1. Introduction

Market Coupling is the most advanced Market Design in the restructuring of the European (EU) electricity market. Market Coupling is the current outcome of a long and possibly never ending process toward an integrated electricity market (Glachant & Ruester, 2014). The path toward Market Coupling developed from an initially very decentralized view of an electricity market that only cared for access to the grid; development toward centralization then came as one progressively became aware of the very particular need for coordination required by electricity. Market Coupling takes some of these requirements into account but still remains far from the central organization that drives the United States (US) Independent System Operator (ISO) system. Market Coupling indeed relies on a separation of the energy market (organized by Power Exchanges) and transmission services (congestion and grid support services) operated by Transmission System Operators (TSOs). As several other electricity markets, Market Coupling is a two tiered system with a day-ahead market followed by a real-time market or, more exactly, a balancing system. The system is zonal in the sense that the European (EU) market is decomposed into zones (often, but not always, corresponding to countries) where Kirchhoff’s laws are ignored and it is assumed that there is no congestion. Power Exchanges operate in national markets and a special supra national organization clears the global energy market on the basis of the bids submitted to the national Power Exchange and a representation of the grid coming from the TSOs (Euphemia Public Description, 2013).

Our work can be related to two streams of previous work. The first stream of literature is that related to the application of stochastic programming models to the electricity markets. Because events can always occur between day ahead and real time, the TSO can encounter in real time network difficulties that have not been anticipated in the day ahead. This phenomenon is enhanced in Market Coupling because market clearing in the day ahead uses an imperfect representation of the grid. In their paper, Smeers, Oggioni, Allevi, and Schaible (2012) assessed the amount of counter trading that might arise in real time as a result of using an approximate description of the grid in the day ahead. This problem takes a further dimension with wind penetration as wind forecast remains imperfect and any market clearing based on such forecasts in the day ahead is bound to be in imbalance in real time. The issue was examined in Oggioni, Murphy, and Smeers (2014) under strong assumptions of perfect coordination among TSOs to organize counter trading. In both cases, the work compared the result of a stylized representation of Market Coupling with an equally stylized representation of nodal pricing. Our second stream for inspiration comes from the computer systems literature (Nair, Adlakha, & Wierman, 2014). In their model, Nair et al. explicitly characterized the impact of growing wind power penetration on electricity Market Design by optimizing the placement of an intermediate market.
between the day ahead and real time, assuming supply and demand balance each other (Nair et al., 2014). The originality of their model stemmed from its capability to derive closed form expressions for the optimal energy procurement strategy by making the analogy with a variant of the well-known newsvendor problem.

Market Coupling is considered as a major achievement on the path to internal energy market; its progress has been discussed at length in EU circles such as the Regulatory Forum in Florence. Surprisingly the literature and in particular the formalized literature on the subject has remained relatively limited in contrast for instance with the extensive literature in the US nodal pricing system (Newbery, Strbac, & Viehoff, 2012).

Our model extends the approach of Nair et al. by introducing competition among the geographic demand markets subject to Market Coupling and characterizing the optimal investment policy in renewable energy supply capacity, which is becoming a major consideration for utilities that are subject to aggressive renewable portfolio standards.

Accurate short-term forecasts of wind farm power output over the next few hours to days are important factors for secure and low cost operations of power systems with high wind power penetration (Morales, Zugno, Pineda, & Pinson, 2014; Papavasiliou & Oren, 2013). According to Girard, Laquaine, and Kariniotakis (2013), it is difficult to quantify the economic benefit of improved forecasting. The recent literature dealing with the placement of wind turbines concludes that the aggregation of wind farms can produce significant effects in terms of variability and cost reduction (Girard et al., 2013) since forecast errors might compensate each other. Furthermore, a portfolio of wind farms is likely to give better results in terms of the trade-off between cost and profit and its variability than relying on a single wind farm (Green & Staffell, 2013). Considering both problems of wind farm expansion and optimal wind farm portfolio generation, Girard et al. checked, using real data from Western Denmark, that power generator revenue is linear with respect to the wind farm capacity factor and that the accuracy of short-term wind power forecasts has only a very small impact on revenue. However, as the authors explained, their results do not quantify the benefit of predictability from a global system point of view. Adopting a more systemic approach, Green showed, using 18 years of hourly wind speed data coming from 120 sites around Great Britain, that careful market analysis is needed if investors are to build optimal portfolios of wind stations (Green & Staffell, 2013). Baringo and Conejo already made the link, dealing with the optimization of a strategic wind power investor who sells wind power in a two tiered market (including a day-ahead and a balancing market) (Baringo & Conejo, 2014).

The main contribution of our work is an analytical treatment of the strategic positioning of suppliers in two tiered electricity markets, where real-time conditions are dictated by the uncertainty of renewable energy supply. A by-product of this analysis is the characterization of the optimal investment policy in renewable energy supply capacity, for suppliers acting as investors in wind power.

According to the literature mentioned above, a careful (simplified) modeling of Market Coupling is the most crucial modeling aspect. We consider a certain number of geographic demand markets, described in Section 2. Since we want to characterize the agents’ general behaviors, we do not consider explicitly Kirchhoff’s laws and aggregate the supply and the demand at the market level. The originality of our approach relies on its capability to cope with competition among suppliers that are operating in spatially separated electricity markets, which was ignored in the previously cited models. After having defined the agents’ roles in Section 2.1, we assume that a Signaling Game occurs over each geographic demand market in the day ahead. Its timing is described in Section 2.2. The link between day-ahead and real-time markets is guaranteed by the existence of a forward position in the day-ahead energy market that compensates for the uncertainty of supply and demand in real time. The Game is solved for two Market Designs: two tiered with exogenous prices in Section 3 and two tiered with endogenous prices in Section 4. Efficient wind farm portfolios are then characterized on the basis of the Markowitz Frontier definition. Its exploration is detailed in Section 5. Contrary to traditional approaches that ignore the occurrence of rare events when performing risk minimization, the Markowitz Frontier is defined in a context of rare events, guaranteeing the robustness of the wind farm distribution. Illustrations based on real wind speed and energy consumption data for France, Germany and Belgium are provided in Section 6.

### 1.1. Specific notation and modeling assumptions

#### Parameters:

- $t_f$: Day-ahead market time of occurrence
- $t_0$: Real-time market time of occurrence
- $N$: Number of geographic demand markets
- $\kappa_{i,j}$: Equivalent capacity between markets $i$ and $j$

#### Probability density functions:

- $\Gamma(a)$: Gamma function evaluated in $a$
- $\Gamma_{inc}(a, x)$: Incomplete Gamma function evaluated in $a, x$
- $N(0; \sigma^2)$: Gaussian density function centered in 0 and of standard deviation $\sigma$

### Market setting:

- $p_f^i$: Day-ahead price for one unit of conventional energy
- $p_0^i$: Real-time price for one unit of conventional energy
- $c_j$: Market $i$ cluster set
- $\rho_{i,j}$: Compensation tariff for wind over-supply
- $\bar{U}_i$: Supplier expected cost
- $\Delta_i$: Conventional generator expected profit
- $s_f^i$: Supply of conventional in the day ahead
- $s_0^i$: Supply of conventional in real time
- $c_f^i$: Marginal cost of conventional at $t_f$
- $c_0^i$: Marginal cost of conventional at $t_0$
- $q_f^i$: Market $i$ demand of conventional at $t_f$
- $q_0^i$: Market $i$ demand of conventional at $t_0$
- $\bar{q}_i$: Market $i$ over-supply of wind at $t_0$
- $r_f^i$: Day-ahead position
- $\gamma_i$: Average wind production of a single wind farm
- $\theta_i$: Number of wind farms
- $\lambda_{i,j}^f$: Bilateral trade between market $i$ and $j$ at $t_f$
- $\lambda_0^i$: Bilateral trade between market $i$ and $j$ at $t_0$
- $\Delta_i$: Sum of market $i$ trades in real time (energy balance)
- $\Delta_i^e$: Sum of market $i$ exports in real time
- $\bar{d}_i$: Market $i$ total demand in real time
- $\tilde{d}_i$: Day-ahead forecast of market $i$ total demand
- $\nu_i$: Forecast error made on the demand prediction
- $\sigma_i^e$: Demand forecast error standard deviation
- $w_i$: Wind power produced in real time
- $\tilde{w}_i$: Day-ahead forecast of wind power production
- $\epsilon_i$: Forecast error on wind power prediction
- $\Delta_i^e$: Difference between wind power and demand forecast errors
- $\sigma_i^e$: Wind power forecast error standard deviation
- $\sigma_{\Delta_i}$: Forecast error difference standard deviation
- $\tilde{\epsilon}_i$: Forecast error on a single wind farm
- $\sigma_i$: Single farm forecast error standard deviation
- $\tilde{\Delta}_i$: Complementary cumulative distribution function associated with $\Delta_i$
Normality of the deviations:
\[ v_i \sim N(0; (s_{\text{dev}}^i)^2), \quad \varepsilon_i \sim N(0; (\sigma^i)^2) \]
\[ \Delta_i \sim N(0; \sigma^2_{\Delta_i}) \]
\[ \Delta_i \text{ is independent of } \Delta_j, \forall j = 1, \ldots, N, \ j \neq i \]

Strategic wind capacity investment problem:
- **T**: Investment problem horizon
- **Z(t, T)**: Market i supplier investment function
- **\( \xi \)**: Construction cost for a single wind farm
- **\( \Xi \)**: Construction cost for the wind farm portfolio

Market modeling simplifications:
- Balancing is offered by a different set of technologies than those used in the day-ahead market, as a proxy of the existing practice in Europe where the power exchange (administered by the power exchange operator) is separated from the operation of the balancing market (administered by TSOs).

