Submitted to *Operations Research*

manuscript (Please, provide the manuscript number!)

**Multi-Area Stochastic Unit Commitment for High Wind Penetration in a Transmission Constrained Network**

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In this paper we present a unit commitment model for studying the impact of large-scale wind integration in power systems with transmission constraints and system component failures. The model is formulated as a two-stage stochastic program with uncertain wind production in various locations of the network as well as generator and transmission line failures. We present a scenario selection algorithm for selecting and weighing wind power production scenarios and composite element failures and we provide a parallel dual decomposition algorithm for solving the resulting mixed integer program. We validate the proposed scenario selection algorithm by demonstrating that it outperforms common reserve rules in a 225 bus model of California with 124 generators and 375 transmission lines. We use our model to quantify generation capacity requirements, operating cost impacts and renewable energy utilization levels for various degrees of wind power integration. We then demonstrate that failing to account for transmission constraints and contingencies can result in significant errors in assessing the economic impacts of renewable energy integration.

*Key words*: unit commitment; stochastic programming; wind power; transmission constraints

1. **Introduction**

The large-scale integration of renewable energy resources such as wind and solar power in power systems is limited by two adverse characteristics of renewable power supply. Renewable supply is highly unpredictable and, in contrast to conventional generators, it is not controllable. Due to the need for maintaining a constant balance between supply and demand in the system and also due to the prohibitively high cost of electricity storage, these two adverse characteristics greatly influence our ability to utilize renewable resources at a large scale by (a) *increasing the cost of power system operations*, (b) *reducing the amount of renewable power that can be absorbed in the network* and, (c) *necessitating the deployment of excess generating capacity that can ensure the reliable operation of*
The focus of this paper is on quantifying these impacts in the presence of transmission network congestion and contingencies.

The operation of power systems under uncertainty can be perceived as a multi-stage decision process, where resources are committed in advance of operations and decisions are updated as an operating interval approaches and conditions in the system are revealed. We follow the approach of [Ruiz et al. (2009b)] in modeling system operations as a two-stage decision process. The first-stage decisions represent the day-ahead commitment of generators based on demand forecasts. Subsequently, uncertainty is revealed and in the second stage the commitment of fast responding units and the dispatch of all committed units is updated in order to respond to system conditions. In deregulated power systems, the first and second stage of the model can be interpreted as simulating the day-ahead and real-time markets respectively.

A stochastic unit commitment model is especially appropriate for quantifying the impacts of renewable energy integration on renewable energy utilization, operating costs and capacity requirements. Renewable power utilization is an explicit decision variable in the problem, operating costs are quantified in the objective function of the problem while reserve capacity requirements are quantified indirectly by the fact that the commitment of generators is an endogenous decision variable in the problem. The estimation of reserve capacity requirements is an especially challenging aspect of our analysis. In order to commit reserves, system operators and analysts often resort to suboptimal ad hoc rules for committing reserve capacity in a deterministic formulation of the unit commitment problem. These commitment policies are not calibrated to an environment with large amounts of renewable resources and may result in inefficiencies and miscalculation of the economic impacts of renewable energy integration.

The work presented in this paper extends the model in [Papavasiliou et al. (2010)] by introducing transmission network constraints as well as generator and transmission line failures. Both features are crucial in accurately assessing the economic impact of renewable energy integration. As we demonstrate in Section 6, ignoring transmission constraints and contingencies can result in a significant overestimation of the capacity value and operating cost savings of wind power. Additionally,
transmission constraints result in shedding renewable power supply due to the fact that the system, often, cannot support power flows during periods of increased renewable energy supply. This phenomenon has been observed in previous studies by Sioshansi and Short (2009) and Morales et al. (2009a) and it undermines the benefits of renewable energy integration. Moreover, transmission constraints greatly complicate the task of determining reserves in various locations of the system and render the use of stochastic unit commitment especially relevant. Arroyo and Galiana (2005) demonstrate that an ad hoc allocation of reserves in various locations of a transmission-constrained model results in suboptimal system performance. The complex influence of transmission constraints on locational reserve requirements is also demonstrated by Galiana et al. (2005) as well as Bouffard et al. (2005).

Although transmission constraints and contingencies cannot be ignored when analysing the economic impacts of renewable energy integration, they also introduce certain modeling and computational challenges that are addressed in this paper. The modeling of wind power production as an aggregate resource located in a single bus is not adequate. Instead, in Section 4 we present a multi-area wind production model that captures both the temporal as well as spatial correlations of wind power production in the various nodes of the network. In Section 5 we address the challenge of scenario selection. As in the case of locational wind power production, the introduction of generation contingencies requires identifying generator failures depending on where they occur in the network. This should be contrasted with modeling the loss of aggregate generation capacity which is a common approach in simpler models without transmission constraints such as Takriti et al. (1996) and Ruiz et al. (2009a). Due to the fact that wind production uncertainty is interleaved with contingencies, an enormous set of scenarios needs to be considered. In order to develop a systematic approach for selecting and weighing scenarios, we use a scenario selection algorithm inspired by importance sampling techniques that select uncertain scenarios on the basis of their likelihood of occurrence and the severity of their impact on operating costs. The algorithm that we present formalizes the intuition used in the scenario selection algorithm of Papavasiliou et al. (2010) and is shown to outperform common deterministic rules in all the case studies that are
presented in Section 6. Even with a limited scenario set, the resulting problem requires the use of decomposition techniques in order to achieve computational tractability. In Section 3 we present an extension of the decomposition algorithm of Papavasiliou et al. (2010) for solving the more general model presented in this paper.

