these cases ([20][21][27]) and other conjectures [22] ; distribution of zeros ([24] [29]); sharp estimates of the extreme zeros [25] for $\alpha > 1$ (i.e., $x_{1,n} = - x_{n,n}$, $x_{n,n}/ [n/C(\alpha)]^{1/\alpha} \xrightarrow[n\to\infty]{} 2$, with the $C(\alpha)$ of (1), but not using nor establishing (1)); proof of expected behaviour of geometric mean of $a_1,\ldots,a_n$, compatible with (1) for $\alpha > 0$ [18]. Much more must clearly be expected...

If the general tools involved (bounds for Freud-Christoffel function [4] [20] [11] [12], function spaces identities [12][18], potential theory methods [18] [19][25]) are fairly powerful, they do not seem to be able to reach (4) naturally. For this reason, the Freud's proof [5] will be expanded here.

FREUD'S EQUATIONS.

The weight function is $w(x) = |x|^\rho \exp(-|x|^{2m})$ $-\infty < x < \infty$, with integer m. The factor $|x|^\rho$ is not essential, but useful if one wants to investigate weight functions $\exp(-x^m)$ on the positive real axis [2] chap. 1 §8,9. As w is an even function, $b_n = 0$. Equations for the a_n's will be obtained by equivalent forms of $\int_{-\infty}^{\infty} (p_n(x)p_{n-1}(x))'w(x)dx$. First, by expanding the derivative of the product, using the orthonormality of the p_n's and the recurrence relation (2), one finds n/a_n. Next, integration by parts, using $w'(x) = (\rho/x - 2m\, x^{2m-1})w(x)$, yields $-(\rho/a_n)\mathrm{odd}(n) + 2m \int_{-\infty}^{\infty} x^{2m-1}\, p_n(x)p_{n-1}(x)w(x)dx$ where the odd-eveness of the p_n's with respect to n has been used (odd(n) = 1 if n is odd; 0 if n is even). Finally, the last integral is found to be a combination of $a_{n-m+1},\ldots,a_{n+m-1}$ from repeated application of the recurrence (2) written in the form $xp_n(x) = a_n p_{n-1}(x) + a_{n+1}p_{n+1}(x)$,

or $\quad x\,[p_0(x),p_1(x),\ldots]^T = A\,[p_0(x),p_1(x),\ldots]^T$, $A = \begin{bmatrix} 0 & a_1 & & \\ a_1 & 0 & a_2 & \\ & a_2 & 0 & \ddots \\ & & \ddots & \ddots \end{bmatrix}$

Let the result be called Freud's equations :

$$(6)\quad F_n(a) = 2m\, a_n (A^{2m-1})_{n,n+1} = n + \rho\ \mathrm{odd}(n), \quad n = 1,2,\ldots$$

where $(X)_{n,m}$ means the n^{th} row - m^{th} column entry of the matrix X.