We invoke rational expectation of suppliers and generators with respect to the outcome of the real-time market, which implies that agents correctly anticipate the congestion patterns of the real-time market. This is an extension of the usual assumption of stochastic programming to a game context. In other words, first stage decisions are optimal with respect to the outcome that they influence. Suppliers then anticipate the outcome of the market (this does not imply anticipating the realization of demand and wind supply, which is inherently uncertain and cannot be inferred by rationality), and since line congestion in real time is an outcome of the market, suppliers are able of predicting real-time congestion patterns and optimally adapting their day-ahead strategy. Conventional generators are subject to market power in each stage: they are subject to the strategic decisions of suppliers.

- We assume, in our paper, that in the balancing phase, excess wind is compensated by a feed-in-tariff\(^1\); whereas under-supply is penalized at the real-time price, higher than the compensation price. This is an extreme case of a monotone kinked deviation price where agents are charged more for a shortage than they are remunerated for excess supply. This form of balancing penalty is common in European balancing systems (Martin, Smeers, & Aguado, 2014; Potential Cross-Border Balancing Cooperation between the Belgian, 2014). Although the exact closed form solutions depend on this choice of penalty, the qualitative results (existence of an equilibrium) extend beyond this particular case.

2. The market model

We consider suppliers (distributors or utility companies) with long term contracts for renewable energy. Given such a long term contract, the suppliers participate in a two tiered market: a day-ahead market containing one or more TSOs and is fully interconnected with the real-time markets. A negative sign indicates an import from \( j \) to \( i \), whereas a positive sign indicates an export from \( i \) to \( j \). Throughout the article, we will use the simplifying notations: \( \Lambda_i \triangleq \sum_{j \neq i} \lambda_{ij}^0 \Lambda_j^0 \triangleq \sum_{j \neq i} (\lambda_{ij}^0)_{i} \), and the following conventions:
\( (x)_+ \triangleq \max\{x; 0\} \) and \( (x)_- \triangleq \max\{-x; 0\} \), \( \forall x \in \mathbb{R} \).

Market Coupling clears energy and transmission in the day ahead with an "implicit auction" of transmission capacity (Hagspiel et al., 2014; Neuhoff, Hobbs, & Newbery, 2011), and is followed by intra-day trading and real-time balancing. Commonly, markets having smaller prices export to markets having higher prices, until a common price is reached or congestion occurs due to limited Available Transmission Capacities at the interconnections (Daxhelet & Smeers, 2007; Kunz, 2013; Smeers et al., 2012). Congestion management remains a controversial issue in the restructured European electricity sector. Congestion occurs when the infrastructure constrains transactions, thereby influencing the long term evolution of generation and consumption (Daxhelet & Smeers, 2007).

Our network model is based on a virtual representation of the European area where each node coincides with a geographic demand market containing one or more TSOs and is fully interconnected with other geographic demand markets. We give an illustration of this representation in Fig. 1 for three geographic demand markets (\( N = 3 \)). Inside each demand market, the balancing markets (managed by the TSOs) are coupled whereas they are split with the \( N - 1 \) other demand markets. We introduce the following definition: geographic demand markets are coupled in real time if, and only if, their real-time prices coincide i.e., no congestion occurs in real time.\(^3\) Otherwise, they are said to be split. We introduce the equivalent interconnection capacity between market \( i \) and market \( j \): \( k_{ij} \in \mathbb{R} \) such that \( k_{ij} < +\infty, i \neq j \). According to Hutchison and Bialek's representation of the European system (Hutchison & Bialek, 2013), for any \( i, j = 1, \ldots, N \).


2. The term "geographic" will be understood in the rest of the article.

3. The case of uncongested markets in real time is studied in Le Cadre and Didier (2014).
\[ i \neq j. \ k_{i,j} \text{ can be either positive or negative; furthermore, we have the relation: } k_{i,j} = -k_{j,i}. \] We assume that all agents correctly anticipate the congestion of a certain set of lines leading to a virtual representation of the European area in \( N \) split demand markets. Such an anticipation has already been done by Borenstein, Bushnell, and Stoft (2000) for two geographic markets sharing a constrained transmission line. As a by-product of congestion anticipation, for any couple of demand markets, real-time trade equals equivalent interconnection capacity minus day-ahead trade i.e., \( \lambda^0_{i,j} = k_{i,j} - \lambda^1_{i,j}, \forall i, j \neq i. \)

2.1. The agents

There is a certain number of geographic demand markets, each characterized by a price insensitive\(^4\) and random demand. Only forecasts are available in the day ahead and the true values of the demand (i.e., its realizations) are revealed in real time. Similarly, wind generation in each geographic market is price insensitive and random: only forecasts are available in the day ahead and the true values (i.e., the random variables realizations) are revealed in real time. We now describe the different categories of agents interacting in each demand market.

- Suppliers (distributors or utility companies) deliver energy to consumers characterized by their aggregated demand. They are price takers in the first Market Design (MD 1) detailed in Section 3. In contrast with standard assumptions, they are not price takers in the second Market Design (MD 2), described in Section 4. In this latter Market Design, they are aware that their decisions modify prices and take that knowledge into account to minimize their procurement cost. Since consumers are exposed to a retail price that is unrelated to wholesale market conditions, we assume an inelastic consumer demand whose real-time level is uncertain in the day-ahead time frame. Additionally, we assume that the suppliers are driven by renewable portfolio standards to invest in renewable capacity. Suppliers have to decide how to compose an optimal wind farm portfolio for participating in the electricity market.

- Conventional energy generators are characterized by their supply function. There is conventional generation in each market. Marginal costs are higher in real time than in the day ahead for a given output level. We assume that generators do not exercise market power, and suppliers buy electricity at marginal cost.

2.2. A Signaling Game

We make the assumption that a clearing price is reached at \( t_f \). Because the trades are limited by the equivalent interconnection capacity, it will be harder to align the market prices at \( t_0 \). We assume that all agents correctly anticipate the clearing price at \( t_f \) and the relation: \( \lambda^0_{i,j} = k_{i,j} - \lambda^1_{i,j}, \forall i, j \neq i. \)

Under congestion anticipation of all the equivalent interconnection capacities, the trades at \( t_f \) are linked to the optimal trades at \( t_0 \) according to the relation: \( \lambda^0_{i,j} = k_{i,j} - \lambda^1_{i,j}, \forall i, j \neq i. \) The distributed optimization of the program described above gives rise to a game belonging to the family of Signaling Games (Sobel, 2000). Here the signal is based on the suppliers’ forward positions which can be expressed as mappings of the suppliers’ wind and demand forecasts of the underlying state of the system (unknown in the day ahead): \((W_t, \hat{d}_t)_{t=1 \ldots N} \).

The bilevel game described above is solved by backward induction: we start by optimizing the generator’s bilateral trades, assuming that the suppliers’ forward positions are fixed. Then, the supplier’s expected cost are optimized by deriving their utility with respect to their purchase of conventional energy.

2.3. Description of the markets

Market \( i \) is defined by:

- \( d_i \), the end users’ total demand of energy at time \( t_0 \). It satisfies the relation: \( d_i = \hat{d}_i - v_i \) where \( \hat{d}_i \) is the forecast made at \( t_f \) of the end users’ total demand of energy at \( t_0 \). \( v_i \) is a random variable, representing the forecast error made on the demand prediction, and distributed according to a Gaussian density function centered in 0 and of standard deviation \( \sigma_v \); \( v_i \sim \mathcal{N}(0; \sigma_v^2) \).

- \( w_i \), the energy produced at time \( t_0 \) by the market renewable energy generators. It satisfies the relation: \( w_i = \check{w_i} - e_i \) where \( \check{w_i} \) is the forecast made at \( t_f \) of the quantity of renewable energy that market \( i \) generator will produce at \( t_0 \). \( e_i \) is a random variable, representing the forecast error made on the prediction of the renewable production, distributed according to a Gaussian density function centered in 0 and of standard deviation \( \sigma_e \); \( e_i \sim \mathcal{N}(0; \sigma_e^2) \). The forecast error on the production of a single wind farm will be denoted \( \hat{e}_i \). Being consistent with the assumption made on \( e_i \) generation, it is distributed according to a Gaussian density function\(^5\) centered in 0 and of standard deviation \( \delta^2 \); \( \hat{e}_i \sim \mathcal{N}(0; \delta^2) \).