2. Unit Commitment and Economic Dispatch

Recently, various studies have utilized unit commitment models for assessing the impacts of wind integration. A variety of modeling approaches and solution strategies are employed in these studies. Ruiz et al. (2009a) use the two-stage stochastic unit commitment model of Ruiz et al. (2009b) to analyze the impact of wind integration in the Colorado power system, without considering transmission constraints. A method employed in Ruiz et al. (2009b) that we also adopt in our paper is to classify generators as either "fast" or "slow" resources. Slow generators are committed in the first stage, while fast generators can be committed in the second stage of decision making. Sioshansi and Short (2009) study the impact of wind integration on the Electricity Reliability Council of Texas with a deterministic unit commitment model that includes transmission constraints. Wang et al. (2008) use a two-stage formulation that ensures a feasible dispatch in the second stage for all possible wind production outcomes given first-stage commitment decisions. Recent work by Constantinescu et al. (2011) also uses the two-stage stochastic programming formulation of Ruiz et al. (2009b). The authors test their model on a system with 10 generators without transmission constraints. Tuohy et al. (2009) develop a stochastic unit commitment model that accounts for load and wind uncertainty and perform a simulation of the Irish power system. Morales et al. (2009b) use a two-stage stochastic programming model for studying wind integration in the IEEE RTS 96 model of Grigg et al. (1999) that includes transmission constraints but not contingencies. The model assumes explicit bids for spinning and non-spinning reserves. A similar two-stage stochastic programming model is used by Bouffard and Galiana (2008) for a case study of wind integration in a test system with 4 generators without transmission constraints and contingencies. With the exception of Wang et al. (2008), none of the aforementioned wind integration studies employ
decomposition techniques, which limits the size of the models that are studied. By comparison, our model uses a dual decomposition algorithm implemented in a parallel environment in order to examine a reduced model of California with 124 generators, 375 lines and 225 buses.

Alternative formulations of the unit commitment problem have been recently proposed in order to address uncertainty. This work is motivated by the fact that system operators tend to operate the system so as to protect against worst-case outcomes, and also by the fact that the stochastic programming formulation requires an excessive amount of information about the underlying uncertainty. Ozturk et al. (2004) formulate a chance-constrained optimization of the unit commitment problem without transmission constraints and ramping constraints. Jiang et al. (2010) use a robust optimization formulation of the unit commitment problem, where locational demand is assumed to obey the polyhedral and cardinal uncertainty models defined in Bertsimas and Sim (2004). The authors present a Benders’ decomposition algorithm for solving the problem with transmission and ramping constraints. A similar formulation is proposed by Bertsimas et al. (2011). However, neither of these models accounts for contingencies explicitly.

In our study we consider a set of generation resources $G$ which is partitioned in a set of slow generators $G_s$ and fast generators $G_f$. Uncertainty is modeled as a discrete set of realizations $S$. In the stochastic unit commitment model, the commitment of slow generators is a first stage decision that cannot be altered in the second stage, whereas the commitment of fast generators and the production level of all generators can be adjusted once a realization $s \in S$ is observed. Analogously, in the deterministic model, which follows Sioshansi and Short (2009), slow generators can only provide slow reserves whereas fast generators can provide slow as well as fast reserves. The notation used in the following models is explained in the appendix.

### 2.1. Stochastic Unit Commitment

The stochastic unit commitment problem can be stated as follows:

\[
(SUC) : \min \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst})
\]

\[\text{s.t.} \] (1)
\begin{equation}
\sum_{l \in LI_n} e_{lst} + \sum_{g \in G_s} p_{gst} = D_{nst} + \sum_{l \in LO_n} e_{lst}, \ n \in N, \ s \in S, \ t \in T \tag{2}
\end{equation}

\begin{equation}
e_{lst} = B_{ls}(\theta_{nst} - \theta_{mst}), l = (m, n) \in L, \ s \in S, \ t \in T \tag{3}
\end{equation}

\begin{equation}
e_{lst} \leq TC_l, l \in L, \ s \in S, \ t \in T \tag{4}
\end{equation}

\begin{equation}
-TC_l \leq e_{lst}, l \in L, \ s \in S, \ t \in T \tag{5}
\end{equation}

\begin{equation}
p_{gst} \leq P_{gst}^+, g \in G, \ s \in S, \ t \in T \tag{6}
\end{equation}

\begin{equation}
P_{gst}^- \leq p_{gst}, g \in G, \ s \in S, \ t \in T \tag{7}
\end{equation}

\begin{equation}
p_{gst} - p_{gs,t-1} \leq R_g^+, g \in G, \ s \in S, \ t \in T \tag{8}
\end{equation}

\begin{equation}
p_{gs,t-1} - p_{gst} \leq R_g^-, g \in G, \ s \in S, \ t \in T \tag{9}
\end{equation}

\begin{equation}
\sum_{q=t-UT_g+1}^{t+DT_g} z_{gq} \leq w_{gt}, g \in G_s, \ t \geq UT_g \tag{10}
\end{equation}

\begin{equation}
\sum_{q=t+1}^{t+DT_g} z_{gq} \leq 1 - w_{gt}, g \in G_s, \ t \leq N - DT_g \tag{11}
\end{equation}

\begin{equation}
\sum_{q=t-UT_g+1}^{t+DT_g} v_{gsq} \leq u_{gst}, g \in G_f, \ s \in S, \ t \geq UT_g \tag{12}
\end{equation}

\begin{equation}
\sum_{q=t+1}^{t+DT_g} v_{gsq} \leq 1 - u_{gst}, g \in G_f, \ s \in S, \ t \leq N - DT_g \tag{13}
\end{equation}

\begin{equation}
z_{gt} \leq 1, g \in G_s, \ t \in T \tag{14}
\end{equation}

\begin{equation}
v_{gst} \leq 1, g \in G, \ s \in S, \ t \in T \tag{15}
\end{equation}

\begin{equation}
z_{gt} \geq w_{gt} - w_{g,t-1}, g \in G_s, \ t \in T \tag{16}
\end{equation}

\begin{equation}
v_{gst} \geq u_{gst} - u_{gs,t-1}, g \in G_f, \ s \in S, \ t \in T \tag{17}
\end{equation}

\begin{equation}
\pi_s w_{gst} = \pi_s w_{gt}, g \in G_s, \ s \in S, \ t \in T \tag{18}
\end{equation}

\begin{equation}
\pi_s v_{gst} = \pi_s v_{gt}, g \in G_s, \ s \in S, \ t \in T \tag{19}
\end{equation}

\begin{equation}
p_{gst}, v_{gst} \geq 0, u_{gst} \in \{0, 1\}, g \in G, \ s \in S, \ t \in T \tag{20}
\end{equation}

\begin{equation}
z_{gt} \geq 0, w_{gt} \in \{0, 1\}, g \in G_s, \ t \in T. \tag{21}
\end{equation}

