Appendix

to

"Should developing countries participate in the Clean Development Mechanism under the Kyoto Protocol? The low-hanging fruits and baseline issues"

by

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We give here the mathematical derivation of some propositions and statements of the paper.

Contents

1. Preliminaries: the second period maximization problem.

We see that the full problem (7) contains several times the same subproblem: we first have to maximize with respect to a variable $e \geq 0$ an expression of the form $Ae^{\gamma} - Be + C$, with $A, B > 0$, and $\gamma < 1$.

The derivative $\gamma A e^{\gamma - 1} - B$ has exactly one positive zero:

$$e = \left[\frac{\gamma A}{B}\right]^{1/[1-\gamma]}, \tag{A.1}$$

which leads to $Be = \gamma A e^{\gamma}$, so that the maximum is

$$[1-\gamma]Ae^{\gamma} + C = \gamma' A^{1/[1-\gamma]} B^{-\gamma/[1-\gamma]} + C, \text{ where } \gamma' = [1-\gamma]\gamma^{\gamma/[1-\gamma]}. \tag{A.2}$$

We apply now this to Π_2.

fig A.1

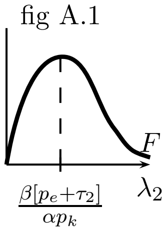
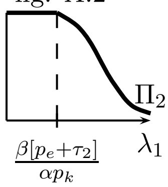


fig. A.2



In $\Pi_2(\lambda_1, \tau_2)$, for a given τ_2, A and B depend on λ_2: $A = \lambda_2^{\beta}$, $B = p_e + \tau_2 + p_k \lambda_2$, and so

$$\Pi_2(\lambda_1, \tau_2) = \max_{\lambda_2 \geq \lambda_1} F(\lambda_2; \tau_2) + C,$$

where

$$\begin{aligned} F(\lambda_2; \tau_2) &:= \max_{e2>0} e_2^{\gamma} \lambda_2 - [p_e + \tau_2 + p_k \lambda_2] e_2 \\ &= \gamma' \lambda_2^{\beta/[1-\gamma]} [p_e + \tau_2 + p_k \lambda_2]^{-\gamma/[1-\gamma]}, \end{aligned} \tag{A.3}$$

$F(\lambda_2; \tau_2)$ increases between $\lambda_2 = 0$ and $\lambda_2 = \beta[p_e + \tau_2]/[\alpha p_k]$, and decreases afterwards (see fig. A.1). Therefore, when $\lambda_1 \leqslant \beta[p_e + \tau_2]/[\alpha p_k]$, the maximum over $\lambda_2 \geqslant \lambda_1$ of $F(\lambda_2; \tau_2)$ is the maximal value of F, i.e.,

$$F\left(\frac{\beta[p_e + \tau_2]}{\alpha p_k}; \tau_2\right) = \gamma''[p_e + \tau_2]^{-\alpha/[1-\gamma]} p_k^{-\beta/[1-\gamma]}, \tag{A.4}$$

where $\gamma'' = \gamma' \beta^{\beta/[1-\gamma]} \alpha^{\alpha/[1-\gamma]} / \gamma^{\gamma/[1-\gamma]} = [1-\gamma]\beta^{\beta/[1-\gamma]}\alpha^{\alpha/[1-\gamma]}$;
whereas, when $\lambda_1 \geqslant \beta[p_e + \tau_2]/[\alpha p_k]$, the maximum over $\lambda_2 \geqslant \lambda_1$ of $F(\lambda_2; \tau_2)$ is merely $F(\lambda_1; \tau_2)$. Thus:

$$\begin{aligned} \Pi_2(\lambda_1, \tau_2) &= \gamma' \lambda_1^{\beta/[1-\gamma]} [p_e + \tau_2 + p_k \lambda_1]^{-\gamma/[1-\gamma]} + C, \quad \text{if } \lambda_1 \geq \beta[p_e + \tau_2]/[\alpha p_k], \\ &= \gamma''[p_e + \tau_2]^{-\alpha/[1-\gamma]} p_k^{-\beta/[1-\gamma]} + C \qquad\quad \text{if } \lambda_1 \leq \beta[p_e + \tau_2]/[\alpha p_k], \end{aligned} \tag{A.5}$$

Remark that $\Pi_2(\lambda_1, \tau_2)$ is a **non increasing** function of λ_1 (see figure A.2).

This also holds for the integral term of (7), as this term is a positive linear combination of functions $\Pi_2(\lambda_1, \tau_2)$ for an interval of values of τ_2.

2. The absolute baseline case.

2.a. Proof of proposition 2.a (existence, unicity and characterization of λ_{a1}).

