Model Predictive Control
for Electric Vehicle Charging

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Abstract—In this paper we present a model predictive control algorithm for scheduling electric vehicle charging. The model accounts for distribution network constraints and seeks to minimize the cost of procuring energy from the real-time market for electric vehicle charging. We present a case study for an IEEE test case distribution system.

I. INTRODUCTION

The central scheduling of electric vehicle charging is becoming increasingly important as electric vehicles are being deployed worldwide. In this paper we present an online algorithm for dispatching electric vehicles in the real-time market, while accounting for distribution network constraints.

Numerous studies have been published recently that consider the coordination of electric vehicle charging in order to account for constraints on the distribution network. The paradigms of electric vehicle control that have been set forth can be categorized as: centralized control where an aggregator manages the dispatch of each electric vehicle individually; decentralized control where electric vehicles control their charging autonomously, for example by responding to a price signal or by immediately charging once they arrive to their destination; and a hybrid approach of centralized and decentralized control whereby the aggregator acts as a market-clearing agent for coordinating the consumption of electric vehicles based on bids that depend on the private information available to individual electric vehicles. The latter approach is advantageous as it decentralizes the computational effort required and imposes minimal requirements on the information that is available to the aggregator.

The model we present follows the electric vehicle charging model presented by Sundström and Binding [2] who account for distribution network constraints. The authors consider a linear programming formulation of the smart charging problem, where the objective is to minimize real-time energy costs in an energy market that clears every 15 minutes. The authors consider three charging policies: eager charging, whereby vehicles charge immediately as they plug in, price-based charging whereby vehicles respond to the real-time electricity price without coordinating in order to prevent grid overloads, and grid-aware price-based charging, where electric vehicles respond to real-time electricity prices while coordinating in order not to overload the distribution grid. The model of Sundström and Binding is used in the EDISON project for integrating electric vehicles in the Danish island of Bornholm [3]. The authors demonstrate that the additional cost of coordinating charging schedules in order to prevent the overloading of the distribution grid amounts to a mere 0.2% cost increase, while yielding substantial benefits in terms of preventing grid overload.

Similar work in this area includes a paper by Galus and Andersson [1]. In this model, the authors use energy hubs for coordinating the consumption of electricity, the charging of plug-in hybrid electric vehicles and heating. Energy hubs communicate with a PHEV manager that receives bids from individual PHEV agents. The PHEV agents hold private information regarding their driving patterns and their valuation for power. The PHEV manager is then responsible for clearing the bids of these agents and provides an aggregate demand to the energy hub agent. The energy hub agent subsequently co-optimizes the dispatch of electricity and natural gas in order to meet both thermal as well as electrical loads.

The possibility of using electric vehicles for providing ancillary services to the grid is discussed by Brooks et al. [4]. The authors discuss the possibility of using electric vehicles for providing services ranging from the hourly time step down to sub-second balancing through frequency response. The authors present an example of electric vehicles responding to the regulation signal of ERCOT every 4 seconds.

A central driving force that motivates the research on large-scale electric vehicle integration in power systems is the potential of electric vehicles to enable the deep integration of renewable energy sources in power systems [7]. This idea is discussed by Brooks et al. [4] who test the ability of a fleet of several hundred electric vehicles to absorb the fluctuations of 3,800 MW of wind power supply in Texas. Similarly, a significant amount of power in the study by Sundström and Binding [2] is provided by renewable generators.

The impact of the large-scale integration of electric vehicles on power system operations from the point of view of the system operator is also an important area of research [5]. This assessment requires a detailed model of both electric vehicle operations as well as the balancing operations of the
II. Model

Distribution network constraints necessitate the centralized management of electric vehicle charging by an aggregator. In the absence of distribution network constraints, electric vehicles could respond independently to a real-time price signal, without coordinating their charging schedules. In this model we assume that the aggregator can measure the loading of each transformer in the distribution system. Following Sundström and Binding [2], we represent the distribution network as a directed graph \( G = (N, E) \). The source node of the network represents the root of the distribution system. Each load point corresponds to a sink node.

A. Nomenclature

We first describe the notation used in the optimization model.

Sets

- \( D \): set of vehicle drivers
- \( T = \{1, \ldots, t_d\} \): set of time periods
- \( N \): set of nodes in the distribution network
- \( E \): set of edges in the distribution network

Decision variables

- \( e_{dt} \): power supply to driver \( d \) in period \( t \)
- \( r_{dt} \): residual energy demand of driver \( d \) in period \( t \)
- \( s_{nt} \): supply in node \( n \) in period \( t \)
- \( f_{lt} \): flow in line \( l \) in period \( t \)

Parameters

- \( \lambda_t \): forecast price of electricity for period \( t \)
- \( E_{dt} \): forecast of energy consumption of driver \( d \) in period \( t \)
- \( L_{dt} \): forecast of location of driver \( d \) in period \( t \)
- \( R_{dk} \): initial condition of residual demand of driver \( d \) in step \( k \)

- \( B \): battery energy capacity
- \( C \): battery power capacity
- \( T_l \): transfer capacity of line \( l \) in period \( t \)
- \( S_n \): limit on injection of power to node \( n \)
- \( D_{nt} \): forecast of non-EV demand of node \( n \) in period \( t \)
- \( LI_n \): set of lines directed in node \( n \)
- \( LO_n \): set of lines directed out of node \( n \)
- \( t_d \): deadline for serving EV demand
- \( \rho \): penalty for each unit of unserved EV demand

B. Model Predictive Control

Model predictive control is an appealing online algorithm for controlling electric vehicle charging due to computational speed. Assuming that aggregators will need to control thousands of electric vehicles in real time and update decisions at the frequency of the real-time energy market (every five to fifteen minutes), computational speed becomes the key constraint for designing online planning algorithms. In order to describe the model predictive control algorithm we first describe the problem that is solved at each interval of operations \( k \):

\[
\begin{align*}
(\text{MPC}_k) : \min \sum_{t \in T} \lambda_t \sum_{d \in D} e_{dt} + \sum_{d \in D} r_{dt} & \\
\text{s.t.} & \quad r_{d0} = R_{dk}, d \in D \\
& \quad r_{dt} = r_{d,t-1} - e_{d,t-1} + E_{d,t-1}, d \in D, t \in T \setminus \{0\} \\
& \quad s_{nt} + \sum_{l \in LI_n} f_{lt} = D_{nt} + \sum_{i \in T, n = i} e_{it} + \sum_{l \in LO_n} f_{lt}, n \in N, t \in T \\
& \quad -T_l \leq f_{lt} \leq T_l, l \in L, t \in T \\
& \quad 0 \leq e_{dt} \leq C \cdot 1_{[L_{dt}, \infty)}, d \in D, t \in T \\
& \quad 0 \leq r_{dt} \leq B, d \in D, t \in T \\
& \quad 0 \leq s_{nt} \leq S_n, n \in N, t \in T
\end{align*}
\]

The objective of the aggregator in Eq. (1) is to minimize operating costs which consist of the cost of procuring energy from the real-time market and the cost of unserved energy. The deadline for serving driver energy demand is assumed to be 6 a.m. Note that the end period of the optimization \( t_d \) is a function of the running interval \( k \). For example, in the beginning of the day, \( k = 0 \), \( t_d = 288 \), for \( k = 1 \) we have \( t_d = 277 \) and so on. Any amount of unserved energy at the end of the horizon incurs a cost of 500 $/MWh. We emphasize that the state variables \( r_{dt}, d \in D, t \in T \), are the remaining energy demands in the beginning of period \( t \). The constraints in Eq. (2) represent the initial condition for the energy demand of each electric vehicle. The dynamics of evolution of the residuals are given in Eq. (3). The power flow balance is given in Eq. (4). The constraints of Eq. (5) represent the flow limits on the network transformers. In Eq. (6) we represent the power charging limits of EV batteries. The indicator function is used to prevent charging when vehicles are located outside the charging network (by convention denoted as node \( N_{-1} \)) or when the vehicles are en route (by convention denoted as node \( N_0 \)). The battery energy capacity is given in the constraints of Eqs. (7). Power supply limits for each node are given in Eq. (8).

The model predictive control algorithm proceeds as follows:

**Step (a):** Set \( k = 0 \).
**Step (b):** Set \( R_{dk} = R_{d,k-1} - \hat{e}_{d,k-1} + \hat{E}_{d,k-1} \). Update \( t_d \).
**Step (c):** Update the forecasts of energy demand over the planning horizon, \( E_{dt}, d \in D, t \in T \); the forecasts for the location of drivers over the planning horizon \( L_{dt}, d \in D, t \in T \); the forecasts of energy demand in each node over the planning horizon \( D_{nt}, n \in N, t \in T \); and the forecasts of real-time energy prices over the planning horizon \( \lambda_t, t \in T \).
**Step (d):** Solve \((\text{MPC}_k)\) and apply the control \( \hat{e}_{dk} = e_{d0}, d \in D \). Set \( k = k + 1 \) and return to step (b).

C. Perfect foresight

The perfect foresight algorithm can predict the outcome of the entire day in advance and is used for benchmarking
III. Case Study

A. Distribution System

We are using a distribution system which is available for IEEE reliability studies [6]. The topology of the system is shown in Fig. 1. The characteristics of the system are described in detail in Allan et al. [6]. In Table I we present detailed information about the static (non-EV) loads in the system. We use the information in Table I in conjunction with typical electricity demand profiles for residential customers, commercial buildings and government buildings from the Southern California Edison dynamic load profiles database to develop consumption profiles for static loads. In particular, we choose a typical static load profile for winter weekdays (01/01/2010), summer weekdays (06/01/2010), winter weekends (01/02/2010), and summer weekends (06/05/2010) from the Southern California Edison database.