The objective of the problem is to minimize expected operating costs, which consist of minimum load costs, startup costs and fuel costs. Equation (2) is a market clearing constraint that requires
balancing the amount of power that flows into and out of each bus. Equation (3) is a linearized, lossless model of the alternating current (AC) power flow equations (Kirchoff’s law) according to which the power flow on a line \( l \) is proportional to the phase angle difference between the two end buses of the line. The proportionality factor \( B_{ls} \) is the susceptance of line \( l \) under scenario \( s \), where \( B_{ls} = 0 \) for scenarios \( s \in S \) in which line \( l \) is out of service, thereby forcing the flow in line \( l \) to equal zero. Constraints (4) and (5) define thermal capacity limits on the transmission lines. Constraints (6) and (7) impose minimum and maximum generator capacity limits. Similarly to the case of transmission line failures, \( P^{+}_{gs} = 0 \) and \( P^{-}_{gs} = 0 \) holds for those scenarios \( s \in S \) in which generator \( g \) is out of service, thereby forcing power production for generator \( g \) to equal zero. The constraints defined by Equations (8) and (9) represent ramping constraints on the rate of change of generator output. Constraints (10) - (13) represent the minimum up and down time constraints of both fast and slow generators. Constraints (14) and (15) place upper bounds on the startup variables. Note that although startup variables are binary, following O’Neill et al. (2010) we are able to model them as continuous variables, thereby reducing the computational complexity of the model. The transition rule for generator startup variables is imposed in Equations (16) and (17). Equations (18) and (19) are the non-anticipativity constraints which imply that, for slow generators, second stage commitment needs to be consistent with the first stage commitment under each scenario \( s \in S \). Integrality and non-negativity constraints are imposed in Equations (20) and (21).

2.2. Deterministic Unit Commitment

A simpler approach to protect the system against uncertainty is to introduce exogenous requirements on excess generation capacity rather than explicitly modeling the ability of the system to observe and respond to uncertainty. More specifically, in contrast to modeling wind production uncertainty, the deterministic formulation imposes exogenous requirements on reserve supply, which is generation capacity available in excess of the production level of generators that can be called upon in the case of renewable supply fluctuations. Likewise, rather than modeling contingencies explicitly, the deterministic model imposes exogenous import constraints that ensure that the
system can withstand the failure of major generation resources within load pockets as well as the failure of major transmission inter-ties. Since the reserve requirements and import limits rely on the judgement of the modeler, the resulting unit commitment policy is expected to deliver inferior performance relative to a stochastic model that explicitly models uncertainty and the ability of the system to respond in the second stage of decision making. For the deterministic formulation, we follow the model of Sioshansi and Short (2009).

\begin{align*}
(DUC) : \min \sum_{g \in G} \sum_{t \in T} (K_g w_{gt} + S_g z_{gt} + C_g p_{gt}) \\
\text{s.t.} \\
\sum_{l \in LI_n} e_{lt} + \sum_{g \in G_n} p_{gt} = D_{nt} + \sum_{l \in LO_n} e_{lt}, n \in N, t \in T \\
e_{lt} &\leq TC_l, l \in L, t \in T \\
-TC_l &\leq e_{lt}, l \in L, t \in T \\
e_{lt} &= B_l(\theta_{nt} - \theta_{mt}), l = (m, n) \in L, t \in T \\
p_{gt} + f_{gt} &\leq P_g^+ w_{gt}, g \in G, t \in T \\
p_{gt} + s_{gt} + f_{gt} &\leq P_g^+, g \in G, t \in T \\
p_{gt} &\geq P_g^- w_{gt}, g \in G, t \in T \\
p_{gt} - p_{g,t-1} + s_{gt} &\leq R_g^+, g \in G, t \in T \\
p_{g,t-1} - p_{gt} &\leq R_g^-, g \in G, t \in T \\
\sum_{g \in G_f} f_{gt} + \sum_{g \in G} s_{gt} &\geq T_{t}^{req}, t \in T \\
f_{gt} &\leq F_{r} g, g \in G, t \in T \\
\sum_{g \in G_f} f_{gt} &\geq F_{r}^{req}, t \in T \\
\sum_{l \in IG_j} \gamma_{jl} e_{lt} &\leq IC_j, j \in IG, t \in T \\
\sum_{q=t+UT_g+1}^{t+DT_g} z_{gq} &\leq w_{gt}, g \in G, t \geq UT_g \\
\sum_{q=t+1}^{t+DT_g} z_{gq} &\leq 1 - w_{gt}, g \in G, t \leq N - DT_g 
\end{align*}
Note that decision variables in the resulting model are not contingent on scenarios, thereby reducing the size of the model. The maximum capacity constraint in Equation (27) is modified to account for the provision of fast reserves and Equation (28) is added to account for the provision of fast and slow reserves. The total reserve requirement is imposed in Equation (32) and an upper limit on the provision of fast reserves is imposed in Equation (33). The fast reserve requirement is imposed in Equation (34) and the import constraints are imposed in Equation (35).

2.3. Economic Dispatch

The deterministic and stochastic unit commitment models represent two different methods for committing slow generation resources in the day-ahead scheduling timeframe. Once slow resources are committed, the performance of the system is tested by performing Monte Carlo simulations of its response to net demand and contingency outcomes, given the unit commitment schedule of slow generators. The economic dispatch of units for each outcome \( c \) requires solving the following problem:

\[
(ED_c) : \min \sum_{g \in G} \sum_{t \in T} (K_g w_{gt} + S_g z_{gt} + C_g p_{gt})
\]

s.t.

\[
\sum_{t \in L_n} e_{lt} + \sum_{g \in G_n} p_{gt} = D_{net} + \sum_{t \in L_n} e_{lt}, n \in N, t \in T
\]

\[
e_{lt} = B_{lt} (\theta_{nt} - \theta_{mt}), l = (m, n) \in L, t \in T
\]

\[
p_{gt} \leq P^+_{gc} u_{gt}, g \in G, t \in T
\]

\[
p_{gt} \geq P^-_{gc} u_{gt}, g \in G, t \in T
\]

\[
p_{gt} - p_{g,t-1} \leq R_g^+, g \in G, t \in T
\]

\[
w_{gt} = w^*_{gt}, g \in G, t \in T
\]
\[ z_{gt} = z^*_g, g \in G_s, t \in T \quad (48) \]

Equations (47) and (48) set the commitment of slow generators to the optimal solution of the unit commitment problem.

### 3. Decomposition Algorithm

The use of decomposition algorithms for solving the stochastic unit commitment problem was pioneered by Takriti et al. (1996), who use a multi-stage stochastic unit commitment formulation for studying load uncertainty and generator failures. The authors use the progressive hedging algorithm of Rockafellar and Wets (1991) to decompose the stochastic formulation to single-period subproblems. In Carpentier et al. (1996) the authors use the augmented Lagrangian algorithm to decompose a multi-stage stochastic program to single-generator subproblems. Nowak and Römisch (2000) develop a Lagrangian decomposition algorithm for optimizing the operations of a hydro-thermal system under load uncertainty. Shiina and Birge (2004) develop a column generation algorithm for decomposing a multistage stochastic program into single-generator subproblems. In this section we present a generalization of the subgradient algorithm that is used in Papavasiliou et al. (2010) for decomposing the stochastic unit commitment problem to individual scenarios.

By dualizing the constraints of Equations (18) and (19) we obtain the following Lagrangian:

\[
\mathcal{L} = \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
+ \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (\mu_{gst} (u_{gst} - w_{gt}) + \nu_{gst} (v_{gst} - z_{gt})) \quad (50)
\]

We can then decompose the Lagrangian to one subproblem for each scenario, that determines optimal second-stage decisions:

\[
(P2_s): \min \sum_{g \in G} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
+ \sum_{g \in G} \sum_{t \in T} \pi_s (\mu_{gst} u_{gst} + \nu_{gst} v_{gst}) \quad (51)
\]

s.t.
Note that the constraint \( v_{gst} \leq 1, g \in G_s \) of Equation 15, although redundant for the model \((SUC)\), is necessary for bounding \( P_{2s} \) when applying the decomposition algorithm. We also obtain a single subproblem that determines optimal first-stage decisions:

\[
(P1) : \min - \sum_{g \in G_s} \sum_{s \in S} \sum_{t \in T} \pi_s (\mu_{gst} w_{gt} + \nu_{gst} z_{gt})
\]

s.t.

\[
(10), (11), (14), (16)
\]

\( w_{gt} \in \{0, 1\}, z_{gt} \geq 0, g \in G_s, t \in T \)

The dual variables are updated as follows:

\[
\mu_{gst}^{k+1} = \mu_{gst}^k + \alpha_k \pi_s (w_{gst}^k - u_{gst}^k), g \in G_s, s \in S, t \in T
\]

\[
\nu_{gst}^{k+1} = \nu_{gst}^k + \alpha_k \pi_s (z_{gst}^k - v_{gst}^k), g \in G_s, s \in S, t \in T,
\]

where \( w_{gt}^k, z_{gt}^k \) are the optimal solutions of \((P1)\) at iteration \( k \) and \( u_{gst}^k, v_{gst}^k \) are the optimal solutions of \((P2_s)\) at iteration \( k \). We could have relaxed only the non-anticipativity constraint on the commitment variables. The advantage of also relaxing the non-anticipativity constraint on the startup variables is that \((P2_s), s \in S\), is a smaller problem, since the constraints on the unit commitment of the slow generators are a part of \((P1)\). An additional advantage of this choice of decomposition is that, at each step, the slow generator unit commitment solutions of the first subproblem can be used for generating a feasible solution to the original problem by solving an economic dispatch problem \((ED_s)\), Equations \((41) - (49)\), for each scenario \( s \in S \). As a result, at each step of the algorithm we get an upper bound on the optimal solution which can be used for terminating the algorithm, as well as a feasible schedule. This should be contrasted with the case where we would
have chosen to relax only the non-anticipative constraints on the unit commitment variables, and not the startup variables.

An important feature of the proposed algorithm is the fact that the second-stage subproblems \((P2_s)\) and the economic dispatch problems \((ED_s)\) can be solved in parallel. As we discuss in Section 6, we have implemented a parallel algorithm for the problem, which has reduced running times sixteenfold. The step size rule follows [Fisher (1985) and Held et al. (1974)] and is presented in [Papavasiliou et al. (2010)].

4. Multi-Area Wind Power Model

Due to the highly nonlinear but static relationship between wind speed and wind power production, it is common in the wind power modeling literature to model wind speed rather than wind power with time series models and then convert speed to power using a static conversion curve. The task of modeling wind speed consists of fitting the data to a parametric or nonparametric distribution, removing seasonal and daily patterns and fitting an appropriate time series model to the underlying "noise" in order to capture the strong temporal dependency of wind speed. Early work on wind power production modeling which follows this approach was performed by [Brown et al. (1984) and Torres et al. (2005)]. Due to the introduction of transmission constraints it is not sufficient to describe the aggregate wind power production in the network, but instead it is necessary to specify the production of wind power in each location of the network. This necessitates the development of a multi-area wind power production model, which needs to faithfully reproduce both the temporal as well as spatial correlations of wind power production that are observed in the data set. The modeling and calibration methodology described in this section follows [Morales et al. (2010)].

Given a multi-area dataset \(y_{kt}\) where \(k\) indexes location and \(t\) indexes time period, the first step is to filter the data set in order to obtain an approximately Gaussian dataset \(y_{kt}^G\). Brown et al. (1984), Torres et al. (2005) and Morales et al. (2010) use this approach for transforming Weibull-distributed wind speed data to Gaussian data, and Callaway (2010) uses a nonparametric transformation. Since no single parametric distribution provides a close fit for the observed data
in all locations in our data set, we fit an empirical distribution \( \hat{F}_k(\cdot) \) to the data for each location \( k \).

We next remove hourly and monthly patterns by subtracting the hourly mean and dividing by the hourly standard deviation in order to obtain a Gaussian stationary data set \( y_{kt}^{GS} \) for each location. The resulting data set \( y_{kt}^{GS} \) can be modeled by an autoregressive process:

\[
y_{kt+1}^{GS} = \sum_{j=0}^{p} \hat{\phi}_{kj} y_{t-j}^{GS} + \hat{\omega}_{kt},
\]

(57)

where \( \hat{\omega}_{kt} \) is the estimated noise and \( \hat{\phi}_{kj}, j \in \{1, \ldots, p\} \) are the estimated coefficients of the autoregressive model. The calibration process is summarized in the following steps:

**Step (a).** Transform the data in order to obtain a Gaussian distribution on the data set:

\[
y_{kt}^G = N^{-1}( \hat{F}_k(y_{kt}) ),
\]

where \( y_{kt} \) is the data, \( y_{kt}^G \) is the transformed data that is Gaussian distributed, \( N^{-1}(\cdot) \) is the inverse of the cumulative distribution function of the normal distribution and \( \hat{F}_k \) is the cumulative function of the fit for the data in location \( k \).

**Step (b).** Remove systematic monthly and diurnal effects:

\[
y_{kt}^{GS} = \frac{y_{kt}^G - \hat{\mu}_{kmt}}{\hat{\sigma}_{kmt}},
\]

where \( y_{kt}^{GS} \) is the transformed data that is Gaussian distributed and stationary, and \( \hat{\mu}_{kmt} \) and \( \hat{\sigma}_{kmt} \) are the sample mean and standard deviation respectively for location \( k \), month \( m \) and hour \( t \).

**Step (c).** Use the Yule-Walker equations [Box and Jenkins 1976] to estimate the autoregressive parameters \( \hat{\phi}_{kj} \) and covariance matrix \( \hat{\Sigma} \) of the estimated noise in the autoregressive model of Equation (57).

Once the parameters of the statistical model are estimated, the inverse process can be performed for simulating wind speed. In order to simulate wind power production, we use a piecewise linear
Table 1  Current and projected capacity of wind power installations (MW)

<table>
<thead>
<tr>
<th>County</th>
<th>Existing</th>
<th>Moderate</th>
<th>Deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altamont</td>
<td>954</td>
<td>954</td>
<td>1,086</td>
</tr>
<tr>
<td>Clark</td>
<td>-</td>
<td>-</td>
<td>1,500</td>
</tr>
<tr>
<td>Imperial</td>
<td>-</td>
<td>-</td>
<td>2,075</td>
</tr>
<tr>
<td>Solano</td>
<td>348</td>
<td>848</td>
<td>1,149</td>
</tr>
<tr>
<td>Tehachapi</td>
<td>1,346</td>
<td>4,886</td>
<td>8,333</td>
</tr>
<tr>
<td>Total</td>
<td>2,766</td>
<td>6,688</td>
<td>14,143</td>
</tr>
</tbody>
</table>

approximation of the aggregate power curve (see the lower right panel of Figure 1). We use wind speed and wind power production data from the 2006 data set of the National Renewable Energy Laboratory (NREL) Western Wind and Solar Integration Study (WWSIS) database described in Potter et al. (2008). We study two wind integration cases. The first represents a moderate energy integration level for wind power corresponding to the 2012 target of California, and the second case represents a deep integration level corresponding to the 2020 targets. Ex post we have observed that the moderate integration case corresponds to approximately 7% energy penetration, while the deep integration case corresponds to approximately 14% energy penetration. In the subsequent analysis we will refer to these cases as moderate and deep integration respectively.

In order to collect data for each case, we examined the interconnection queue of the California ISO until 2020 (CAISO 2010), and placed individual wind generators in our model by matching the geographical locations of planned wind power installations with the corresponding wind park data in the WWSIS data set. In Table 1 we present the locations of existing wind generation capacity, as well as capacity for the moderate and deep integration cases. In Figure 1 we compare the load duration curve of our calibrated model to the raw data set for the deep integration case.

5. Scenario Selection

The challenge of selecting scenarios for the stochastic unit commitment problem is to discover a small number of representative outcomes that properly guide the stochastic program to produce a unit commitment schedule that improves average costs, as compared to a schedule determined by solving a deterministic unit commitment model. The basic tradeoff that needs to be balanced in dispatching fast reserves is the flexibility that fast units offer in utilizing renewable generation
Figure 1 In reading order: load duration curves for Altamont, Clark County, Imperial, Solano and Tehachapi, and power curve at the Tehachapi area for deep integration.

versus their higher operating costs. Fast generators are fueled by gas, which has a relatively high marginal cost. In addition, the startup and minimum load costs of these units are similar to those of slow units, however their capacity is smaller; hence, their startup and minimum load cost per unit of capacity is greater than that of slow generators. The advantage of largely relying on fast units is that the system is capable of discarding less renewable power, which results in significant savings in fuel costs. Unlike fast generators which can shut down on short notice in the case of increased renewable power generation, slow generators cannot back down from their minimum generation levels and therefore require the waste of excess renewable energy in order to stay online.

The introduction of transmission constraints complicates scenario selection considerably due to
the fact that the fast reserves are not readily accessible when certain transmission lines are congested and the availability of renewable resources may be limited due to transmission constraints. The further introduction of composite outages introduces the complication of protecting the system against very low probability outcomes that can severely impact system reliability.

The stochastic unit commitment literature has relied extensively on the scenario selection and scenario reduction algorithms proposed by Dupacova et al. (2003) and their faster variants that were proposed by Heitsch and Römisch (2003). The effectiveness of these algorithms in the stochastic unit commitment problem was first demonstrated by Gröwe-Kuska et al. (2002) who apply the algorithms for scheduling hydro and thermal units in a German utility. Morales et al. (2009b) propose a variant of the scenario reduction algorithm of Heitsch and Römisch (2003) that removes the scenarios that cause the least change in the second stage costs of the optimization problem.

Despite the theoretical justification of the scenario reduction algorithm of Dupacova et al. (2003) and Heitsch and Römisch (2003), as discussed in Papavasiliou et al. (2010) these algorithms perform poorly in a unit commitment model without transmission constraints and contingencies, where wind power production is the only stochastic input. This is attributed to two reasons. Firstly, the algorithms of Dupacova et al. (2003), Heitsch and Römisch (2003) are not guaranteed to preserve the moments of hourly wind generation. Due to the predominant role of fuel costs in the operation of the system, the accurate representation of average wind supply in the case of large-scale wind integration is crucial for properly guiding the weighing of scenarios. Moreover, the modeler cannot specify certain scenarios which are deemed crucial. For example, the realization of minimum possible wind output throughout the entire day needs to be considered explicitly as a scenario. Otherwise, there is the possibility of under-committing resources and accruing overwhelming costs from load shedding in economic dispatch. In order to overcome these drawbacks, Papavasiliou et al. (2010) generate a large number of samples from the statistical model of the underlying process and select a subset of samples based on a set of prescribed criteria that are deemed important. The authors then assign weights to each scenario such that the first moments of hourly wind output are matched as closely as possible (scenario selection that strives to match certain statistical properties
of the underlying process has been proposed in previous literature, e.g. by Hoyland and Wallace (2001).

The introduction of transmission constraints and contingencies complicates the task of scenario selection considerably, as it is not clear how to extend the scenario selection algorithms of Dupacova et al. (2003) and Papavasiliou et al. (2010) to networks with multi-area wind production and how to account for network component failures. Assuming independence among net load outcomes and contingencies, which is a very reasonable assumption, one natural approach for generating scenarios would be to decouple the selection of contingencies from the selection of net load scenarios. The previously discussed algorithms could then be adapted for selecting multi-area net load scenarios, and these scenarios could be multiplexed with a set of significant contingencies. One natural question that this approach raises is which contingencies to select and how to weigh them relative to each other. For example, the failure of any given generator has a likelihood of 1%, however in a network with 124 generators the chances of a single-generator failure are approximately 36.0%. The question arises, then, if a scenario includes the failure of a single generator, how should that scenario be weighed against other scenarios? Which generator failures should we include in the scenario set? Should we consider composite failures, for example the failure of multiple generators, multiple lines, or generators and lines in the same scenario? It becomes clear that a methodical approach for scenario selection is needed.