2.a.1. *The equation for λ_{a1}: the maximum condition.*

We now come to the full problem (4), with an absolute baseline:

$$\max_{e_1\ge 0,\lambda_1\ge 0}\Bigg\{e_1^{\gamma}\lambda_1^{\beta}-[p_e+p_k\lambda_1]e_1+\tau_1[\overline{e}_1-e_1]$$
$$+\rho\int_{\tau_{\min}}^{\tau_{\max}}f(\tau_2)\max_{\substack{e_2\ge 0\\ \lambda_2\ge\lambda_1}}\left\{e_2^{\gamma}\lambda_2^{\beta}-[p_e+p_k\lambda_2]e_2+\tau_2[\tilde{e}_2-\delta[\overline{e}_1-e_1]-e_2]\right\}d\tau_2\Bigg\}.$$

Emphasizing the dependence in e_1, using (2):

$$\max_{\lambda_1\ge 0}\Bigg\{\max_{e_1\ge 0}\left\{e_1^{\gamma}\lambda_1^{\beta}-[p_e+p_k\lambda_1+\tau_1-\rho\delta\tilde{\tau}_2]e_1\right\}$$
$$+\rho\int_{\tau_{\min}}^{\tau_{\max}}f(\tau_2)\max_{\lambda_2\ge\lambda_1}\left\{\max_{e_2\ge 0}\left\{e_2^{\gamma}\lambda_2^{\beta}-[p_e+p_k\lambda_2+\tau_2]e_2\right\}\right\}d\tau_2\Bigg\}+\tau_1\overline{e}_1+\tilde{\tau}_2\rho[\tilde{e}_2-\delta\overline{e}_1]$$

which is first solved with respect to e_1:

$$\max_{\lambda_1\ge 0}\left\{\Pi_{a1}(\lambda_1,\tau_1):=F(\lambda_1;\tau_1-\rho\delta\tilde{\tau}_2)+\rho\int_{\tau_{\min}}^{\tau_{\max}}f(\tau_2)[\max_{\lambda_2\geqslant\lambda_1}F(\lambda_2;\tau_2)]d\tau_2\right\}+\tau_1\overline{e}_1+\tilde{\tau}_2\rho[\tilde{e}_2-\delta\overline{e}_1], \tag{A.6}$$

using the definition of F in (A.3).

Recall that the first term is an increasing function of λ_1 as long as $\lambda_1\leqslant\lambda_{\text{ref}}$, where

$$\lambda_{\text{ref}}:=\frac{\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]}{\alpha p_k}, \tag{A.7}$$

and that the second term contains only non-increasing functions of λ_1.
Moreover, if λ_1 is small enough, so that the condition seen above $\lambda_1\leqslant\beta[p_e+\tau_2]/[\alpha p_k]$ holds for all the τ_2's in the integral, i.e., if $\lambda_1\leqslant\beta[p_e+\tau_{2\min}]/[\alpha p_k]$, all the Π_2's are constant functions of λ_1, and the net result is a function Π_{a1} behaving like the first term F plus a constant.

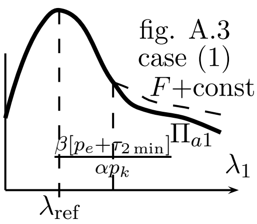


fig. A.3
case (1)

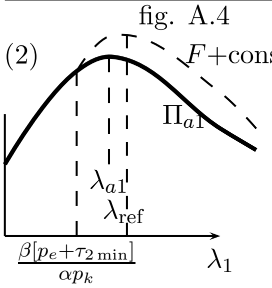


fig. A.4
case (2)

When $\lambda_1>\beta[p_e+\tau_{2\min}]/[\alpha p_k]$, the sum is F plus an actually decreasing function of λ_1, and is smaller than the continuation of the rule F plus a constant (shown by a dashed line in the figures A.3 and A.4 nearby, whereas the actual Π_{a1} [solid line] is even smaller). Therefore,

(1) If $\tau_1\le\tau_{2\min}+\rho\delta\tilde{\tau}_2$, Π_{a1} reaches its maximum value at $\lambda_1=\lambda_{\text{ref}}=\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$, as this latter value is smaller than $\beta[p_e+\tau_{2\min}]/[\alpha p_k]$, which is the place where a more complicated behaviour occurs, but we don't have to care, as the maximum has already been encountered.

(2) If $\tau_1>\tau_{2\min}+\rho\delta\tilde{\tau}_2$, the actual maximum of Π_{a1} occurs between $\beta[p_e+\tau_{2\min}]/[\alpha p_k]$ and $\lambda_{\text{ref}}=\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$, as the λ_1-derivative is still positive at the first bound (F is still increasing, and the integral term still contains constant values with respect to λ_1), and is negative at the second bound (the integral now contains actually decreasing functions of λ_1).

We have therefore established parts (i) and (iii) of Proposition 2.a, i.e., that the maximum is reached at $\lambda_{a1} = \lambda_{\rm ref}$ in case (1); that $\dfrac{\beta[p_e + \tau_{2\,\min}]}{\alpha p_k} < \lambda_{a1} < \lambda_{\rm ref}$ in case (2).

