if it is present in the sum, will therefore be multiplied by $u_{j_1} + \dots + u_{j_m}$ (the indexes playing the role of k), and another sum of the u's, the indexes playing the role of n. The admissible values of n will be such that, after some permutation,

$n = j_1,\ j_1 - 1 \leqslant j_2,\ j_2 - 1 \leqslant j_3, \dots, j_{m-1} - 1 \leqslant j_m \leqslant j_1 + 1$, i.e.,

$j_1 \leqslant j_2 + 1 \leqslant \dots \leqslant j_m + m - 1 \leqslant j_1 + m$. If a permutation is valid, so are all the circular permutations $(j_2, \dots, j_m, j_1), \dots,$ of this one, until the initial permutation is recovered, which will happen after m, or m/2, or m/3...steps, producing the corresponding multiple of $u_{j_1} + \dots + u_{j_m}$:

$$(J(a)u,u) = 4m \sum_{j_1,\dots,j_m \text{ admissible}} a_{j_1}^2 \dots a_{j_m}^2\, c(j_1,\dots,j_m)(u_{j_1} + \dots + u_{j_m})^2 > 0.$$

For instance (m = 4), the complete factor of $a_n^4\ a_{n+1}^4$ is $6(u_n + u_{n+1})^2$,

$4(u_n + u_{n+1})^2 = (u_n + u_n + u_{n+1} + u_{n+1})^2$ coming from the 4 circular permutations of (n,n,n+1,n+1) and $2(u_n + u_{n+1})^2 = (u_n + u_{n+1} + u_n + u_{n+1})(u_n + u_{n+1})$ coming from the 2 different circular permutations of (n,n+1,n,n+1). For $j_1 = j_2 = \dots = j_m = n$, there is only one permutation, whence the factor $(u_n + \dots + u_n)u_n = m\, u_n^2$ and the lower bound in the lemma.

A more elegant, but less explicit proof is given in [16].

See eq. (15) of [31] for an ingenious simplification.

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