5.1. Importance Sampling

The scenario selection algorithms that we propose in this section are inspired by importance sampling. Importance sampling is a statistical technique for reducing the number of Monte Carlo simulations that are required for estimating the expected value of a random variable within a certain accuracy. For an exposition see Mazumdar (1975) and Infanger (1992). As Pereira and Balu (1992) report, this technique has been used in reliability analysis in power systems with composite generation and transmission line failures, where the estimated random variable is a reliability metric (e.g. loss of load probability or expected load not served).
Given a sample space $\Omega$ and a measure $p$ on this space, importance sampling defines a measure $q$ on the space that reduces the variance of the observed samples of the random variable $C$, and weighs each simulated outcome $\omega$ by $p(\omega)/q(\omega)$ in order to unbias the simulation results. The measure $q$ is ideally chosen such that it represents the contribution of a certain outcome to the expected value that is being computed, i.e.

$$q^*(\omega) = \frac{p(\omega)C(\omega)}{\mathbb{E}_p C}$$  \hfill (58)

Of course, it is not possible to determine this measure since $\mathbb{E}_p C$ is the quantity we wish to compute. Nevertheless, the intuition of selecting samples according to their contribution to the expected value can be carried over to scenario selection. For example, in Papavasiliou et al. (2010) the authors include the wind power production outcome with the lowest aggregate production over the entire day. Although the likelihood of this outcome is very low, its impact on system costs can be extremely high, making its contribution to expected cost $p(\omega)C(\omega)$ significant.

### 5.2. Proposed Algorithm

The extension of the intuition of importance sampling to the case of scenario selection is straightforward: if the ideal measure $q^*$ of Equation (58) were closely approximated by a measure $q$, then selecting a small number of outcomes according to this measure and weighing them according to $p(\omega)/q(\omega)$ would provide an accurate estimate of the expected cost. Therefore, samples selected according to $q$ can be interpreted as representative scenarios that need to be weighted according to $p(\omega)/q(\omega)$ relative to each other in order not to bias the result.

We proceed by generating an adequately large subset of the sample space $\Omega_S = \{\omega^1, \ldots, \omega^M\}$ and we calculate the cost of each sample against a deterministic unit commitment policy $C_D(\cdot)$. Since

$$\bar{C} = \frac{1}{M} \sum_{i=1}^{M} C_D(\omega_i)$$

provides an accurate estimate of expected cost, we interpret the sample space of the system as $\Omega_S$ and the measure as the uniform distribution over $\Omega_S$, hence $p(\omega) = M^{-1}$ for all $\omega \in \Omega_S$. We then obtain $q(\omega_i) = C_D(\omega_i)/\bar{C}$, $i = 1, \ldots, M$, and each selected scenario is weighed according to $\pi_s = p(\omega)/q(\omega)$, hence $\pi_s/\pi_s' = C_D(\omega^s)/C_D(\omega^s')$ for each pair of selected scenarios.
ω, ω′ ∈ Ō. Hence, the proposed algorithm selects scenarios with a likelihood that is proportional to their cost impact, and discounts these scenarios in the stochastic unit commitment in proportion to their cost impact in order not to bias the stochastic unit commitment policy. We therefore propose the following algorithm:

**Step (a).** Define the size $N$ of the reduced scenario set $\hat{\Omega} = \{\omega^1, \ldots, \omega^N\}$.

**Step (b).** Generate a sample set $\Omega_S \subset \Omega$, where $M = |\Omega_S|$ is adequately large. Calculate the cost $C_D(\omega)$ of each sample $\omega \in \Omega_S$ against the best deterministic unit commitment policy and the average cost $\bar{C} = \frac{1}{M} \sum_{i=1}^{M} C_D(\omega_i)$.

**Step (c).** Choose $N$ scenarios from $\Omega_S$, where the probability of picking a scenario $\omega$ is $C_D(\omega)/\bar{C}$.

**Step (d).** Set $\pi_s = C_D(\omega)^{-1}$ for all $\omega \in \hat{\Omega}$.

### 5.3. Discussion

The proposed algorithm ensures that, as long as the cost impacts of all selected scenarios are of the same order of magnitude, which is commonly the case, then so are the weights in the stochastic unit commitment formulation. As a result, each scenario influences the first-stage decisions. This should be contrasted with the case where the probabilities of certain scenarios are very small compared to the probabilities of other scenarios. In that case, as we can see in Equation (53), the influence of scenarios with very small probability is minimal in the objective function of $(P1)$, and consequently these scenarios will not tend to influence the optimal solution of $(P1)$ and therefore the first-stage decision, i.e. the commitment of slow generators. In addition, from Equations (55) and (56) we see that scenarios with small probability exhibit very small changes in the values of their dual multipliers, which further supports the argument that these scenarios do not influence $(P1)$. In fact, we observed that the stochastic unit commitment policy remained completely unaffected by scenarios that were 100 times less likely to occur than their competing scenarios in the stochastic unit commitment formulation. Therefore, the inclusion of these scenarios introduced superfluous computational load. This is guaranteed not to happen with our proposed scenario
selection algorithm.

The rationale of the proposed scenario selection method suggests a heuristic for deciding on the appropriate number of scenarios to include in the stochastic unit commitment formulation. Namely, the modeler should select a certain confidence interval and select $N$ to be such that the moving average $\bar{C}_n = \frac{\sum_{i=1}^{n} C_D(\omega_i)}{n}$ lies within the interval.

An additional appealing feature of the proposed algorithm is that it selects a rich set of multi-area net load outcomes. This should be contrasted to the case where we would multiplex net load scenarios selected according to the scenario selection algorithms of Dupacova et al. (2003), Gröwe-Kuska et al. (2002), Papavasiliou et al. (2010) with contingency scenarios. Moreover, in contrast to the scenario selection method proposed in Papavasiliou et al. (2010), the scenario selection algorithm proposed in this paper does not depend on the judgement of the modeler for specifying criteria that are deemed important. The proposed procedure can be applied to a broad setting of problems under uncertainty in a straightforward fashion. In Section 6 it is shown that the resulting stochastic unit commitment policy outperforms common deterministic rules for four case studies of uncertainty, one of which is the case study that was performed in Papavasiliou et al. (2010).

