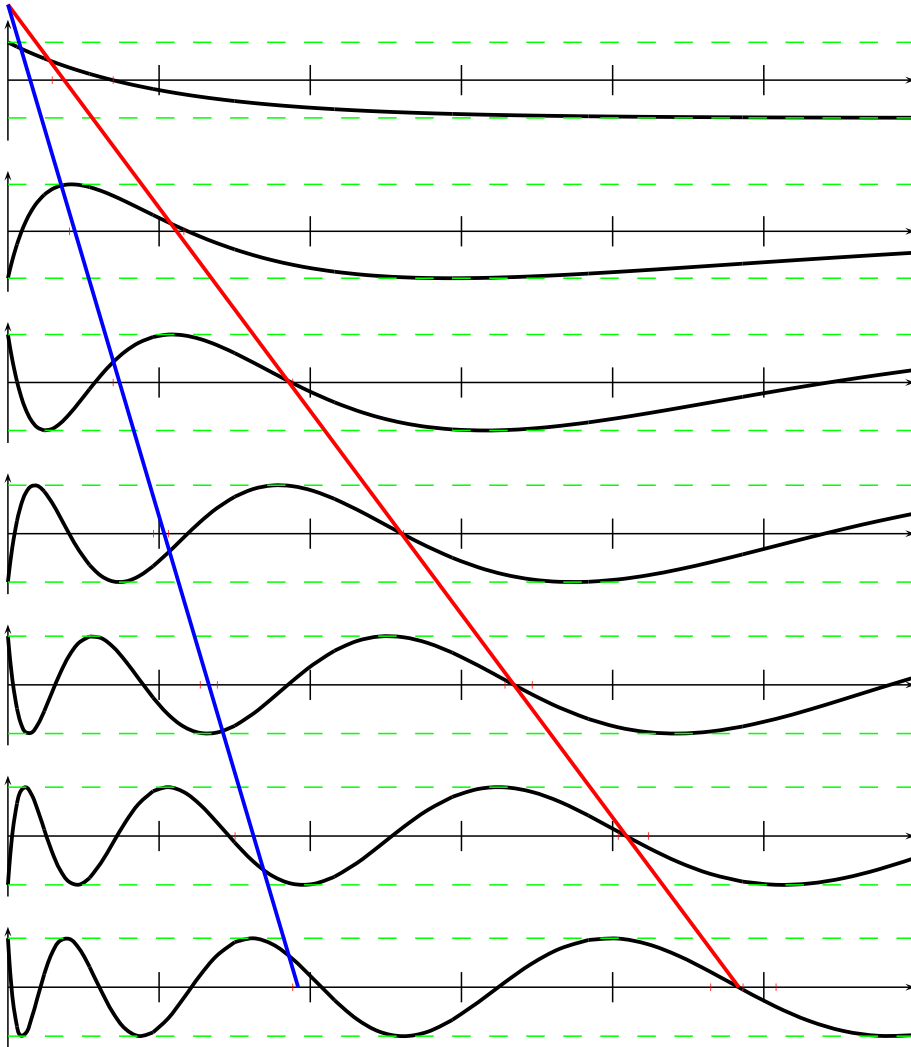
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When we look at the best approximation error, the most conspicuous feature is equioscillation.

From well known exactly solvable minimax approximation problems, let us try

$$e^{-t} - \frac{P_n(t)}{Q_n(t)} = (-1)^n E_n \cos((2n+1)\theta_n(t))$$

and we look where $\theta_n(t) = \pi/2$ and $\pi/3$,



Conjecture (which we hope to prove some day!)

$$\theta_n((n + 1/2)u) \rightarrow_{n \rightarrow \infty} \theta(u). \quad (1)$$

$u = t/(n + 1/2)$	$\theta_4(t)/\pi$	$\theta_5(t)/\pi$	$\theta_6(t)/\pi$
0.01000	0.06373	0.06368	0.06365
0.04000	0.12700	0.12691	0.12685
0.06250	0.15832	0.15820	0.15814
0.09000	0.18936	0.18921	0.18913
0.16000	0.25031	0.25014	0.25004
0.25000	0.30940	0.30920	0.30629
0.36000	0.36615	0.36134	0.36581
0.49000	0.42003	0.41983	0.41973
0.64000	0.47047	0.47040	0.47036
0.81000	0.51700	0.51718	0.51726
1.00000	0.55937	0.55983	0.56008
2.56000	0.73251	0.73333	0.73381
4.00000	0.79290	0.79323	0.79342
9.00000	0.86694	0.86694	0.86693
16.00000	0.90147	0.90143	0.90140
25.00000	0.92163	0.92159	0.92156
100.00000	0.96111	0.96108	0.96107

The u -plane, with lines $\text{Re } \theta_n = \text{constant}$ (cyan), and $\text{Im } \theta_n = \text{constant}$ (magenta), with light colours to dark colours when $n = 4, 5, 6$:

