Multi-Stage Stochastic Economic Dispatch under Renewable Energy Supply Uncertainty

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Outline

• Motivation
• Stochastic dual dynamic programming
• Stochastic multi-period dispatch of pumped hydro resources
  • Renewable supply model
  • Stochastic multi-period economic dispatch
• A Case Study of Germany
Motivation
Multi-Stage Stochastic Economic Dispatch

- Economic dispatch: function of
  - re-dispatching online units
  - mobilizing fast-start units
  in intra-day and real time for purpose of load following
- Should be distinguished from day-ahead unit commitment
- Increasing sophistication of MSED:
  - Multi-period look-ahead
  - ‘Rough’ consideration of uncertainty by securing flexible ramp capacity (e.g. California ISO and Midwest ISO)
- Typical time scale: 5-minute time step, 15-minute look-ahead, solved 7.5 minutes in advance of operations
Multi-Period Decision Making Under Uncertainty

- At each stage:
  - Observe uncertainty ($\omega_t$)
  - Make a decision ($u_t$) given realized uncertainty and state of the system ($x_t$)
  - Incur cost ($c_t(x_t, u_t, \omega_t)$)
  - Step forward

- Goal: minimize expected cost over optimization horizon

- Bellman’s principle of optimality: at each stage, decide $x_t$ such that present cost plus expected cost-to-go is minimized

- Dynamic programming algorithm: solve the problem by recursively computing cost-to-go -> avoids redundancy in computations
Applying An Effective Solution ...

• Multi-stage stochastic linear programming
  • Specific class of problems for multi-period decision making under uncertainty
  • Natural formulation for various problems in power system planning / operations

• Stochastic Dual Dynamic Programming (SDDP)
  • Scalable: go-to solution for medium-term hydro-thermal planning
  • Extensive theoretical analysis (representation of uncertainty, convergence, binary decisions)
... To An Emerging Problem

- Multi-period stochastic economic dispatch
  - Limited analysis
  - Highly relevant (e.g. rooftop solar, distributed storage, pumped hydro)
  - Naturally cast as a multi-stage stochastic linear program

- Our research agenda:
  - Can dynamic programming solve MSED?
  - Is MSED an interesting problem?
Stochastic Dual Dynamic Programming
Multi-Stage Stochastic Linear Programming

\[
\min_x \sum_{t=1}^{H} \sum_{\omega[t] \in \Omega[t]} p_{t,\omega[t]} c_t^T x_{t,\omega[t]}
\]

Set of possible history of events up to stage \( t \): **Non-scalable**

\[
T_{t,\omega_t} x_{t-1,\omega[t],A(\omega[t])} + W_t x_{t,\omega[t]} = h_{t,\omega_t}, t \in T, \omega[t] \in \Omega[t]
\]

\[
x_{t,\omega[t]} \geq 0, t \in T, \omega[t] \in \Omega[t]
\]
Decomposing the Problem

Solution approach: break the overall problem by time stage $t$, and uncertainty realization $k \rightarrow$ small linear program $NLDS_{t,k}$ for each $t, k$

The value function $\tilde{V}_t(x)$ is piecewise linear affine, question is how to ‘discover’ it
Describing Uncertainty in a Lattice

• A lattice is a graphical description of **Markovian** uncertainty:
  • Nodes: realization of uncertainty \( h_t(\omega_t) \)
  • Arcs: transition probability \( \mathbb{P}[\omega_t | \omega_{t-1}] \)

• **Serial independence**: specific class of lattices where \( h_t(\omega_t) \) is distributed independently of history: \( \mathbb{P}[\omega_t | \omega_{t-1}] = \mathbb{P}[\omega_t], \forall \omega_{t-1}, \forall t \)

Serial independence \( \Rightarrow \) value functions are shared across lattice nodes
Stochastic Dual Dynamic Programming Algorithm

- **Forward pass**
  - Generates trial decisions
  - Determines **probabilistic** upper bound
  - Determines lower bound

- **Backward pass**
  - Generates optimality cuts that approximate cost-to-go $\tilde{V}_t(x)$

- **MATLAB open-source implementation:**
  [https://web.stanford.edu/~lcambier/fast/](https://web.stanford.edu/~lcambier/fast/)
  - User-defined decomposition subproblem
  - User-defined input lattice
Stochastic Multi-Period Dispatch of Pumped-Hydro Resources
Renewable Supply Model

- Multiplicative model of forecast error/renewable production ratio:
  - Capture heteroscedasticity of data
  - Capture inter-temporal dependence of forecast error
  - Compatible with SDDP format (serial independence)

\[ y_{t+1} = (c + \phi \cdot y_t) \cdot \eta_t \]
\[ p_t = RF_t \cdot y_t \]

Serial independence (\( \Rightarrow \) value functions are shared across lattice nodes)
Stochastic Multi-Period Economic Dispatch

- Objective: minimize load shedding and fuel cost
- Coupling constraint: power balance
- $l_{sn}$: load shedding
- $c_g$: fuel cost
- $D_l$: load
- $pd_g$: pumping demand
- $pp_g$: pumping production
- $p_g$: power production
- $RF_g$: renewable forecast
- $y_g$: renewable forecast / realization ratio
- $f_k$: power flow over line

$$\min \frac{1}{4}(\sum_{n \in N} (VOLL \cdot l_{sn}) + \sum_{g \in G} c_g)$$

$$\sum_{l \in L_n} D_l + \sum_{l \in L_n} pd_g + \sum_{k \in (\cdot, n)} f_k = \sum_{g \in G} p_g +$$

$$\sum_{g \in PH_n} pp_g + \sum_{g \in GR_n} RF_g y_g + l_{sn} + \sum_{k \in (\cdot, n)} f_k, n \in N$$
Stochastic Multi-Period Economic Dispatch (II): Conventional Generators