The capacity of each transformer for the network of Fig. 1 is assumed to be 16 MVA in Allan et al. [6]. In order to enhance the significance of distribution network constraints, we assume that the capacity of all transformers leading to a load point is equal to 2 MW and that the capacity of each transformer in the root of the distribution network is equal to 28 MW.

B. Driving Behavior

In this case study we assume that the aggregator is coordinating 3000 vehicles. We assume that each time step lasts 5 minutes. The driving speed of all vehicles is assumed to be equal to 60 miles per hour. The assumed mileage of electric power is assumed to be 4 miles per kWh. This translates to 1.25 kWh of electricity consumption per time step for each vehicle that is en route. The geographical distance between feeders is assumed equal to the difference between load point indices.

In order to construct driving behavior data, we consider four day types. We differentiate between summer weekdays, summer weekends, winter weekdays and winter weekends.

For weekdays, the following assumptions hold:

- Each driver resides in one of the residential feeders.
- The chances of being located in any of these feeders is proportional to the population of the feeder.

For weekends, the following assumptions hold:

- Each driver is randomly assigned to one of the work locations. The chances of being located in any of the work locations is proportional to the average consumption of the given working location.
- Each driver has a 30% probability of leaving home after work for leisure. If drivers leave, they go to their nearest commercial location. The assignment of commercial locations to home locations is given in Table II.
- The departure times in the morning are distributed uniformly between 6.30 a.m. and 11 a.m.
- The departure times in the afternoon from work are distributed uniformly between 3 p.m. and 8 p.m.
- The departure times in the evening from home to leisure are distributed uniformly between 6 p.m. and 10 p.m.
- The departure times in the night from leisure to home are distributed uniformly between 8 p.m. and midnight.

![Fig. 1. A schematic of the test distribution system.](http://www.sce.com/AboutSCE/Regulatory/loadprofiles/loadprofiles.htm)
- Each driver has a 20% probability of staying home, a 60% probability of moving to the nearest leisure location and a 20% probability of being absent from the service territory.
- If the driver is moving out of the service territory, the distance driven is 30 minus the index of the home feeder.
- The departure time for leisure is distributed uniformly between 8 a.m. and 4 p.m.. The departure time for returning home is distributed uniformly between 2 p.m. and 10 p.m..

C. Forecasts

We assume that the aggregator forecasts real-time prices, driving patterns and static load demand by using data dating one week in the past. More sophisticated forecasting methods can be employed and tested against this benchmark hypothesis.

D. Results

The following results are based on the price realization for January 9th, 2009. We do not present results for summer days as these are not publicly available currently. Since 2009, the California electricity market enacted convergence bidding. This has had a significant impact on day-ahead and real-time electricity prices, therefore our analysis focuses on the most recent price data available. In Fig. 2 we present the evolution of real-time electricity prices and in Fig. 3 we present the evolution of total energy costs to the aggregator. In Fig. 4 we present the evolution of cumulative energy demand in the system. Two drawbacks of the model predictive control algorithm become apparent. Firstly, demand spikes occur during periods of low prices. Although these spikes do not violate any of the model constraints, they are expected to affect the distribution system adversely and it may be possible to prevent them by adding additional constraints to \((MPC_k)\) (e.g. constraints on the ramp rate of vehicles). Moreover, the algorithm delays a portion of the charging until the end of the charging deadline. This behavior may be alleviated by developing more accurate price forecasting models. In Fig. 5 we present the power consumption at each location of the distribution network. In Fig. 6 we present the line flow through the 2000 MVA transformers of the network. We note that for the selected day the transformer capacity constraints are not active. This observation suggests that the centralized scheduling problem could first be relaxed by ignoring distribution constraints, and enforcing violated distribution constraints iteratively. This approach is implemented by Sundström and Binding [2]. The authors develop cuts on the centralized cost minimization problem by identifying maximum flow cuts on the distribution network, and limiting the amount of electric vehicle charge through these cuts.

IV. CONCLUSIONS AND FUTURE WORK

We have presented a model predictive control algorithm for dispatching electric vehicles in the real-time energy market. We have also described a test system for evaluating the performance of the algorithm that is based on a IEEE test
case distribution system. The algorithm accounts for distribution network constraints and uses a simple forecasting methodology for determining a control strategy with a one-day lookahead forecast of electricity prices, driving patterns and distribution network energy demand.

The test case that we have developed can be used for evaluating the performance of alternative smart charging schemes and forecasting models. In future work we are interested in decentralizing the proposed model per electric vehicle by relaxing distribution network constraints in order to increase the computational speed and reduce the memory requirements of the proposed online algorithm. The use of a stochastic optimization model that better hedges the charging policy of electric vehicles is also an interesting area of future work. We are also interested in enforcing additional operational constraints that prevent aggregate vehicle consumption from spiking during periods of low electricity prices, as well as constraints that limit the switching frequency of individual vehicle batteries. The extension of the model to incorporate day-ahead bidding (as in Papavasiliou and Oren [7]) and the co-optimization of energy and ancillary services is also an interesting direction of future work.

REFERENCES