The relation between \( e_i \) and \( \hat{e}_i \) will be discussed explicitly in Section 2.5.

- The forecast error vector for wind production and demand: \((e_i, v_i)^T \) is also supposed to be a Gaussian random vector. According to Sinden (2007), wind power output in the United Kingdom (UK) has a weak positive correlation to current electricity demand patterns i.e., \( \rho_{[w,v]} \sim 0.6 \). This implies that \( \Delta_i \equiv e_i - v_i \), which is the difference between renewable production and demand forecast errors, is distributed according to a Gaussian distribution function centered in 0 and of variance \( \sigma^2_{\Delta} = \sigma_e^2 + \sigma_v^2 \). The rest of the article, we will let \( \Delta_i \sim \mathcal{N}(0, \sigma^2_{\Delta}) \). \( \hat{F}_{\Delta} \) will represent the associated complementary cumulative distribution function. \( \Delta_i \) is supposed to be independent of any \( \Delta_j, \forall j \neq i \) i.e., the prediction errors made on one geographic market are independent of the ones made on the other geographic markets.

- \( s_i \) (resp. \( s_{i}^0 \)) market i supply of conventional energy in day-ahead (resp. real-time) markets.

- \( c_i^f (s_i^0) = a_i + b_i \hat{s}_i^f \) (resp. \( c_i^f (s_i^0) = a_i^0 + b_i^0 \hat{s}_i^f \)) the marginal cost function of conventional energy produced by market \( i \) and purchased at \( t_f \) (resp. \( t_0 \)), with \( a_i^0 > a_i > 0 \) and \( b_i^0 > b_i > 0 \) guaranteeing that the marginal cost on the real-time market remains larger than on the day-ahead market.

\[^4\] We do not consider demand side management in the present article. Models dealing with decentralized demand response integration through distributed learning approaches can be found in Le Cadre and Bedo (2014). In Le Cadre and Mercier (2012), the end user’s demand is price responsive and storage is possible either at the end users’ level, or at the microgrid aggregator level.

\[^5\] Other density functions might be considered without adding any changes in the derived theoretical results except in the numerical illustrations where the Gaussian assumption greatly simplified the computations.

\[^6\] \( \mathbf{t} \) stands for the transpose of the vector.

\[^7\] Data analysis ran on 66 onshore weather recording sites for the period 1970–2003 in the UK showed a correlation of 0.28 (Sinden, 2007). This is the value that we will use in the simulations.

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than in the day-ahead market for a given output level. Indeed, balancing is often supplied from machines that are reserved on a long-term basis, and these are the older machines. The two supply curves are then different because they come from different machines. If so, the current formulation that separates day-ahead and real-time machines, the latter having a higher marginal cost than the former, may be seen as a simplification made to preserve analytical tractability but is also in line with EU type systems where there are separated fleets of capacity for balancing and for the day-ahead market. Balancing capacity is contracted one year in advance (ELIA, 2014a; 2014b; 2014c; Potential Cross-Border Balancing Cooperation between the Belgian, 2014). In addition, since the balancing market is pay-as-bid there is anyway no way of empirically verifying or refraining whether strong links between day-ahead market and balancing system exist. Our modeling approach is an alternative that facilitates the development of our analysis and approximates non-convexities associated with real-time operations such as the wear and tear of machinery in emergency operation (emergency startup costs, wear due to emergency ramping, etc.).

- \( q_i^f \) (resp. \( q_i^0 \)) market \( i \) demand of conventional energy in day-ahead (resp. real-time) markets.

The amounts of conventional energy purchased by market \( i \) at \( t_f \) and at \( t_0 \) are defined as follows: \( q_i^f = (d_i - \hat{w}_i + r_i) \) and \( q_i^0 = (d_i - w_i^0 - q_i^f) \), where \( r_i \) is a position in the day-ahead (lower cost) market. This day-ahead position is determined by the energy supplier in market \( i \) for the consumers’ demand \( d_i \) to be satisfied at \( t_0 \) at the lowest possible cost; taking into account the uncertainty of supply at \( t_0 \). Market \( i \) knows \( d_i \) and \( w_i \). Hence it is equivalent for the supplier to determine \( q_i^f \) or \( r_i \). The hypothesis that \( q_i^f > 0 \) holds as long as the demand exceeds the average wind capacity.\(^8\) In the rest of the article, we will assume that: \( q_i^f = d_i - \hat{w}_i + r_i \). The over-supply of conventional production in real time is defined as: \( q_i^0 = (\Delta_i - r_i) \). Depending on the market mechanism, excess wind supply can be:

- Curtailed and compensated based on a feed-in-tariff. However, feed-in-tariffs are highly controversial and many countries wish that renewable energy generators directly participate to the market (Percebois, 2014; Rössner, 2014). We focus on this mechanism in this article.
- Bought by hydro-power generators and stored in their Energy Transfer by Pumping Stations. Hydro-power generators will inject extra supply later into the system in case of peak demand. This mechanism would require to introduce dynamicity in the agents’ decisions and explicitly model the storage capabilities of the hydro-power generators. It is therefore out of the scope of this article.
- Re-injected into the system. If the ramp down of conventional generators (especially, gas and coal) is not committed in the day ahead (the startup costs being so high that the generators prefer producing power in excess than turning off their units), negative prices might appear. This mechanism is also out of the scope of this article because it would require to explicitly model the unit-commitment and the start-up cost of the conventional generators.

2.4. Suppliers’ expected cost and generators’ expected profits

We let \( p^f \) be the day-ahead price, \( p_i^0 \) be market \( i \) real-time price and \( p_i^0 < p_i^f \); the compensation price in case of excess wind. We define \( U_i \), as the expected cost at \( t_f \), that the supplier has to pay for its end user energy consumption. The supplier is penalized paying the real-time price per missing unit and is compensated on the basis of \( p_i^f \) per wind unit in excess (Pritchard et al., 2010):

\[
U_i = q_i^f p^f + \mathbb{E}[q_i^0 p_i^0 - q_i^0 p_i^f] 
\]

We let \( \Pi_i \) be the expected profit at \( t_f \) of market \( i \) conventional energy generator. It is defined as the difference between the price paid by all the markets for the purchase of conventional energy and the cost of the energy. We assume that all the supply is sold at each time. Then:

\[
\Pi_i = s_i^f p^f - \int_0^{t_f} c_i^f(s) ds + \mathbb{E} \left[ \sum_{j \in m}(\lambda_i^0(q_{ij}^0) + p_j^0) \right] + \mathbb{E} \left[ \int_0^{t_f} c_i^0(s) ds \right]
\]

2.5. Renewable energy modeling

The renewable wind energy production of market \( i \) is a function of the number of wind farms and of their concentration which is characterized by their spatial distribution over market \( i \) geographic area. To determine the renewable energy procurement for market \( i \), we use the model of Nair et al. (2014). For market \( i \), we introduce:

- \( \alpha_i \), the average wind production of a single wind farm over the geographic area of market \( i \).
- \( \gamma_i \), the number of wind farms over market \( i \) geographic area.
- \( \theta_i \) in \( \{1/2; 1\} \) (resp. \( 1/2; 1\) [resp. \( 1-\theta_i \in \{0; 1/2\} \) ), a constant capturing the concentration (resp. the scattering) of the wind farm locations over market \( i \) geographic area. The more (resp. the less) concentration, the more (resp. the less) correlation there is between the wind farm productions.

We suppose that, at \( t_f \), \( \alpha_i \) is the best forecast of wind energy procurement of a wind farm (Nair et al., 2014). Then: \( w_i(y_i) = \alpha_i \). The forecast error will depend on the wind farm concentration too, and we choose the coefficient \( \theta_i \) so that \( \epsilon_i(y_i) = \gamma_i \hat{\epsilon}_i \) where \( \hat{\epsilon}_i \) is already introduced, represents the forecast error for the production of a single wind farm. We propose the following interpretation for the scaling of \( \theta_i \); if the wind farms are co-located they will all produce the same quantity of energy at the same time i.e., their productions are strongly correlated. This is the case when \( \theta_i = 1 \). This implies in turn that: \( \epsilon_i = \gamma_i \hat{\epsilon}_i \) and that: \( \hat{\epsilon}_i = \epsilon_i - \gamma_i \hat{\epsilon}_i \). On the contrary, if they are spatially distributed so that their productions are independent from one another i.e., uncorrelated, and under the assumption that the forecast errors are distributed according to Gaussian distribution functions, the Central Limit Theorem tells us that: \( \sigma_i^f = \sqrt{\theta_i \sigma_i^0} \) (Nair et al., 2014). Therefore, the wind farm productions are independent from one another if, and only if, \( \theta_i = 1/2 \). That is the worst forecast error distribution functions, it can be interpreted as an approximation for \( \gamma_i \) large enough. Finally, in case where \( \theta_i = 1/2; 1 \), the wind farms are randomly located over the market geographic area and their spatial distribution is intermediate between perfect independence and co-location. With these notations, we obtain:

\[
w_i(y_i) = \hat{w}_i(y_i) - \epsilon_i(y_i) = \alpha_i \gamma_i - \gamma_i \hat{\epsilon}_i = \gamma_i \hat{\epsilon}_i
\]

In the rest of the article, for the sake of simplicity, the dependence of \( w_i, \sigma_i^f \) and \( \epsilon_i \) on \( y_i \) will be omitted.

In the following sections, we solve the Signaling Game described in Section 2.2 for two Market Designs.

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\(^8\) It is sufficient to impose an upper-bound on the degree of wind penetration that we consider i.e., \( \Delta_i > \gamma_i \) to ensure that this inequality holds.
3. MD 1: two tiered with exogenous prices

In this first Market Design, the prices are supposed exogenous and such that:

\[ 0 < p^l < p^u, \quad \forall i = 1, \ldots, N \]

Going backward, we start with Step (ii) in the bilevel game described in Section 2.2. Derivating market \( i \) generator’s expected profit with respect to \( \lambda_{i,j}^0 \) and assuming that \( \lambda_{i,j}^0 > 0 \), we obtain:

\[ \frac{\partial p_i}{\partial \lambda_{i,j}^0} = -p^l + q^l_i + b_i^l (\delta_i - \bar{w}_i + r_i + \sum_{k \neq i} \kappa_{i,k} (\Delta_i - \lambda_i) + p_i^0 - q_k^0 - p_i^0 \lambda_j \sigma_k \Delta_i - \lambda_i + \lambda_j) \]

We will now prove that market \( i \) generator’s optimal trades with the other markets is either zero or is reached at the saturation of the equivalent capacities.

Reasoning by contradiction, if there exists a \( \lambda_{i,j}^0 > 0 \) such that \( \frac{\partial p_i}{\partial \lambda_{i,j}^0} = 0 \), this would mean that \( p_i^0 \) can be expressed exclusively as a function of market \( i \) parameters. Suppose that there exists another market \( k \neq j \) and \( k \neq i \) such that \( \lambda_{i,k}^0 > 0 \), then necessarily, \( p_i^0 \) and \( p_j^0 \) should coincide. But, by assumption, real-time prices in different geographic demand markets do not coincide. Therefore, if there are more than two geographic demand markets, the maximum of \( \Pi_i \) is reached at one boundary i.e., either \( \lambda_{i,j}^0 = 0 \) or \( \lambda_{i,j}^0 = \kappa_{i,j} \).

We now turn to Step (i) in the bilevel game described in Section 2.2. Market \( i \) supplier’s expected cost takes the form:

\[ U_i = q_i^l p^l + \mathbb{E} \left[ q_i^l p_i^0 - q_i^l p^l \right] \]

\[ = (\bar{d}_i - \bar{w}_i + r_i) p^l + p_i^0 \mathbb{E} \left[ (\Delta_i - r_i) \right] - p_i^0 \mathbb{E} \left[ (\Delta_i - r_i) \right] \]

We solve Step (i) of the optimization program described in Section 2.2. In Step (i), each market \( i \) supplier determines independently and simultaneously the quantity of energy to purchase, \( q_i^l \), or, equivalently, its day-ahead position, \( r_i \), so as to minimize its expected procurement cost:

\[ \min_{r_i} U_i \]

The decision variable \( r_i \) is in fact defined on a smaller space than the real coordinate space. Indeed, we need to have \( r_i \geq 0 \) since otherwise this means that a supplier could be short in the day ahead, something that one may find unrealistic given that conventional plants are more expensive in real time.

Derivating market \( i \) expected cost with respect to \( r_i \), we obtain:

\[ \frac{\partial U_i}{\partial r_i} \]

\[ = 0 \Leftrightarrow r_i = \frac{p_i^l - \bar{p}_i^l}{p_i^0 - \bar{p}_i^l} \]

Since \( \frac{\partial U_i}{\partial r_i} > 0 \) for all \( r_i \) that coincides with a minimum for \( U_i \). Furthermore, market \( i \)'s optimal position being independent of the other markets' optimal positions, Eq. (4) leads to a unique Nash Equilibrium for the market positions.

4. MD 2: two tiered with endogenous prices

In the following subsections, we derive analytically the endogenous prices in the day-ahead and in the real-time markets.

The global day-ahead market is characterized by the equilibrium between the supply and the demand: \( q_{tot}(N) = \sum_{i=1}^{N} q_i^l = \sum_{i=1}^{N} \Delta_i N \) which is the global quantity of conventional energy exchanged on day-ahead markets. Furthermore, for any market \( i \), we suppose, at \( t_0 \), that the difference between the supply and the demand for conventional energy in market \( i \) coincides with the sum of bilateral trades with the other markets:

\[ \sum_{j=1, \ldots, N, j \neq i} \lambda_{i,j}^0 = q_i^l - q_i^0 \Leftrightarrow \Delta_i = q_i^l - q_i^0 \quad (5) \]

We make the assumption that the prices \( p^l \) and \( p^u \) paid by market \( i \) suppliers for the energy purchased at \( t_f \) and \( t_0 \) respectively equal the marginal costs. This assumption is justified in Section 2.3 by the decoupling of the cost functions at the day-ahead and real-time markets and the assumption of perfect competition which holds within each geographic demand market. In two-stage market models, represented by stochastic programming, such an assumption does not hold [Pritchard et al., 2010]. There are two major reasons why we do not work with a two stage stochastic program at the generation side. One is the analytical character of our work. Our model assumes suppliers that obey a two-stage optimization paradigm with market power. Adding generators that also solve a two-stage optimization problem and finding the equilibrium between the suppliers and these generators is not amenable to an analytical solution. But, we also believe that this additional technical complexity would bring little in terms of realism in the EU context where the day-ahead market and the balancing are quite different systems that can in no way be modeled or approximated by two-stage stochastic program. The 2013 Survey on Ancillary Services Procurement and Electricity Balancing Market Design by ENTSO-E [Survey on Ancillary Services Procurement & Balancing Market Design, 2014] shows that there is no relation in EU mind between what takes place in the energy market and in balancing. The maps contained in the Survey on Ancillary Services Procurement and Balancing Market Design also show that quite different systems can prevail in neighboring countries that are part of the Market Coupling, with the consequence that it is simply unrealistic to formulate the coupling of the day ahead and real time as a stochastic program. This separation is in fact our justification for our modeling of the system as two separate markets. It implies that: \( p_i^0 = c_f^l (s_i) \) and \( p_i^0 = c_f^u (s_i) \). Furthermore, we assume that a clearing price is reached at \( t_f \) i.e., \( p_i^l = p_i^0 = p_i^0 \), \( \forall i, j = 1, \ldots, N, i \neq j \) meaning that all the markets are integrated in a single one at that time. Because the transfers are limited by the available transmission capacities, it will be harder to align the market prices at \( t_0 \) for the markets clear then \( p_i^0 = p_i^0 \neq p_i^0 \), \( \forall i, j = 1, \ldots, N, i \neq j \); otherwise there exists at least one market \( i \in \{1, \ldots, N\} \) in which the supplier pays \( p_i^0 \neq p_i^0 \) for \( j \in \{1, \ldots, N\} \) and \( j \neq i \). This raises a combinatorial problem since all the possible combinations of split/coupled markets should be included in the model. But, the number of countries involved in the Central Western Europe area is currently limited to 6 (Netherlands, Belgium, France, Luxembourg, Germany and Italy since February 2015). Therefore, the number of combinations of split/coupled markets remains reasonable.

4.1. Derivation of the coupling price

We set: \( A^f = \sum_{i=1}^{N} n_i^f q_i^f > 0 \) and \( B^f = \sum_{i=1}^{N} \frac{1}{b_i^f} > 0 \).

**Lemma 1.** The coupling price for the day-ahead market is: \( p_i^f = \sum_{i=1}^{N} \frac{n_i^f}{b_i^f} A^f \).

**Proof of Lemma 1.** Using the assumption of the supply and demand equilibrium guaranteed by the day-ahead market rules, we have:

\[ q_i^f(N) = \sum_{i=1}^{N} q_i^f = \sum_{i=1}^{N} s_i^f = \sum_{i=1}^{N} \frac{p_i^f - q_i^0}{b_i^f} \]

under the assumption that \( p_i^f = c_f^l \).

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We infer from the following equations the day-ahead price on the coupling zone.

4.2. Derivation of the split market prices

We set: $A_i^0 \triangleq \frac{B_i^0}{B_i}$, $B_i^0 > 0$ and $B_i^0 \triangleq \frac{1}{B_i^0} > 0$. As in the proof of Lemma 1, we infer the real-time price on the ith split market.

Lemma 2. The N markets being split in N geographic areas, at time $t_0$, market i price for the real-time market is: $p_i^r = \frac{\sum_{j=1}^{n} \frac{1}{B_i^j} q_{ij}^r + \sum_{i=1}^{n} A_i^0 \sum_{j=1}^{n} \frac{q_{ij}^r}{B_i^j}}{B_i}$

Proof of Lemma 2. Using the real-time market rules defined through Eq. (5), we have: $q_i^r = \sum_{j=1}^{n} q_{ij}^r + \sum_{i=1}^{n} \frac{q_{ij}^r}{B_i}$. This implies that: $q_i^r = \sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}$. Then: $p_i^r = q_i^r \frac{\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}}{B_i}$ by definition of $A_i^0$.

Day-ahead and real-time prices being now determined analytically, we let $r_{-i}^j$ be a $N - 1$ dimensional vector containing the day-ahead positions of all the suppliers except market i supplier.

Proposition 3. The sum of the bilateral trades of market i in the real-time market, $\Lambda_i$, can be expressed as a linear function in $r_i$ and $r_{-i}$.

Proof of Proposition 3. By definition: $q_i^r = -\sum_{j=1}^{n} q_{ij}^r = -\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}$ and $p_i^r = \frac{q_i^r}{B_i}$. This implies, in turn, that: $\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i} = q_i^r - q_i^r = q_i^r - \frac{q_i^r}{B_i} = q_i^r - \frac{\sum_{j=1}^{n} q_{ij}^r}{B_i}$ using the definition of $q_i^r$ obtained in Lemma 1. Then, under congestion anticipation: $\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i} = \sum_{j=1}^{n} k_{ij} - q_i^r = \sum_{j=1}^{n} k_{ij} - \frac{q_i^r}{B_i} = (1 - \frac{1}{B_i}) \sum_{j=1}^{n} \frac{q_{ij}^r}{B_i} = \sum_{j=1}^{n} (\frac{\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}}{B_i} - \frac{1}{B_i} \frac{q_i^r}{B_i})$.

This means that there exists a linear function $\lambda_{i,j} \in \mathbb{R}$ such that $\Lambda_i = \lambda_{i,j} r_i r_{-i}$.

As a corollary of Proposition 3, we obtain: $\frac{\lambda_{i,j}}{B_i} = \frac{\partial}{\partial r_i} (\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}) = \frac{\partial}{\partial r_i} (\frac{\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}}{B_i}) = \frac{1}{B_i} \frac{\sum_{j=1}^{n} \frac{q_{ij}^r}{B_i}}{B_i} - \frac{1}{B_i} \frac{q_i^r}{B_i}$.

4.3. Solving the Signaling Game

The Signaling Game assumes that the supplier substitutes the day-ahead and real-time prices with linear functions of $r_i$ and $r_{-i}$ (this was proved in Proposition 3), and optimizes with respect to $r_{-i}$ given $r_{i}$. Day-ahead positions and bilateral trades cannot be derived analytically since, as we will see, it is intractable to inverse the complementary cumulative distribution function $F_{k_{ij}}$ in the fixed point equation defining the Nash Equilibrium in the day-ahead positions. However, we provide conditions guaranteeing the existence and uniqueness of a Nash Equilibrium for the day-ahead positions and detail algorithmically how day-ahead positions and bilateral trades at the optimum should be computed.

4.3.1. Optimization of the bilateral trades

Assuming that the agents correctly anticipate the congestion of the lines, we observe that the supply of conventional energy on day-ahead and real-time markets can be expressed as functions of the bilateral trades in real time: $s_i = \sum_{j=1}^{n} p_{ij} + (\sum_{j=1}^{n} \sum_{k=1}^{n} k_{ij,k} - \sum_{j=1}^{n} \sum_{k=1}^{n} k_{ij,k} + \sum_{j=1}^{n} \sum_{k=1}^{n} k_{ij,k} - \sum_{j=1}^{n} \sum_{k=1}^{n} k_{ij,k})$ + $\frac{1}{B_i} \sum_{k=1}^{n} \frac{q_{ij}^r}{B_i}$.

Proposition 4. The optimal bilateral trade between market $i$ and market $j$ can be expressed as a linear function in $r_i$, $r_{-j}$. $\sum_{i=1}^{n} \frac{q_{ij}^r}{B_i}$. Furthermore, since $\sum_{i=1}^{n} \frac{q_{ij}^r}{B_i} = \sum_{i=1}^{n} \frac{q_{ij}^r}{B_i}$.

Proof of Proposition 4. Substituting $s_i$ and $s_j$ described above in the conventional energy generators’ expected profit given in Eq. (2), we obtain:

$$\Pi_i = \left( \sum_{j=1}^{n} \left( \frac{1}{B_i} \left( \sum_{j=1}^{n} (\frac{1}{B_i} \left( \sum_{j=1}^{n} \frac{q_{ij}^r}{B_i} + \frac{1}{B_i} \frac{q_i^r}{B_i} \right) - \frac{1}{B_i} \frac{q_i^r}{B_i} \right) \right) \right) + \sum_{j=1}^{n} \left( \frac{1}{B_i} \left( \sum_{j=1}^{n} \frac{q_{ij}^r}{B_i} + \frac{1}{B_i} \frac{q_i^r}{B_i} \right) \right) \right) \right)$$

We observe that the first line of the above equation vanishes since, by assumption, $p_i^r = \frac{1}{B_i} \frac{q_i^r}{B_i}$. Furthermore, since $\sum_{i=1}^{n} \frac{q_{ij}^r}{B_i} = \sum_{i=1}^{n} \frac{q_{ij}^r}{B_i}$ and after a few simplifications, we obtain: $\frac{\partial \Pi_i}{\partial \lambda_{i,j}} = \frac{1}{B_i} \frac{q_i^r}{B_i}$.
This means that $\lambda^0_{i,j}$ can be expressed exclusively as a linear function of $\Delta_i$, $r_i$, $\mathbb{E}(\Delta_k - r_k)_{+}$, $B_i$, and $\mathbb{E}(\Delta_i - r_i)_{+}$. But, we proved in Proposition 3 that there exists a linear function $\psi_i : \mathbb{R}^N \to \mathbb{R}$ such that $\lambda_i = \psi_i(r_i, \mathcal{A}_i)$. As a result, the optimum $\lambda^0_{i,j}$ can be expressed as a linear function in $r_i$, $\mathcal{A}_i$, $\mathbb{E}(\Delta_i - r_i)_{+}$, $B_i$, and $\mathbb{E}(\Delta_i - r_i)_{+}$.

\[ \lambda^0_{i,j} = B_i \left( \sum_{k} B_k c_i^k (\mathcal{A}_k) \right) \left( \sum_{k} B_k c_i^k (\mathcal{A}_k) \right) + \mathbb{E}(\Delta_i - r_i)_{+} - \frac{d_i\partial_{\mathcal{A}_i}}{B_i} - \frac{d_i\partial_{\mathcal{A}_i}}{B_i} + \frac{1}{B_i} \left( \frac{1}{B_i} \sum_{k} (d_k - \lambda^0_{k,j}) + \frac{1}{B_i} (\lambda^0_{i,j})_{+} \right) \right) \]
\(\alpha_i(c)\) are supposed constant and such that at each time period \(t\):
\[d_i(c, t) = \hat{d}_i(c) - \nu_i(c, t)\]
\[w_i(c, t) = \hat{w}_i(c) - \epsilon_i(c, t)\] (8) (9)

We assume that the geographic market \(i\) clusters are defined so that there is no correlation among the cluster forecast errors because correlation among wind farms decreases with the distance (Girard et al., 2013) but that, inside each cluster, there remains a positive correlation between the wind production and the demand forecast error. Making the parallel with the previous notations, we have: \(d_i(t) = \sum_{c \in C_i} d_i(c, t)\) and \(\alpha_i = \sum_{c \in C_i} \alpha_i(c)\). Clustering may be performed through one of the automatic partitioning algorithms used in Machine Learning (Kogan, 2007), as illustrated for Germany in Section 6.

We let \(\Delta_i(c, t)\) be the difference between the wind power supply and the demand forecast errors in market \(i\) cluster \(c\), at time period \(t\). Transposing Section 2.3 assumptions to a finer scale (i.e., clusters instead of geographic markets), we assume that \((\Delta_i(c, t))_{c \in C_i}\) is distributed according to a \([c_i]\)-dimensional Gaussian density function centered in the zero \([c_i]\)-dimensional vector and with a diagonal variance–covariance matrix having on its principal diagonal all the variances \((\sigma_i(c))^2\), \(\forall c \in C_i\) and zeros everywhere else since the covariance between any \(\Delta_i(c, t), \Delta_j(c', t)\), \(\forall c, c' \in C_i\), \(c \neq c'\) vanishes. Furthermore, over each cluster \(c \in C_i\), the wind production and the demand forecast errors are correlated due to the assumption that \((\tilde{\epsilon}_i(c, t), \nu_i(c, t))^T\) is a Gaussian random vector centered in \((\mu_i, \sigma_i(c))^T\) and of variance–covariance matrix:

\[
\left( \frac{(\tilde{\alpha}_i(c))^2}{E[\tilde{\epsilon}_i(c, t)\nu_i(c, t)]} \right) \begin{pmatrix} \sigma_i(c)^2 & \text{E}[\tilde{\epsilon}_i(c, t)\nu_i(c, t)] \\ \text{E}[\tilde{\epsilon}_i(c, t)\nu_i(c, t)] & \sigma_i(c)^2 \end{pmatrix}
\]

Modern Portfolio Theory is an alternative to the traditional method of analyzing each investment’s individual merits. When investors look at each investment’s individual merits, they are analyzing one investment without worrying about the way the different investments will perform relative to each other. On the other hand, Modern Portfolio Theory places a large emphasis on the correlation between the investments. Markowitz defines as efficient the portfolios which are characterized by a maximum expected revenue for a fixed risk (or, equivalently, for a minimum risk for a fixed expected revenue) (Markowitz, 1952). Risk and volatility are treated as the same thing: Markowitz uses risk as a measurement of the likelihood that an investment still goes up and down in value, and how often and by how much. The theory assumes that investors prefer to minimize risk. The Efficient Frontier, also called the Markowitz Frontier (MF), is then defined as the set of all the portfolios which are efficient. In this article, the supplier applies Modern Portfolio Theory to determine the wind farm portfolio that minimizes its expected investment while minimizing its risk conditionally to the occurrence of rare events (Marling & Emanuelsson, 2012) caused by discrepancies between forecasts and realizations. Note that since wind generator directly participates to the market and excess wind is curtailed, only under-supply is considered as a rare event.

In the numerical illustrations we will test two assumptions on the construction cost function for the wind farm portfolio on the cluster \(c \in C_i\); either it is linear in the number of wind farms: \(\Xi(\gamma_i(c)) = \xi \gamma_i(c)\) or it is quadratic: \(\Xi(\gamma_i(c)) = \xi \gamma_i(c)^2\) with \(\xi > 0\) representing the construction cost for a single wind farm. The supplier applies Modern Portfolio Theory to minimize risk. The Efficient Frontier, also called the Markowitz Frontier (Mathieu, 2013), is characterized by a maximum expected revenue for a fixed risk (or, equivalently, for a minimum risk for a fixed expected revenue) (Markowitz, 1952). Risk and volatility are treated as the same thing: Markowitz defines as efficient the portfolios which are characterized by a maximum expected revenue for a fixed risk (or, equivalently, for a minimum risk for a fixed expected revenue) (Markowitz, 1952). Risk and volatility are treated as the same thing: Markowitz uses risk as a measurement of the likelihood that an investment still goes up and down in value, and how often and by how much. The theory assumes that investors prefer to minimize risk. The Efficient Frontier, also called the Markowitz Frontier (MF), is then defined as the set of all the portfolios which are efficient. In this article, the supplier applies Modern Portfolio Theory to determine the wind farm portfolio that minimizes its expected investment while minimizing its risk conditionally to the occurrence of rare events (Marling & Emanuelsson, 2012) caused by discrepancies between forecasts and realizations. Note that since wind generator directly participates to the market and excess wind is curtailed, only under-supply is considered as a rare event.

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equals zero, variance is a quadratic operator) and the fact that there is no dependence between two time consecutive forecast error differences, we infer that:

$$\text{Var}(\mathcal{Z}(i, T) | \Delta t(t) \geq r_t(t)) = \sum_{c \in C} \sigma^2_{\Delta t}(c) \sum_{t} p^0_t(t)^2$$

$$= \sum_{c \in C} \left( (\sigma_t^2(c))^2 \gamma_c(t) \gamma_t(c)^{2\beta(c)} - 2 \gamma_c(t) \gamma_t(c)^{\beta(c)} \mathbb{E}[\tilde{\xi}_t(c) v_t(c)] + \sigma_t^2(c)^2 \right) \sum_{t} p^0_t(t)^2$$

using Eq. (10). In the rest of the article, we let:

$$\text{RVar}(i, T) \triangleq \text{Var}(\mathcal{Z}(i, T) | \Delta t(t) \geq r_t(t))$$

be the risk of market $i$ supplier.

The expectation of the supplier’s investment is:

$$\mathbb{E}[\mathcal{Z}(i, T)] = \sum_{t} \left( \sum_{c \in C} \tilde{d}_c(t) + \tilde{F}_t^{-1} \left( \frac{p^0_t(t)}{p^1_t(t)} \right) \right) p^1_t(t)$$

$$- \sum_{t} \left( \sum_{c \in C} \alpha_t(c) \gamma_t(c) \right) p^0_t(t)$$

$$+ \sum_{t} p^0_t(t) \mathbb{E} \left[ \left( \Delta_t(t) - \tilde{F}_t^{-1} \left( \frac{p^1_t(t)}{p^0_t(t)} \right) \right) \right]$$

$$+ \sum_{c \in C} \mathbb{E} \left( \gamma_t(c) \right)$$

Derivating $\text{RVar}(i, T)$ with respect to $\theta_t(c)$ and solving

$$\frac{\partial \text{RVar}(i, T)}{\partial \theta_t(c)} = 0,$$

we obtain:

$$\theta_t(c) = \frac{\ln \left( \mathbb{E}[\tilde{\xi}_t(c) v_t(c)] / \mathbb{E}[\gamma_t(c)] \right)}{\ln \gamma_t(c)}.$$ 

But $\frac{1}{2} \leq \theta_t(c) \leq 1$ which is equivalent to

$$\frac{\mathbb{E}[\tilde{\xi}_t(c) v_t(c)]}{\mathbb{E}[\gamma_t(c)]} \leq \gamma_t(c) \leq \left( \frac{\mathbb{E}[\tilde{\xi}_t(c) v_t(c)]}{\mathbb{E}[\gamma_t(c)]} \right)^2.$$ 

Using the fact that $\frac{\mathbb{E}[\tilde{\xi}_t(c) v_t(c)]}{\mathbb{E}[\gamma_t(c)]} \leq \gamma_t(c)$, we check that $\theta_t(c)$ coincides with a minimum for $\text{RVar}(i, T)$ indeed:

$$\frac{d^2 \text{RVar}(i, T)}{d \theta_t(c)^2} = 2 \gamma_t(c)^{\beta(c)} \ln \gamma_t(c)^2 \left[ 2 \gamma_t(c)^{\beta(c)} \gamma_t(c) - \mathbb{E}[\tilde{\xi}_t(c) v_t(c)] \right] > 0.$$ 

**MD 2:** In Eqs. (8) and (9), $\tilde{d}_t(c)$ and $\tilde{w}_t(c)$ do not depend on time. This implies that in the algorithms described in Section 4.3, the optimal positions are also dependent on time; contrary to MD 1, where the optimal positions are functions of the day-ahead and real-time price ratio, which is time dependent. Using this observation, we infer that the day-ahead price is time independent; indeed:

$$p^1_t = \sum_j \left( d_j - \tilde{w}_j + r_j \right) + A^t / L.$$ 

In case of endogenous prices, market $i$ supplier investment takes the following form:

$$\mathcal{I}(i, T) = \sum_t \left( \sum_{c \in C} \left( \tilde{d}_c(t) - \tilde{w}_c(t) \right) + r_t \right) p^1_t(t)$$

$$+ \sum_t \left( \sum_{c \in C} \left( \mathbb{E}[\tilde{\xi}_t(c) v_t(c)] - r_t \right) - r_t \right) p^0_t(t)$$

$$+ \sum_{c \in C} \mathbb{E} \left( \gamma_t(c) \right).$$

(11)

Compared to MD 1, we observe one additional difficulty: at the optimum $r_t$ relies on $A^t$ and the bilateral trades rely on the decisions of the other markets.

Market $i$ supplier’s risk is similar to the one derived for MD 1:

$$\text{RVar}(i, T) = \text{Var}(\mathcal{I}(i, T) | \Delta t(t) \geq r_t(t)) = \sigma^2_{\Delta t} \sum_{t} p^0_t(t)^2.$$ 

Therefore, the same result holds about the optimal wind farm concentration. In MD 1 and MD 2, the MF is completely described by the set of equations summarized in the statement of Theorem 7. \Box

5.2. In the $(\mathbb{E}[\mathcal{Z}(i, T)], \text{RVar}(i, T))$ plane

The representation of the MF in the $(\mathbb{E}[\mathcal{Z}(i, T)], \text{RVar}(i, T))$ plane is not straightforward. Indeed, in case of endogenous prices, the optimal number of wind farms to construct in each cluster cannot be computed analytically. The optimal positions and the optimal bilateral trades can only be obtained algorithmically, as explained in Section 4.3.