2.a.2. *Existence of λ_{a1}: the first order condition.*

In order to establish more properties of λ_{a1} when $\tau_1 > \tau_{2\,\min} + \rho\delta\tilde{\tau}_2$ (case (2) above) we get a closer look at the equation for λ_{a1} from the λ_1-derivative

$$\frac{\partial \Pi_{a1}(\lambda_1, \tau_1)}{\partial \lambda_1} = \frac{\partial}{\partial \lambda_1}\left\{F(\lambda_1; \tau_1 - \rho\delta\tilde{\tau}_2) + \rho\int_{\tau_{2\,\min}}^{\tau_{2\,\max}} f(\tau_2)[\max_{\lambda_2 \geqslant \lambda_1} F(\lambda_2; \tau_2)]d\tau_2\right\} = 0,$$

or

$$\frac{\partial F(\lambda_1; \tau_1 - \rho\delta\tilde{\tau}_2)}{\partial \lambda_1} + \rho\int_{\tau_{2\,\min}}^{\alpha p_k\lambda_1/\beta - p_e} f(\tau_2)\frac{\partial F(\lambda_1; \tau_2)}{\partial \lambda_1}d\tau_2 = 0,$$

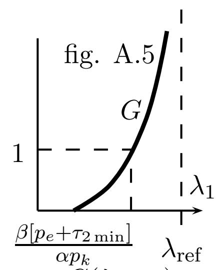
as we saw in (A.5) that $\Pi_2(\lambda_1, \tau_2)$ is a constant function with respect to λ_1 while $\lambda_1 \leqslant \beta[p_e + \tau_2]/[\alpha p_k]$, and leaves therefore no trace in the integral of the λ_1-derivative, so that we only have to consider the derivative when $\lambda_1 > \beta[p_e + \tau_2]/[\alpha p_k]$, or $\tau_2 < \alpha p_k\lambda_1/\beta - p_e$ in the integral. Remark that the upper bound in the integral is actually larger than the lower bound, i.e., $\alpha p_k\lambda_1/\beta - p_e > \tau_{2\,\min}$, as $\lambda_1 > \beta[p_e + \tau_{2\,\min}]/[\alpha p_k]$ (case (2) above).

We now perform the λ_1-derivative of $F(\lambda; \tau) = \text{const. } \lambda_2^{\beta/[1-\gamma]}[p_e + \tau + p_k\lambda_2]^{-\gamma/[1-\gamma]}$:

$$\beta\lambda_1^{\beta/[1-\gamma]-1}[p_e + \tau_1 + p_k\lambda_1 - \rho\delta\tilde{\tau}_2]^{-\gamma/[1-\gamma]} - \gamma p_k\lambda_1^{\beta/[1-\gamma]}[p_e + \tau_1 + p_k\lambda_1 - \rho\delta\tilde{\tau}_2]^{-\gamma/[1-\gamma]-1}$$
$$+\rho\int_{\tau_{2\,\min}}^{\alpha p_k\lambda_1/\beta - p_e} f(\tau_2)\{\beta\lambda_1^{\beta/[1-\gamma]-1}[p_e+\tau_2+p_k\lambda_1]^{-\gamma/[1-\gamma]} - \gamma p_k\lambda_1^{\beta/[1-\gamma]}[p_e+\tau_2+p_k\lambda_1]^{-\gamma/[1-\gamma]-1}\}d\tau_2 = 0,$$

The condition on λ_1 amounts to

$$[\beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1][p_e + \tau_1 + p_k\lambda_1 - \rho\delta\tilde{\tau}_2]^{-1/[1-\gamma]}$$
$$+ \rho\int_{\tau_{2\,\min}}^{\alpha p_k\lambda_1/\beta - p_e} f(\tau_2)[\beta[p_e + \tau_2] - \alpha p_k\lambda_1][p_e + \tau_2 + p_k\lambda_1]^{-1/[1-\gamma]}d\tau_2 = 0.$$



Remark that the first term is positive, while the second term is negative when $\beta[p_e + \tau_{2\,\min}]/[\alpha p_k] < \lambda_1 < \lambda_{\rm ref} = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$.

Managing to keep only positive terms:

$$G(\lambda_1, \tau_1) = 1,$$

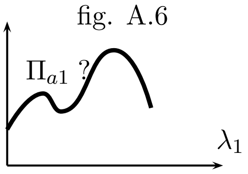
where

$$G(\lambda_1, \tau_1) = \rho\int_{\tau_{2\,\min}}^{\alpha p_k\lambda_1/\beta - p_e} f(\tau_2)\frac{\alpha p_k\lambda_1 - \beta[p_e + \tau_2]}{\beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1}\,\frac{[p_e + \tau_1 + p_k\lambda_1 - \rho\delta\tilde{\tau}_2]^{1/[1-\gamma]}}{[p_e + \tau_2 + p_k\lambda_1]^{1/[1-\gamma]}}d\tau_2. \tag{A.8}$$

$G(\lambda_1, \tau_1) = 0$ when $\lambda_1 = \beta[p_e + \tau_{2\,\min}]/[\alpha p_k]$, and $G(\lambda_1, \tau_1) \to +\infty$ when $\lambda_1 \to \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, so there must be at least one intermediate value of λ_1 where $G = 1$ (see fig. A.5). This is another way to ensure parts (i) and (iii) of Proposition 2.a, i.e., the existence of a solution $\beta[p_e + \tau_{2\,\min}]/[\alpha p_k] < \lambda_1 < \lambda_{\rm ref} = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$.

2.a.3. *Unicity of the solution of the first order condition.*

fig. A.6



When λ_1 maximizes (7), the equation $G = 1$ is certainly solved. The converse is not clear, as solutions of $G = 1$, i.e., of $\partial\Pi_{a1}/\partial\lambda_1 = 0$, may be any kind of stationary point for (7) (see fig. A.6). The situation is non ambiguous when G is a monotonous function of λ_1, as $G = 1$ can then have at most one root. We will look for a condition ensuring that $G(\lambda_1, \tau_1)$ is an increasing function of λ_1. Thus, we have to consider

$$\frac{\partial G(\lambda_1,\tau_1)}{\partial\lambda_1} = \rho \int_{\tau_{2\,\min}}^{\alpha p_k\lambda_1/\beta - p_e} f(\tau_2)[\tau_1 - \rho\delta\tilde{\tau}_2 - \tau_2]\frac{p_k}{1-\gamma}$$

$$\left[\frac{\alpha\beta[1-\gamma]}{[\alpha p_k\lambda_1 - \beta[p_e+\tau_2]][\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1]} - \frac{1}{[p_e+\tau_1+p_k\lambda_1-\rho\delta\tilde{\tau}_2][p_e+\tau_2+p_k\lambda_1]}\right]$$

$$\frac{\alpha p_k\lambda_1 - \beta[p_e+\tau_2]}{\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1}\frac{[p_e+\tau_1+p_k\lambda_1-\rho\delta\tilde{\tau}_2]^{1/[1-\gamma]}}{[p_e+\tau_2+p_k\lambda_1]^{1/[1-\gamma]}}d\tau_2$$

$\tau_1 - \rho\delta\tilde{\tau}_2 - \tau_2$ is positive in the whole integration interval, as $\tau_2 < \alpha p_k\lambda_1/\beta - p_e$, so $\lambda_1 > \beta[p_e+\tau_2]/[\alpha p_k]$, and we saw that $\lambda_1 < \beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$.

We also see that the ratio $\dfrac{\alpha p_k\lambda_1 - \beta[p_e+\tau_2]}{\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1}$ is positive for τ_2 in the (open) integration interval.

To be sure that G is increasing, it remains to show that the big intermediate factor is positive too, or that

$$\alpha\beta[1-\gamma] > \frac{\alpha p_k\lambda_1 - \beta[p_e+\tau_2]}{p_e+\tau_2+p_k\lambda_1}\,\frac{\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1}{p_e+\tau_1+p_k\lambda_1-\rho\delta\tilde{\tau}_2}$$

on the whole τ_2−interval, i.e., at $\tau_2 = \tau_{2\,\min}$, as the right-hand side is a decreasing function of τ_2. And as we do not know more about λ_1 that $\beta[p_e+\tau_{2\,\min}]/[\alpha p_k] < \lambda_1 < \beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$, we take the first fraction of the right hand side at its largest possible value, which is reached at $\lambda_1 = \beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$, and so for the second fraction, but this one at $\lambda_1 = \beta[p_e+\tau_{2\,\min}]/[\alpha p_k]$.

The sufficient condition of unicity is then

$$\alpha\beta[1-\gamma] > \frac{\alpha^2\beta^2[\tau_1-\rho\delta\tilde{\tau}_2-\tau_{2\,\min}]^2}{[\alpha[p_e+\tau_{2\,\min}] + \beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]][\beta[p_e+\tau_{2\,\min}] + \alpha[p_e+\tau_1-\rho\delta\tilde{\tau}_2]]}$$

which depends only on known values. The condition can be reworked as

$$1-\gamma > \frac{\alpha\beta x^2}{[\gamma+\beta x][\gamma+\alpha x]},$$

where $x = \dfrac{\tau_1-\rho\delta\tilde{\tau}_2-\tau_{2\,\min}}{p_e+\tau_{2\,\min}}$, or $\gamma < \dfrac{1}{1+\dfrac{\alpha\beta}{\gamma^2}\,\dfrac{x^2}{1+x}}$, or, as $\alpha\beta$ is always smaller than $\gamma^2/4$, a stronger sufficient condition

$$\gamma < \frac{1+x}{1+x+x^2/4}. \tag{A.9}$$

For a given x (or, equivalently, τ_1), returns to scale must be sufficiently decreasing (γ sufficiently low), and conversely. So that part (ii) of Proposition 2.a is established.

2.b. **Proof of Proposition 2.b (behaviour of e, k, and y with respect to τ_1).**

We now discuss e, k, and y, which are positive powers of λ_1 divided by powers of $p_k\lambda_1 + p_e + \tau_1 - \rho\delta\tilde{\tau}_2$: constant $\lambda_1^a/[p_k\lambda_1 + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b$, where the coefficients a and b are

	a	b
$e^{1-\gamma}$	β	1
$k^{1-\gamma}$	$1-\alpha$	1
$y^{1-\gamma}$	β	γ

Remark that $a < b$.

Things are simple when $\tau_1 \le \tau_{2\,\min} + \rho\delta\tilde{\tau}_2$, as exact formulas are available:
$\lambda_1 = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, so that

$$\frac{\lambda_1^a}{[p_k\lambda_1 + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b} = \text{ constant } [p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^{a-b}$$

is obviously a decreasing function of τ_1.