6. Results

In this section we present simulation results for a model of the California power system with 225 buses, 124 generators and 375 transmission lines. A schematic description of the model showing the wind sites, zones and import constraints of the network is presented in Section B of the appendix. The model is also used in Yu et al. (2010) and Papavasiliou et al. (2010). As we mention in Section 4 we study three levels of wind integration, the zero wind integration case as well as a case of moderate and deep wind integration. We also perform the deep integration case study for the case where transmission constraints and contingencies are not accounted for, in order to quantify the impact of these effects on the analysis. The latter case is denoted as Deep-Simple. As we discuss in the Introduction, our focus is on quantifying the degree to which renewable resources can be utilized, operating costs and the amount of capacity that needs to be committed in order to operate
the system reliably. By contrast to Sioshansi and Short (2009), Ruiz et al. (2009a) and Tuohy et al. (2009), rather than simulating an entire year of operations on a rolling horizon basis we focus our analysis on eight day types that represent weekdays and weekends for each season in order to reduce computational load.

6.1. Data

We do not use the wind production data from Yu et al. (2010) since our wind production model is more detailed. Since we are using 2006 wind production data from the NREL database, we also use load data from the same year, which is publicly available at the CAISO Oasis database CAISO (2011). The average load in the system is 27,298 MW, with a minimum of 18,412 MW and a peak of 45,562 MW. The net load profile for each day type, which needs to be served by thermal generators and wind power, is shown in Figure 2.

We use a more general model for thermal generators within CAISO, with 124 generators, compared to the model in Yu et al. (2010), which uses 23 aggregated thermal generators. The value of lost load is set to 5,000 $/MW-h. The number of generators and the capacity for each fuel type are shown in Table 2. The last two rows of Table 2 describe how the fossil fuel generation mix is partitioned into fast and slow generators. The entire thermal generation capacity of the system is 28,381.5 MW.
Table 2  Generation mix for the test case

<table>
<thead>
<tr>
<th>Type</th>
<th>No. of units</th>
<th>Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear</td>
<td>2</td>
<td>4,499</td>
</tr>
<tr>
<td>Gas</td>
<td>88</td>
<td>18,745.6</td>
</tr>
<tr>
<td>Coal</td>
<td>6</td>
<td>285.9</td>
</tr>
<tr>
<td>Oil</td>
<td>5</td>
<td>252</td>
</tr>
<tr>
<td>Dual fuel</td>
<td>23</td>
<td>4,599</td>
</tr>
<tr>
<td>Import</td>
<td>22</td>
<td>12,691</td>
</tr>
<tr>
<td>Hydro</td>
<td>6</td>
<td>10,842</td>
</tr>
<tr>
<td>Biomass</td>
<td>3</td>
<td>558</td>
</tr>
<tr>
<td>Geothermal</td>
<td>2</td>
<td>1,193</td>
</tr>
<tr>
<td>Wind (moderate)</td>
<td>5</td>
<td>6,688</td>
</tr>
<tr>
<td>Wind (deep)</td>
<td>10</td>
<td>14,143</td>
</tr>
<tr>
<td>Fast thermal</td>
<td>82</td>
<td>9,156.1</td>
</tr>
<tr>
<td>Slow thermal</td>
<td>42</td>
<td>19,225.4</td>
</tr>
</tbody>
</table>

6.2. Relative Performance of Policies

In this section we discuss the relative performance of the stochastic and deterministic unit commitment policies. The results are obtained by running the economic dispatch model against 1000 Monte Carlo outcomes of wind power production and contingencies, with a probability of generator failure of 1% (Pereira and Balu (1992)) and a probability of transmission line failure of 0.1% (Grigg et al. (1999)). Wind production outcomes, generator failures and transmission line failures are assumed to be independent. We consider two deterministic policies. The first policy places a total reserve requirement equal to a fraction of forecast peak load for the day and a fast reserve requirement equal to half of the total reserve requirement. In order to determine the best policy of this type, we find the fraction of peak load that yields the best performance. The other deterministic policy that we consider is a variant of a reserve commitment policy that was recently proposed in Piwko et al. (2010). The authors propose a heuristic approach for committing spinning reserves, "the 3+5 rule", which requires the system to carry hourly spinning reserve no less than 3% of hourly forecast load plus 5% of hourly forecast wind power. This rule is adapted in our model by setting this as the fast reserve requirement and setting the total reserve requirement at twice the level of the fast reserve requirement. The results are shown in Table 3 where the best peak-load policy is highlighted in italic font and the best deterministic policy is highlighted in bold font. N/A denotes that the specific policy was not evaluated in order to avoid superfluous computation.
We note that the best peak-load policy outperforms the 3+5 rule for all but the deep integration case study without transmission constraints and contingencies. We also consider a perfect foresight policy that commits and dispatches resources under perfect forecast of uncertain outcomes. The perfect foresight policy bounds the attainable cost of any unit commitment rule.

The relative performance of stochastic unit commitment with respect to the determinsitic policies and the perfect foresight policy for the four case studies are presented in Figure 3. The results are presented in terms of the relative cost of each policy relative to the stochastic unit commitment policy for each of the eight day types. In the last two rows of Table 4 we present the absolute cost of the stochastic unit commitment policy as well as its gains over the best deterministic policy relative to the perfect foresight policy. We note that the stochastic policy benefits range between 32.4% to 46.7% of the potential benefits of perfect forecasting, with higher wind integration resulting in higher benefits due to the increased uncertainty in the system. Moreover, the introduction of transmission constraints affects both the relative as well as absolute gains of stochastic unit commitment, which supports the argument that stochastic unit commitment is especially valuable for the determination of locational capacity requirements.

### 6.3. Renewables Utilization, Operating Costs and Capacity Requirements

In Table 4 we present summary results for renewable energy waste, operating costs and capacity requirements for the four case studies under consideration. Renewable energy losses are negligible relative to total renewable energy production, although accounting for transmission constraints and contingencies results in a twentyfold increase in the estimated loss of renewable power production in the case of deep integration. Operating costs decline steeply as the level of renewable power penetration increases, due to the decrease in fuel costs, which are the predominant cost in the system.
Failing to account for transmission constraints and contingencies results in an underestimation of operating costs by 33.0% relative to the costs when accounting for these factors. The significant cost increase resulting from transmission constraints can be attributed to increased shedding of freely available energy but also to the reduced flexibility of dispatching units in the system.