• Piecewise affine fuel cost
• Technical minimum/maximum
• Ramp rate limits
• $U_g$: Unit commitment (on/off) decision
• $A_{g,m}, B_{g,m}$: cost function parameters
• $RU_g, RD_g$: ramp up/down limit
• $c_g$: fuel cost
• $p_g$: power production

\[
c_g \geq F_g(A_{g,m}U_g + B_{g,m}p_g), g \in G, m = 1, \ldots, 3
\]

\[
PMin_g U_g \leq p_g \leq PMax_g U_g, g \in G
\]

\[
p_g - p_{g,t-1} \leq RU_g U_g + MTL_g (1 - U_{g,t-1}), g \in G
\]

\[
p_{g,t-1} - p_g \leq RD_g U_g + MTL_g (1 - U_{g,t-1}), g \in G
\]

Unit commitment is fixed to the solution of a day-ahead unit commitment model
Stochastic Multi-Period Economic Dispatch (III): Pumped Hydro Units

- Storage dynamics
- Pumped hydro consumption limits
- Pumped hydro production limits
- $s_g$: stored energy
- $\eta_{g,m}$: pumped hydro unit efficiency
- $DMax_g$: power consumption limit
- $PMax_g$: power production limit

$s_g = s_{g,t-1} + 0.25(\eta_gpd_g - pp_g), g \in PH$

$pd_g \leq DMax_g, g \in PH$

$pp_g \leq PMax_g, g \in PH$
Stochastic Multi-Period Economic Dispatch (IV): Power Flow

- Fix reference bus angle
- Linearized DC power flow
- Line flow limits
- $\theta_m$: bus angle
- $B_k$: line susceptance
- $f_k$: line power flow
- $TC_k$: line flow limit

\begin{align*}
\theta_{hub} &= 0 \\
f_k &= B_k (\theta_m - \theta_n), \quad k = (m, n) \in K \\
-TC_k \leq f_k \leq TC_k, \quad k \in K
\end{align*}
A Case Study of Germany
German System

- Two-step simulation of German market:
  - Weekly clearing of reserve + energy exchange: unit commitment model with weekly horizon (September 22-28, 2014)
  - Real-time balancing: economic dispatch model for Thursday, with a horizon of 24 hours and a time step of 15 minutes
- Lattice: 96 stages, 10 nodes per stage
- 292 generators, 228 buses, 312 lines
- Assume fast-start resources at every node with marginal cost of 100 – 500 €/MWh
Convergence

• Run time for obtaining a **policy**: 4.3 hours
• Run time for obtaining a decision, given a history of uncertainty: sub-second
Policy Comparison

<table>
<thead>
<tr>
<th></th>
<th>Slow unit cost ($10^3 \text{ €}$)</th>
<th>Fast-start cost ($10^3 \text{ €}$)</th>
<th>Total cost ($10^3 \text{ €}$)</th>
<th>$\sigma$ total cost ($10^3 \text{ €}$)</th>
<th>Fast-start energy (MWh)</th>
<th>Excess energy (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect foresight</td>
<td>17380</td>
<td>850</td>
<td>18231</td>
<td>291</td>
<td>7204</td>
<td>1809</td>
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<tr>
<td>Stochastic programming</td>
<td>17423</td>
<td>955</td>
<td>18378</td>
<td>305</td>
<td>7511</td>
<td>1855</td>
</tr>
<tr>
<td>Deterministic</td>
<td>17373</td>
<td>1221</td>
<td>18594</td>
<td>350</td>
<td>8688</td>
<td>1879</td>
</tr>
</tbody>
</table>

- Deterministic dispatch: fixed pumped hydro schedule to day-ahead solution
- Benefits of perfect foresight relative to stochastic programming: 0.8%
- Benefits of stochastic programming relative to deterministic dispatch: 1.2%
Adaptiveness of Stochastic Programming Dispatch
# Effects of Transmission and Ramping

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Total cost ($10^3$ €)</th>
<th>$\sigma$ total cost ($10^2$ €)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Perfect foresight (no transmission)</td>
<td>14797</td>
<td>97</td>
</tr>
<tr>
<td>Stochastic programming (no transmission)</td>
<td>14828</td>
<td>100</td>
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<tr>
<td>Deterministic (no transmission)</td>
<td>14867</td>
<td>105</td>
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<tr>
<td>Lookahead 1-step (no transmission)</td>
<td>14865</td>
<td>105</td>
</tr>
<tr>
<td>SP hydro-only (no transmission)</td>
<td>14830</td>
<td>101</td>
</tr>
<tr>
<td>Perfect foresight (no ramp / transmission)</td>
<td>14796</td>
<td>97</td>
</tr>
<tr>
<td>Stochastic programming (no ramp / transmission)</td>
<td>14828</td>
<td>100</td>
</tr>
<tr>
<td>Deterministic (no ramp / transmission)</td>
<td>14856</td>
<td>105</td>
</tr>
</tbody>
</table>
Some Observations

• Transmission constraints have major impact on results: in the absence of transmission constraints, all three policies attain very similar performance

• Incremental cost of ramp constraints is negligible => are flexible ramp products all that important?

• Incremental benefit of look-ahead is minimal => are short-term lookaheads in real-time markets as important as controlling pumped hydro optimally?

• Performance of ‘SP hydro-only’ very close to stochastic programming optimal => are short-term lookaheads in real-time markets as important as controlling pumped hydro optimally?
Price Behavior

• Tendency of pumped hydro storage to level out prices over time periods and realizations
Thank you

For more information

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