In each cluster $c \in C$, the supplier can construct a fixed number of wind farms $\gamma_t(c)$. For each combination of $(\gamma_t(c))_{i,c}$, we derive the optimal wind farm portfolio concentration $(\theta_t(c))_{i,c}$ using Theorem 7. We substitute the resulting $(\gamma_t(c), \theta_t(c))_{i,c}$ in $(\sigma_{\Delta t})$, as derived in Eq. (10) and in the Nash Equilibrium in the positions and bilateral trades $(r_t^*, A_t^*)$, obtained through the algorithm detailed in Section 4.3.

We substitute $(r_t^*, A_t^*)$ and $(\Delta t(t))$, in the real-time price:

$$p^0_t(t) = \frac{\mathbb{E}[(\Delta_t(t) - r_t^*)^2] + \mathbb{E}[(\Delta_t(t) - r_t^*)]}{\mathbb{E}[(\Delta_t(t) - r_t^*)^2] + \mathbb{E}[(\Delta_t(t) - r_t^*)]}.$$

From Eq. (12). From this, we can infer the supplier’s risk using Eq. (12).

At the same time, we note that the expectation of market $i$ supplier investment can be simplified to give:

$$\mathbb{E}[\mathcal{I}(i, T)] = \frac{T}{B^t} \left( \tilde{d}_t - \tilde{w}_t + r_t \right) \left( \sum_j \left( d_j - \tilde{w}_j + r_j + A^t / L \right) \right)$$

where, as detailed in Appendix A:

$$\mathbb{E}[(\Delta_t(t) - r_t^*)] = \mathbb{E}[(\Delta_t(t) - r_t^*)]$$

and

$$\mathbb{E}[(\Delta_t(t) - r_t^*)] = \mathbb{E}[(\Delta_t(t) - r_t^*)]$$

where $\gamma = \mathbb{E}[\tilde{\xi}_t(c) v_t(c)] / \mathbb{E}[\gamma_t(c)]$, $\Gamma(1/2, r_t^*)$ is the incomplete Gamma function evaluated at $a \in \mathbb{R}_+$ and $\Gamma(1/2, r_t^*) = 1 / 2 \sqrt{\pi} \Gamma(1/2, r_t^*)$.

6. Numerical illustrations for three geographic demand markets: France, Germany and Belgium

In the numerical illustrations, we consider three geographic demand markets: France, Germany and Belgium. Wind farm portfolio optimization is restricted to the French area, since our energy consumption data focus on this country.

The marginal cost parameters are based on Chao and Peck’s six node toy network (Chao & Peck, 1998): for France (Fr) we take $a_{Fr} = 42.5, b_{Fr} = 4250, b_{Fr} = 0.025$; for Germany (Ge) we take $a_{Ge} = 15, b_{Ge} = 1500, b_{Ge} = 0.05$ and for Belgium (Be) $a_{Be} = 10, b_{Be} = 1000, b_{Be} = 0.05$.

The equivalent interconnector capacities are set so that $K_{Be, Ge} = 2$ (gigawatt), $K_{Be, Fr} = 6$ (gigawatt) and $K_{Ge, Fr} = 5$ (gigawatt).

6.1. Description of the data and clustering of the geographic demand markets

For Germany, our database is made of time series of 75 sensors located all over Germany, providing one year wind speed measures (from 03/19/2013 until 03/18/2014) with one measure per hour
We partition Germany and France’s wind speed sensors in (b) and (c) respectively; each marker symbol being associated with a specific class. The exact GPS coordinates of the sensors are depicted by circles in Fig. 2(a). We use two Machine Learning techniques to partition the sensors based on the mean and variance of their wind speed time series: firstly, \( k \)-means algorithm clusters data by separating samples in an a priori determined number of groups, minimizing a criterion known as the inertia of the groups. The optimal number of classes for the sensors (4) has been estimated a priori using an unsupervised clustering method known as affinity propagation (Scikit-Learn). Secondly, one-class Support Vector Machine (SVM) can be used as a type of unsupervised learning algorithm, for novelty detection: given a set of samples, it will detect the soft boundary of that set so as to classify new points as belonging to that set or not. Both techniques give identical (or, at least, very close) classes, which are represented in Fig. 2(a) and (b). The convex hull of these sensor classes gives an approximation to the clusters geographic area for Germany.

For France and Belgium, we use data provided by the Météociel Platform for the year 2013. Each country is clustered using one of the automatic partitioning tools, described above for Germany, applied on the historical wind speed measurements. We infer that France (resp. Belgium) can be partitioned in 8 (resp. 2) clusters, as represented in Fig. 2(c) for France. The geographic coordinates of the cluster areas can be found in Biégala and Coulondre (2011). We associate an index with each French cluster area. The wind power production at the various points in the clusters is then inferred from the wind speed measurements using the Betz limit.\(^{10}\) Finally, over each cluster, the wind power productions are averaged. The mean and standard deviations are then calculated on the basis of each cluster average time series of wind power production. We consider a constant average and standard deviation for wind power production throughout the time horizon. The numerical values of mean consumption and consumption standard deviation per French cluster are detailed in Table 1.

Estimated wind production for Germany and Belgium is fixed so that: \( \hat{w}_{Ge} = 15 \) (gigawatt) and \( \hat{w}_{Be} = 10 \) (gigawatt). The forecast error difference standard deviations are set so that: \( \sigma_{\Delta w_{Ge}} = \sigma_{\Delta w_{Be}} = 5 \) (gigawatt). For France, if 10 turbines were placed in each cluster, \( \hat{w}_{Fr} = 4.97 \) (gigawatt) and the forecast error difference standard deviation would be: \( \sigma_{\Delta w_{Fr}} = 5.79 \) (gigawatt).

For estimating the variance associated to the demand forecast, we use a database containing one year (2013) power measurements (in kW) for each French region, with a granularity of one measure per 30 minutes (French Regional Consumption Data). The French regions are then mapped to the French clusters, as defined in RTE report (Biégala & Coulondre, 2011). Then, over each cluster, we take the average of these time series to obtain one value per day. In Fig. 3, we compare the empirical distribution function of one French cluster (PACA region) energy consumption with the best fit Gaussian density

\(^{10}\) Source for Betz limit: http://www.wind-power-program.com/betz.htm.
Table 1

<table>
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<th>Clust. ind.</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conso. stand. dev.</td>
<td>5.048</td>
<td>1.196</td>
<td>0.496</td>
<td>0.654</td>
<td>2.010</td>
<td>1.032</td>
<td>0.953</td>
<td>1.874</td>
</tr>
</tbody>
</table>

Fig. 3. Energy consumption empirical distribution function (in kilowatt hour) compared to the best fit Gaussian density function for PACA region.

Fig. 4. In (a) (resp. (b)), we represent the optimal position $r_i$ in case of exogenous prices (resp. endogenous prices) as a function of the forecast error difference standard deviation $\sigma_{\Delta_i}$ and of the exogenous price ratio $\frac{p_0}{p_f}$ (resp. of the wind forecast $\hat{w}_i$).

Fig. 5. Over each French cluster, the Markowitz Frontier for the wind farm portfolio is represented as a function of the number of turbines and of the concentration between the wind farms over the cluster. Each of the 8 French clusters is characterized by a specific color.

6.2. Optimal forward positions and Markowitz Frontier representations for France

In Fig. 4(a), we plot market $i$ optimal position for the two tiered market with exogenous prices (MD 1) described in Section 3 as a function of the exogenous price ratio $\frac{p_0}{p_f}$ and of the forecast error standard deviation $\sigma_{\Delta_i}$. In Fig. 4(b), we plot the optimal position for the two tiered market with endogenous prices (MD 2) and positions described in Section 4 as a function of the wind forecast $\hat{w}_i$ and of the forecast error standard deviation $\sigma_{\Delta_i}$. The endogenous prices corresponding to the interval of variation of $\hat{w}_i$ and $\sigma_{\Delta_i}$ are such that $p_f$ belongs to the interval $[0; 95; 1.30]$ and $p_f$ to the interval $[3.52; 4.16]$. For the exogenous prices, we choose $p_0 = 4$ which is close to the center of the interval of variation of the real-time endogenous price. The day-ahead endogenous price then varies in the interval $[0.15; 1.35]$. The endogenous and exogenous prices are now in the same range of values. We observe that in case of exogenous prices, $\frac{p_0}{p_f}$ weakly impacts the optimal position whereas $\sigma_{\Delta_i}$ clearly makes the position increase; in case of endogenous prices, both $\hat{w}_i$ and $\sigma_{\Delta_i}$ make the position increase. Reciprocally, the increase of the position makes $\frac{p_f}{p_f}$
In (b), we plot all the portfolio combinations and the MF for $\sum c_i = 24 \times 10^3$ and $36 \times 10^3$. In (d), we plot the portfolio combinations and the MF for $\sum c_i = 1$ and a quadratic cost of construction.

The linearity of the cost functions, the inelasticity of demand and day-ahead Market Coupling are necessary assumptions in our analysis. The relaxation of these assumptions is an interesting direction of future research but will lead us to the limits of analytical tractability.