When $\tau_1 > \tau_{2\,\min} + \rho\delta\tilde{\tau}_2$, things are less explicit.

We discuss first variations of λ_1 with respect to τ_1. As G must keep a fixed value $G = 1$, $dG = [\partial G/\partial\lambda_1]d\lambda_1 + [\partial G/\partial\tau_1]d\tau_1 = 0$, so, $d\lambda_1/d\tau_1 = -[\partial G/\partial\tau_1]/[\partial G/\partial\lambda_1]$. As seen above, G being an increasing function of λ_1, $\partial G/\partial\lambda_1 > 0$ (see fig. A.5).

For a fixed λ_1, $G(\lambda_1, \tau_1)$ will increase or decrease according to the sign of $\partial G/\partial\tau_1$ which, from (A.8), is the sign of

$$\frac{\partial}{\partial\tau_1}\frac{[p_e + \tau_1 + p_k\lambda_1 - \rho\delta\tilde{\tau}_2]^{1/[1-\gamma]}}{\beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1} = \gamma[\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]-[1-\beta]p_k\lambda_1]\frac{[p_e + \tau_1 + p_k\lambda_1 - \rho\delta\tilde{\tau}_2]^{\gamma/[1-\gamma]}}{[\beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2] - \alpha p_k\lambda_1]^2}.$$

So, we see that G decreases when τ_1 increases for a fixed λ_1, as long as $\lambda_1 \ge \beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[p_k[1-\beta]]$.

Therefore, λ_1 will increase with τ_1 as long as $\lambda_1 \ge \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[p_k[1 - \beta]]$, and decreases if λ_1 is lower than this bound.

Let us derivate $\lambda^a/[p_k\lambda + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b$ with respect to τ_1. The sign of the derivative is the sign of

$$[[a-b]p_k\lambda_1 + a[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]]\frac{d\lambda_1}{d\tau_1} - b\lambda_1.$$

For e, $a = \beta$ and $b = 1$, the sign of $d\lambda_1/d\tau_1$ is exactly the sign of $[1-\beta]p_k\lambda_1 - \beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]$, as seen above. So, e is a decreasing function of τ_1, and this settles the first part of Proposition 2.b.

For k and y, it will be shown here that these functions of τ_1 are smaller than their values at $\lambda_{\text{ref}} = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, i.e., that

$$\frac{[\lambda_{a1}]^a}{[p_k\lambda_{a1} + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b} < \frac{\lambda_{\text{ref}}^a}{[p_k\lambda_{\text{ref}} + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b},$$

knowing that $\lambda_{a1} < \lambda_{\text{ref}}$. Indeed, $\lambda^a/[p_k\lambda_1 + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b$ is an increasing function of λ_1 up to $\lambda_1 = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$ if

$$a[p_k\lambda_1 + p_e + \tau_1 - \rho\delta\tilde{\tau}_2] - bp_k\lambda_1 = [[a-b]\beta/\alpha + a][p_e + \tau_1 - \rho\delta\tilde{\tau}_2] \ge 0,$$

or $a\gamma \ge b\beta$, which holds for k and y. Now, the upper bound $\lambda_{\text{ref}}^a/[p_k\lambda_{\text{ref}} + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^b = $ const. $[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^{a-b}$ is a decreasing function of τ_1, so that the remaining part of Proposition 2.b is established.

3. The relative baseline case.

We rewrite (4) with a relative baseline:

$$\max_{e_1\geq 0,\lambda_1\geq 0}\left\{e_1^\gamma\lambda_1^\beta-[p_e+p_k\lambda_1]e_1+\tau_1\left[\frac{\overline{e}_1}{\overline{y}_1}-\frac{e_1}{y_1}\right]\right.$$
$$\left.+\rho\int_{\tau_{\min}}^{\tau_{\max}}f(\tau_2)\max_{\substack{e_2\geq 0\\ \lambda_2\geq\lambda_1}}\left\{e_2^\gamma\lambda_2^\beta-[p_e+p_k\lambda_2]e_2+\tau_2[\tilde{e}_2-\delta[\overline{e}_1-e_1]-e_2]\right\}d\tau_2\right\}.$$

Emphasizing the dependence in e_1, using (1) and (2):

$$\max_{\lambda_1\geq 0}\left\{\max_{e_1\geq 0}\left\{\left[1+\tau_1\frac{\overline{e}_1}{\overline{y}_1}\right]e_1^\gamma\lambda_1^\beta-[p_e+p_k\lambda_1+\tau_1-\rho\delta\tilde{\tau}_2]e_1\right\}\right.$$
$$\left.+\rho\int_{\tau_{\min}}^{\tau_{\max}}f(\tau_2)\max_{\lambda_2\geq\lambda_1}\left\{\max_{e_2\geq 0}\left\{e_2^\gamma\lambda_2^\beta-[p_e+p_k\lambda_2+\tau_2]e_2\right\}\right\}d\tau_2\right\}+\tilde{\tau}_2\rho[\tilde{e}_2-\delta\overline{e}_1]$$

which is first solved with respect to e_1:

$$\max_{\lambda_1\geq 0}\left\{\Pi_{r1}(\lambda_1,\tau_1):=\left[1+\tau_1\frac{\overline{e}_1}{\overline{y}_1}\right]^{1/[1-\gamma]}F(\lambda_1,\tau_1-\rho\delta\tilde{\tau}_2)\right.$$
$$\left.+\rho\int_{\tau_{\min}}^{\tau_{\max}}f(\tau_2)\max_{\lambda_2\geq\lambda_1}F(\lambda_2,\tau_2)\,d\tau_2\right\}+\tilde{\tau}_2[\tilde{e}_2-\delta\overline{e}_1]\qquad\text{(A.10)}$$

where, as before, F is given by (A.3).

