of X (and unicity of the solution appears as a byproduct). For a nonlinear F, the role of X is played by the Jacobian operator (or Fréchet derivative)

$$J(a) = [\partial F_n(a)/\partial a_k].$$

The importance of J^{-1} appears also in Newton's algorithm, producing a new estimate of the solution from an old one by $a^* - (J(a^*))^{-1}(F(a^*) - F(a))$, a very powerful method for the actual computation of the solution [10] (a slightly different form will be used, for reasons that will appear very soon).

Here is how the programme is fulfilled.

<u>THEOREM</u> The equations (6) have only one positive solution; the solution satisfies (1) ($\alpha = 2m$).

As announced, information on the invertibility of the Jacobian operator is essential.

<u>LEMMA</u> The matrix $J(a) = [a_k \, \partial F_n(a)/\partial a_k]$, $n,k = 1,2,\ldots$, is symmetric and positive definite for any positive sequence $\{a_n\}$. In this case,

$$(J(a)u,u) = \sum_{n=1}^{\infty} \sum_{k=1}^{\infty} u_n u_k a_k \, \partial F_n(a)/\partial a_k \geqslant 4m^2 \sum_{n=1}^{\infty} a_n^{2m} u_n^2$$ holds for any finite real sequence $\{u_n\}$.

<u>Proof of the theorem.</u> The matrix J(a) of the lemma is actually the Jacobian operator of F with respect to the variables $\log a_n$. To appreciate the distance between two sequences a' and a" in term of F(a") - F(a'), we join them by the rectilinear path $\log a_n(t) = \log a_n' + t(\log a_n'' - \log a_n')$, $0 \leqslant t \leqslant 1$, n=1,2,..., and integrate J on this path :

$$F_n(a'') - F_n(a') = \int_0^1 \sum_{k=n-m+1}^{n+m-1} a_k \, \partial F_n(a)/\partial a_k \; \delta_k \, dt \quad , \quad \delta_k = \log a_k'' - \log a_k' ,$$

knowing that $F_n(a)$ depends only on $a_{n-m+1},\ldots,a_{n+m-1}$ $(a_0 = a_{-1} = \ldots = 0)$. In order to introduce the quadratic form of the lemma,

$$(9) \quad \sum_{n=1}^{N} \delta_n (F_n(a'') - F_n(a')) = \int_0^1 \sum_{n=1}^{N} \sum_{k=n-m+1}^{\max(n+m-1,N)} a_k \, \partial F_n(a)/\partial a_k \; \delta_k \, \delta_n \, dt + \int_0^1 \sum_{n=N-m+2}^{N} \sum_{k=N+1}^{n+m-1} a_k \frac{\partial F_n(a)}{\partial a_k} \delta_k \, \delta_n \, dt.$$