**Appendix A. Proof of Proposition 5**

By substitution of the day-ahead and real-time prices at equilibrium obtained in Lemmas 1 and 2 in market $i$ expected procurement cost, we obtain:

$$U_i = q_i \sum_j q_j^i + A_i^r$$

Using Proposition 3 and the fact that $E[q_i^r] = E[(\Delta_i - r_i)_+] = E[\Delta_i - r_i][\Delta_i \geq r_i]$ and $E[q_i^0] = E[(\Delta_i - r_i)_-] = E[(\Delta_i - r_i)][\Delta_i \leq r_i]$, we obtain:

$$\frac{\partial U_i}{\partial r_i} = \sum_j q_j^i + \frac{A_i^r}{B_i^r} + q_i^0 \frac{\partial}{\partial r_i} \left( \frac{1}{B_i^r} E[(\Delta_i - r_i)^2][\Delta_i \geq r_i] \right)$$

Since the forecast error differences $\Delta_i$ are distributed according to Gaussian distribution functions centered in 0 and of standard deviation $\sigma_{\Delta_i}$, it is possible to express the first and second derivatives of...


Using Eqs. (13), (14) and (15), we obtain:

\[
\mathbb{E}[(\Delta_i - r_i)] \mid \{\Delta_i \geq r_i\} = \int_{r_i}^{\infty} \Delta f_{\Delta_i}(\Delta) d\Delta - r_i \int_{r_i}^{\infty} f_{\Delta_i}(\Delta) d\Delta = \frac{\sigma_{\Delta_i}}{\sqrt{2\pi}} \exp\left(\frac{-r_i^2}{2\sigma_{\Delta_i}^2}\right) - r_i \hat{F}_{\Delta_i}(r_i)
\]

\[
\frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i)] \mid \{\Delta_i \geq r_i\} = -\hat{F}_{\Delta_i}(r_i)
\]

\[
\frac{\partial^2}{\partial r_i^2} \mathbb{E}[(\Delta_i - r_i)] \mid \{\Delta_i \geq r_i\} = f_{\Delta_i}(r_i)
\]

Using the same principle, we obtain:

\[\frac{\partial}{\partial r_i} \mathbb{E}[(\Delta_i - r_i)^2] \mid \{\Delta_i \geq r_i\} = 2 \hat{F}_{\Delta_i}(r_i)\]

Using Eqs. (13), (14) and (15), we obtain:

\[\frac{\partial^2 U_i}{\partial r_i^2} = \frac{2}{B} + b_i \left(3 - \frac{1}{B/b_i}\right) \hat{F}_{\Delta_i}(r_i) + \sum_{j \neq i} \lambda_{j-i} \left(\frac{r_j}{B_j} - A_i\right) f_{\Delta_i}(r_i)\]

\[= -\frac{f_{\Delta_i}(r_i)}{B_i^0} \frac{1}{\sigma_{\Delta_i}} \left[\left(3 - \frac{1}{B_i/b_i}\right)\sigma_{\Delta_i}^2 + r_i \left(A_i^0 - B_i^0 p_i^0 + \sum_{j \neq i} \lambda_{j-i}^0 r_j\right)\right]\]

using the fact that \(\frac{df_{\Delta_i}(r_i)}{dr_i} = -\frac{r_i}{\sigma_{\Delta_i}^2}\). The function \(\mathbb{E}[(\Delta_i - r_i)]\) is decreasing on \([r_i^0, +\infty]\) and increasing on \([0, r_i^0]\). Furthermore, the number of points where \(\hat{U}_i = 0\) depend on the value of \(\frac{\partial^2 U_i}{\partial r_i^2} < 0\).

1. **Case 2(a):** \(\frac{\partial^2 U_i}{\partial r_i^2} < 0\)

Then \(\frac{\partial^2 U_i}{\partial r_i^2} > 0\) for \(r_i \in \mathbb{R}\). This implies that \(U_i\) is convex on \(\mathbb{R}\). Therefore it admits a unique minimum on \([0, +\infty]\).

2. **Case 2(b):** \(\frac{\partial^2 U_i}{\partial r_i^2} > 0\)

Then two sub-cases should be distinguished depending on the sign of \(\frac{\partial^2 U_i}{\partial r_i^2} \mid_{r_i = r_i^0}\).

(a) **Case 2(b)(i):** \(\frac{\partial^2 U_i}{\partial r_i^2} < 0\)

There exists a unique \(r_i^1 \in [r_i^0, +\infty]\) such that \(\frac{\partial^2 U_i}{\partial r_i^2} < 0\) on \([0, r_i^1]\) and \(\frac{\partial^2 U_i}{\partial r_i^2} > 0\) on \([r_i^1, +\infty]\). This implies that \(r_i^1\) is decreasing on \([0, r_i^1]\) and increasing on \([r_i^1, +\infty]\). Hence \(r_i^1 = 0\) and \(r_i^1 = +\infty\) only if \(r_i^1\) is strictly increasing on \(\mathbb{R}\), reaching its minimum in \(r_i^0\) or \(r_i^0 > r_i^0\). Therefore, there exists a unique \(r_i^0\) maximizing \(U_i\).

Case 2: \(\frac{\partial^2 U_i}{\partial r_i^2} < 0\)

This second case corresponds to the case where the quantity of imports is high compared to the quantity of exports, for geographic market \(i\).

Derivating three times \(U_i\) with respect to \(r_i\), we obtain:

\[\frac{\partial^3 U_i}{\partial r_i^3} = \frac{b_i}{2} \frac{3}{B/b_i} f_{\Delta_i}(r_i) - \frac{A_i^0 - B_i^0 p_i^0}{B_i^0} + \sum_{j \neq i} \lambda_{j-i}^0 r_j f_{\Delta_i}(r_i)\]

\[= -\frac{f_{\Delta_i}(r_i)}{B_i^0} \frac{1}{\sigma_{\Delta_i}} \left[\left(3 - \frac{1}{B_i/b_i}\right)\sigma_{\Delta_i}^2 + r_i \left(A_i^0 - B_i^0 p_i^0 + \sum_{j \neq i} \lambda_{j-i}^0 r_j\right)\right]\]

We set: \(r_i^0 \equiv \left(3 - \frac{1}{B_i/b_i}\right) \sigma_{\Delta_i}^2\). Then, we note that:

- If \(r_i < r_i^0\) then \(r_i < 3 - \frac{1}{B_i/b_i}\sigma_{\Delta_i}^2 + r_i (A_i^0 - B_i^0 p_i^0 + \sum_{j \neq i} \lambda_{j-i}^0 r_j) > 0\). This implies that \(3 - \frac{1}{B_i/b_i}\sigma_{\Delta_i}^2 < 0\).

- Identically, if \(r_i > r_i^0\) then \(3 - \frac{1}{B_i/b_i}\sigma_{\Delta_i}^2 > 0\).

Both of these observations imply that \(\frac{\partial^3 U_i}{\partial r_i^3} > 0\) and \(\frac{\partial^3 U_i}{\partial r_i^3} < 0\). Furthermore, the number of points where \(\frac{\partial^3 U_i}{\partial r_i^3} = 0\) depend on the value of \(\frac{\partial^2 U_i}{\partial r_i^2} < 0\).
The case \( \frac{\partial^2 U_i}{\partial U_i^2} \bigg|_{|U_i|=0} > 0 \) should be avoided since it might give rise to a large number of equilibria \( (2^N) \) for Program 3. Therefore, in the case where \( \sum_j \lambda_{i-j}^0 < \frac{p_i^0}{B_i} - A_i^0 \), it might be reasonable to impose some conditions on the problem parameters so that Case 2(b)(ii) is avoided in other words:

\[
\frac{\partial^2 U_i}{\partial U_i^2} \bigg|_{|U_i|=0} = \frac{2}{B_i} + \frac{1}{2B_i^2} \left( 3 - \frac{1}{B_i} \right) + \sum_j \lambda_{i-j}^0 \left( \frac{p_i^0}{B_i} - A_i^0 \right) f_{\Delta_i}(0) < 0
\]

\[\iff \quad f_{\Delta_i}(0) > -\frac{2B_i}{B_i^2} + \frac{1}{2} \left( 3 - \frac{1}{B_i} \right) \sum_j \lambda_{i-j}^0 \left( \frac{p_i^0}{B_i} - A_i^0 \right) \]

\[\sigma_{\Delta_i} < -\frac{1}{\sqrt{2\pi}} \left[ \frac{\sum_j \lambda_{i-j}^0 \left( \frac{p_i^0}{B_i} - A_i^0 \right)}{\frac{1}{B_i^2} - 3} \right] \]

Therefore, to avoid Case 2(b)(ii), the standard-deviation-related deviation to the knowledge of \( \Delta_i \) should be smaller than \( \frac{1}{\sqrt{2\pi}} \left[ \frac{\sum_j \lambda_{i-j}^0 \left( \frac{p_i^0}{B_i} - A_i^0 \right)}{\frac{1}{B_i^2} - 3} \right] \).

References