Optimal emissions of period 1 with respect to λ_1 and τ_1 are given by

$$e_{r1}=\left[\lambda_1^\beta\frac{[1+\tau_1\overline{e}_1/\overline{y}_1]\gamma}{p_e+\tau_1+p_k\lambda_1-\rho\delta\tilde{\tau}_2}\right]^{1/[1-\gamma]}$$

Note that $\overline{e}_1$ and $\overline{y}_1$ are the solutions of the elementary problem when $\tau_1=0$: $\lambda=\beta[p_e-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$, $e^{1-\gamma}=\gamma\lambda^\beta/[p_e+p_k\lambda-\rho\delta\tilde{\tau}_2]=\beta\lambda^{\beta-1}/p_k$, or

$$\overline{e}_1=\alpha^{[1-\beta]/[1-\gamma]}\beta^{\beta/[1-\gamma]}[p_e-\rho\delta\tilde{\tau}_2]^{[\beta-1]/[1-\gamma]}p_k^{-\beta/[1-\gamma]},\qquad\text{(A.11)}$$

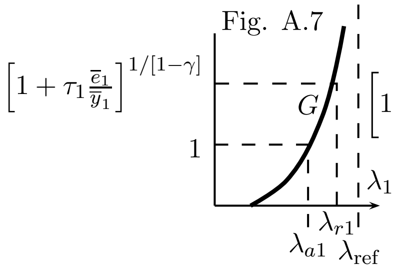
$e/y=e^{1-\gamma}\lambda^{-\beta}=\beta/[p_k\lambda]$, or

$$\frac{\overline{e}_1}{\overline{y}_1}=\frac{\alpha}{p_e-\rho\delta\tilde{\tau}_2}.\qquad\text{(A.12)}$$

3.a. Proof of proposition 3.a (existence, unicity and characterization of λ_{r1}).

As seen before, the integral does not depend on λ_1 if $\tau_1\leq\tau_{2\min}+\rho\delta\tilde{\tau}_2$, and the maximum is reached at $\lambda_1=\lambda_{\text{ref}}=\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]/[\alpha p_k]$, as in the absolute case.

Fig. A.7



When $\tau_1>\tau_{2\min}+\rho\delta\tilde{\tau}_2$, we consider the λ_1-derivative which is

$$\left[1+\tau_1\frac{\overline{e}_1}{\overline{y}_1}\right]^{1/[1-\gamma]}[\beta[p_e+\tau_1-\rho\delta\tilde{\tau}_2]-\alpha p_k\lambda_1][p_e+\tau_1+p_k\lambda_1-\rho\delta\tilde{\tau}_2]^{-1/[1-\gamma]}$$
$$+\rho\int_{\tau_{2\min}}^{\alpha p_k\lambda_1/\beta-p_e}f(\tau_2)[\beta[p_e+\tau_2]-\alpha p_k\lambda_1][p_e+\tau_2+p_k\lambda_1]^{-1/[1-\gamma]}d\tau_2=0,$$

or

$$G(\lambda_1, \tau_1) = \left[1 + \tau_1 \frac{\overline{e}_1}{\overline{y}_1}\right]^{1/[1-\gamma]},$$

with the same G as above, in (A.8). As before, $G(\lambda_1, \tau_1) = 0$ when $\lambda_1 = \beta[p_e + \tau_{2\min}]/[\alpha p_k]$, and $G(\lambda_1, \tau_1) \to +\infty$ when $\lambda_1 \to \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, so there must be at least one intermediate value of λ_1 where the equation for λ_1 is satisfied: $\beta[p_e + \tau_{2\min}]/[\alpha p_k] < \lambda_1 < \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$ (see fig. A.7).

As the right-hand side is larger than 1, and if G is an increasing function of λ_1 (remember the sufficient condition (A.9) $\gamma \leq [1 + x]/[1/x + x^2/4]$, which is still valid here), we have $\beta[p_e + \tau_{2\min}]/[\alpha p_k] < \lambda_{a1} < \lambda_{r1} < \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, and the whole Proposition 3.a is established.

