a) $w(x)p_n^2(x)$ is negligible outside an interval $S_n = [\alpha_n, \beta_n]$ where bounds for α_n and β_n can be given by the extreme abscissaeof the maximal values of $|x|^{2n}w(x)$ (see [11] §6, [12], [14] for conditions on w and precise formulation); $|a_n|$ and $|b_n|$ are also bounded by $c^t \max(|\alpha_n|, |\beta_n|)$; most of the zeros of p_n are in S_n.

b) If $w(x)p_n^2(x)$ is assumed to be approximately equioscillating on S_n (this seems to hold, up to a factor $[(x-\alpha_n)(\beta_n-x)]^{-1/2}$ [22] §2),

(3) $$\underset{\alpha_n,\ \beta_n}{\text{maximum}} \quad \log(\beta_n-\alpha_n)/4 + \frac{1}{2n\pi}\int_{-1}^{1}(1-x^2)^{-1/2}\log w\left(\frac{\alpha_n+\beta_n}{2}+\frac{\beta_n-\alpha_n}{2}x\right)dx$$

gives sharp estimates of α_n and β_n [19]. Many works on these subjects suggest a connection with the Szegö's theory of orthogonal polynomials on a bounded interval. A promising extension of Szegö's estimates is $\log p_n(z) = \sum_{k=1}^{n} \log\left(z-\frac{\alpha_k+\beta_k}{2}+[(z-\alpha_k)(z-\beta_k)]^{1/2}\right) + o(n)$ for nonreal z. When $z = x + i\varepsilon$ is almost real ($\varepsilon > 0$), the imaginary part is close to π times the number of zeros of p_n between x and β_n. A fair estimate of the density of zeros of p_n is therefore $\pi^{-1} \sum_{k\leq n,\, x\in[\alpha_k,\beta_k]} [(x-\alpha_k)(\beta_k-x)]^{-1/2}$, with α_k and β_k given by (3).

c) Important simplifications in proofs and increase of knowledge occur if the recurrence coefficients behave smoothly

(4) $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = 1 \quad , \quad \lim_{n\to\infty} \frac{b_n}{a_n} \text{ exists.}$$

Then, one has

(5) $$\alpha_n \sim x_{1,n} \sim b_n - 2a_n \quad , \quad \beta_n \sim x_{n,n} \sim b_n + 2a_n ,$$

where α_n and β_n agree with (3), and general asymptotic behaviour, [21] distribution of zeros ([24], [29]) can be investigated with accuracy.

In 1973, Freud and Nevai initiated intensive study of the case $w(x) = |x|^\rho \exp(-|x|^\alpha)$, $\alpha > 0$. Advances that have been made include : inequalities and bounds ([4][11] [13] [20][23]); launching of the conjecture (1) by Freud [5] with a proof for $\alpha = 4$ and 6 ($\alpha = 2$ is almost classical [2] p. 157 [26]); asymptotics for these polynomials in