3.b. Discussion of Result 3.b (behaviour of e, k, and y with respect to τ_1).

e_1, $k_1 = \lambda_1 e_1$, and $y_1 = e_1^\gamma \lambda_1^\beta$ now contain a power of $1 + \tau_1 \frac{\overline{e}_1}{\overline{y}_1}$ too, so e_1, k_1, and y_1 write like constant $\lambda_1^a [p_k\lambda_1 + p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^{-b} \left[1 + \tau_1 \frac{\overline{e}_1}{\overline{y}_1}\right]^c$, where the coeficients a, b, c are

	a	b	c
$e^{1-\gamma}$	β	1	1
$k^{1-\gamma}$	$1-\alpha$	1	1
$y^{1-\gamma}$	β	γ	γ

We look at the variation with respect to τ_1 assuming $\lambda_1 = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, which is exact when $\tau_1 \leq \tau_{2\min} + \rho\delta\tilde{\tau}_2$, and found numerically to be very close for larger τ_1's.

Then, the above expresion for e_1, k_1, y_1 becomes: constant $[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^{a-b}[1 + \tau_1\overline{e}_1/\overline{y}_1]^c$, $=$ constant $[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]^{a-b}[p_e + \alpha\tau_1 - \rho\delta\tilde{\tau}_2]^c$.

The τ_1-derivative at $\tau_1 = 0$ has the sign of $a - b + \alpha c = \gamma - 1, 0$, and $\alpha[\gamma - 1]$ for e_1, k_1, and y_1. Whereas for large τ_1, the behaviour is τ_1^{a-b+c} always increasing in the long run ($\beta, 1-\alpha$, and β), which are indications of a $U-$shape behaviour of e_1, k_1, and y_1 with respect to τ_1. This $U-$shape behaviour is indeed confirmed numerically.

Finally, e_1 and y_1, computed assuming $\lambda_{r1} = \lambda_{\text{ref}}$, are minimal when $[a - b + \alpha c][p_e - \rho\delta\tilde{\tau}_2] + \alpha[a - b + c]\tau_1 = 0$, so argmin $y_1 = \alpha$ argmin e_1.

This closes the discussion of result 3.b.

3.c. Proof of Proposition 3.c (comparison of y and k under both baselines).

We already saw that k_1 and y_1 are increasing functions of λ_1, so $k_{r1} \geq k_{a1}$ and $y_{r1} \geq y_{a1}$ hold, as $\lambda_{a1} \leq \lambda_{r1} \leq \lambda_{\text{ref}} = \beta[p_e + \tau_1 - \rho\delta\tilde{\tau}_2]/[\alpha p_k]$, and as there is a further factor $1 + \tau_1\overline{e}_1/\overline{y}_1 > 1$ in the relative case.

Note that for e_1, the expected result is not established, but the factor $1 + \tau_1\overline{e}_1/\overline{y}_1$ present in the formula for e_{1r} makes the inequality $e_{r1} \geq e_{a1}$ valid in most cases. Indeed, the inequality is verified numerically.

3.d. Proof of Proposition 3.d (comparison of Π_1 under both baselines).

Π^*_{r1} versus Π^*_{a1}:

We show that $\Pi^*_{r1} \leq \Pi^*_{a1}$ while $\dfrac{\tau_1}{p_e - \rho\delta\tilde{\tau}_2}$ is smaller than the smallest positive root of

$$X - \frac{1-\gamma}{\alpha}\left[[1 + \alpha X]^{1/[1-\gamma]} - 1\right][1 + X]^{-\alpha/[1-\gamma]} = 0. \tag{A.13}$$

From (A.6), (A.10), and (A.3), we may write for a given τ_1:

$\Pi_{a1}(\lambda_1) = H_a(\lambda_1) + K(\lambda_1)$, where $H_a(\lambda_1) = F(\lambda_1; \tau_1 - \rho\delta\tilde{\tau}_2) + \tau_1\overline{e}_1$;

and $\Pi_{r1}(\lambda_1) = H_r(\lambda_1) + K(\lambda_1)$, where $H_r(\lambda_1) = \left[1 + \tau_1 \dfrac{\overline{e}_1}{\overline{y}_1}\right]^{1/[1-\gamma]} F(\lambda_1; \tau_1 - \rho\delta\tilde{\tau}_2)$;

and where $K(\lambda_1)$ is the integral term $K(\lambda_1) = \rho \int_{\tau_{\min}}^{\tau_{\max}} f(\tau_2) \max_{\lambda_2 \geq \lambda_1} F(\lambda_2, \tau_2) d\tau_2 + \tilde{\tau}_2 \rho[\tilde{e}_2 - \delta\overline{e}_1]$.

Then,

$$
\begin{aligned}
&\Pi_{a1}(\lambda_{a1}) - \Pi_{r1}(\lambda_{r1}) - [\Pi_{a1}(\lambda_{\text{ref}}) - \Pi_{r1}(\lambda_{\text{ref}})] \\
&\qquad = -\int_{\lambda_{a1}}^{\lambda_{\text{ref}}} \Pi'_{a1}(\lambda_1) d\lambda_1 + \int_{\lambda_{r1}}^{\lambda_{\text{ref}}} \Pi'_{r1}(\lambda_1) d\lambda_1 \\
&\qquad = -\int_{\lambda_{a1}}^{\lambda_{r1}} \Pi'_{a1}(\lambda_1) d\lambda_1 - \int_{\lambda_{r1}}^{\lambda_{\text{ref}}} \Pi'_{a1}(\lambda_1) d\lambda_1 + \int_{\lambda_{r1}}^{\lambda_{\text{ref}}} \Pi'_{r1}(\lambda_1) d\lambda_1 \\
&\qquad \geqslant -\int_{\lambda_{r1}}^{\lambda_{\text{ref}}} [H'_a(\lambda_1) + K'(\lambda_1)] d\lambda_1 + \int_{\lambda_{r1}}^{\lambda_{\text{ref}}} [H'_r(\lambda_1) + K'(\lambda_1)] d\lambda_1 = \int_{\lambda_{r1}}^{\lambda_{\text{ref}}} [H'_r(\lambda_1) - H'_a(\lambda_1)] d\lambda_1 \\
&\qquad \geqslant 0,
\end{aligned}
$$

as Π'_{a1} is negative between λ_{a1} and λ_{r1}, $\Pi_{a1} = H_a + K$ being a decreasing function of λ_1 when $\lambda_1 > \lambda_{a1}$ (see fig. A.4), so that $\int_{\lambda_{a1}}^{\lambda_{r1}} \Pi'_{a1}(\lambda_1) d\lambda_1 \leqslant 0$.

Finally, H'_r and H'_a are positive as H_r and H_a, both of the form const. $F+$ const. (see section 1), reach their maximum at $\lambda_1 = \lambda_{\text{ref}}$, and $H'_r = [1 + \tau_1 \overline{e}_1/\overline{y}_1]^{1/[1-\gamma]} H'_a > H'_a$.

So,

$$\Pi_{a1}(\lambda_{a1}) - \Pi_{r1}(\lambda_{r1}) \geq \Pi_{a1}(\lambda_{\text{ref}}) - \Pi_{r1}(\lambda_{\text{ref}}).$$

We even have an equality while $\tau_1 \leqslant \tau_{2\,\min} + \rho\delta\tilde{\tau}_2$, as $\lambda_{a1} = \lambda_{r1} = \lambda_{\text{ref}}$ in this case.

We compute now the right-hand side in order to estimate the τ_1-interval where this lower bound is positive. It is

$$
\begin{aligned}
\Pi_{a1}(\lambda_{\text{ref}}) - \Pi_{r1}(\lambda_{\text{ref}}) &= \tau_1 \overline{e}_1 - \left[\left[1 + \tau_1 \frac{\overline{e}_1}{\overline{y}_1}\right]^{1/[1-\gamma]} - 1\right] \gamma' \lambda_1^{\beta/[1-\gamma]} [p_e + \tau_1 + p_k \lambda_1 - \rho\delta\tilde{\tau}_2]^{-\gamma/[1-\gamma]} \\
&= \tau_1 \alpha^{[1-\beta]/[1-\gamma]} \beta^{\beta/[1-\gamma]} [p_e - \rho\delta\tilde{\tau}_2]^{[\beta-1]/[1-\gamma]} p_k^{-\beta/[1-\gamma]} \\
&\quad - \left[\left[1 + \frac{\alpha\tau_1}{p_e - \rho\delta\tilde{\tau}_2}\right]^{1/[1-\gamma]} - 1\right] \gamma' \lambda_{\text{ref}}^{\beta/[1-\gamma]} [p_e + \tau_1 + p_k \lambda_{\text{ref}} - \rho\delta\tilde{\tau}_2]^{-\gamma/[1-\gamma]} \\
&= \text{const.} \left\{ \frac{\tau_1}{p_e - \rho\delta\tilde{\tau}_2} - \frac{1-\gamma}{\alpha} \left[\left[1 + \frac{\alpha\tau_1}{p_e - \rho\delta\tilde{\tau}_2}\right]^{1/[1-\gamma]} - 1\right] \left[1 + \frac{\tau_1}{p_e - \rho\delta\tilde{\tau}_2}\right]^{-\alpha/[1-\gamma]} \right\} \\
&= \text{const.} \left\{ X - \frac{1-\gamma}{\alpha} \left[[1 + \alpha X]^{1/[1-\gamma]} - 1\right] [1 + X]^{-\alpha/[1-\gamma]} \right\}, \qquad \text{(A.14)}
\end{aligned}
$$

from (A.11), (A.12), and $\overline{\lambda} = \dfrac{\beta[p_e - \rho\delta\tilde{\tau}_2]}{\alpha p_k}$, and where $X = \dfrac{\tau_1}{p_e - \rho\delta\tilde{\tau}_2}$.

The lower bound $\Pi_{a1}(\lambda_{\text{ref}}) - \Pi_{r1}(\lambda_{\text{ref}})$ is positive for small positive τ_1, as the Taylor expansion of (A.14) with respect to X about $X = 0$ starts with const. $\dfrac{\alpha[2-\gamma]}{2[1-\gamma]} X^2 > 0$, but the bound behaves like $-X^{[1-\alpha]/[1-\gamma]} < 0$ for large X, so there is a finite lowest positive root, however comfortably large if γ is not too close to 1. An empirical formula for a valid interval is $0 < X < 6\sqrt{1-\gamma}$.