Capacity requirements, which are the most important factor in analyzing the economics of renewable energy integration, present the most interesting results. We note that the moderate integration case reduces capacity requirements by a mere 1.2% of the installed wind capacity, whereas the capacity requirements for the deep integration case are the same as for the moderate integration scenario, indicating that the excess wind capacity cannot contribute to capacity savings. Most importantly, we note that failing to account for transmission constraints results in an overestimation of the capacity credit of wind power production by 39.8% relative to the 1.2% capacity credit when these features are accounted for. This strongly supports the argument that the inclusion of transmission constraints and contingencies is crucial for accurately assessing the impact of large scale renewable energy integration.
<table>
<thead>
<tr>
<th></th>
<th>Deep-Simple</th>
<th>No Wind</th>
<th>Moderate</th>
<th>Deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE daily waste (MWh)</td>
<td>100</td>
<td>0</td>
<td>890</td>
<td>2,186</td>
</tr>
<tr>
<td>Cost ($M)</td>
<td>5.012</td>
<td>11.508</td>
<td>9.363</td>
<td>7.481</td>
</tr>
<tr>
<td>Capacity (MW)</td>
<td>20,744</td>
<td>26,377</td>
<td>26,068</td>
<td>26,068</td>
</tr>
<tr>
<td>Daily savings ($)</td>
<td>38,628</td>
<td>104,321</td>
<td>198,199</td>
<td>188,735</td>
</tr>
<tr>
<td>Forecast gains (%)</td>
<td>32.4</td>
<td>35.4</td>
<td>41.9</td>
<td>46.7</td>
</tr>
</tbody>
</table>

### 6.4. Computational Performance

The stochastic unit commitment algorithm was implemented in the Java callable library of CPLEX 11.0.0, and parallelized using the Parallel Virtual Machine (PVM) on a network of 16 DELL Poweredge 1850 servers (Intel Xeon 3.4 GHz, 1GB RAM). \((P_1_s), s \in S\) and \((P_2)\) were run for 120 iterations. For the last 40 iterations, \((ED_s)\) was run for each \(s \in S\) in order to obtain a feasible solution and an upper bound for the stochastic unit commitment problem. The average elapsed time on a single machine was 43,776 seconds. The MIP gap for \((P_1_s), s \in S\) and \((P_2)\) was set to \(\epsilon_1 = 1\%\), and the MIP gap for obtaining a feasible schedule from \((ED_s)\) was set to \(\epsilon_2 = 0.1\%\). The sum of the optimal solutions of the first and second subproblem yield a lower bound \(LB\) on the optimal cost, whereas the optimal solution of the feasibility run results in an upper bound \(UB\). The average gap, \(\frac{UB-LB}{LB}\), that we obtained is 1.39\%. However, to estimate an upper bound on the optimality gap it is also necessary to account for the MIP gap \(\epsilon_1\) that is introduced in the solution of \((P_1)\) and \((P_2_s), s \in S\). The average upper bound on the optimality gap, \(\frac{UB-(1-\epsilon_1)LB}{(1-\epsilon_1)LB}\), is 2.41\%.

### 7. Conclusions

In this paper we present a two-stage stochastic unit commitment model that can be used for assessing the impact of wind power integration on renewable energy utilization, operating costs and generation capacity requirements. We present a scenario selection algorithm inspired by importance sampling that is shown to outperform common deterministic reserve rules for four case studies, and a subgradient algorithm for solving the resulting stochastic program. The benefits of stochastic unit commitment relative to deterministic reserve rules are shown to range between 32.4\%-46.7\% of the potential gains under perfect forecasting. Renewable energy waste is negligible across all case studies. The increased integration of wind power significantly reduces operating costs due to
fuel cost savings. Failing to account for contingencies and transmission results in an underestimation of operating costs by 33.0%. Capacity requirements decrease negligibly as renewable energy integration increases when transmission constraints and contingencies are accounted for. Failing to account for transmission constraints and contingencies features results in an overestimation of wind capacity credit by 39.8%, relative to 1.2% when these features are accounted for.

**Appendix A: Notation**

*Sets*

- $G$: set of all generators, $G_s$: subset of slow generators, $G_f$: subset of fast generators
- $S$: set of scenarios
- $T$: set of time periods
- $L$: set of lines
- $N$: set of nodes
- $G_n$: set of generators that are located in bus $n$

$$LI_n = \{ l \in L : l = (k, n), k \in N \}, \quad LO_n = \{ l \in L : l = (n, k), k \in N \}$$

- $IG$: set of import groups, $IG_j$: set of lines in import group $j$

*Decision variables*

- $u_{gst}$: commitment, $v_{gst}$: startup, $p_{gst}$: production of generator $g$ in scenario $s$, period $t$
- $\theta_{nst}$: phase angle at bus $n$ in scenario $s$, period $t$
- $w_{gt}$: commitment, $z_{gt}$: startup of slow generator $g$ in period $t$
- $s_{gt}$: slow reserve, $f_{gt}$: fast reserve provided by generator $g$ in period $t$
- $e_{lst}$: power flow on line $l$ in scenario $s$, period $t$

*Parameters*

- $\pi_s$: probability of scenario $s$
- $K_g$: minimum load cost, $S_g$: startup cost, $C_g$: marginal cost of generator $g$
- $D_{nst}$: demand in bus $n$, scenario $s$, period $t$
- $P_{gs}^+, P_{gs}^-$: minimum and maximum capacity of generator $g$ in scenario $s$
$R_g^+, R_g^-$: minimum and maximum ramping of generator $g$

$UT_g$: minimum up time, $DT_g$: minimum down time of generator $g$

$N$: number of periods in horizon

$T_t^{req}$: total reserve requirement, $F_t^{req}$: fast reserve requirement in period $t$

$B_{ls}$: susceptance of line $l$ in scenario $s$

$TC_l$: maximum capacity of line $l$

$FR_g$: fast reserve limit of generator $g$

$IC_j$: maximum capacity of import group $j$

$\gamma_{jl}$: polarity of line $l$ in import group $j$

**Appendix B: The WECC Model**

In Figure 4 we present a schematic diagram of the Western Electricity Coordinating Council (WECC) model that is studied in Section 6. The dashed boxes represent load and generation pockets. The thick solid lines represent the import constraints that are defined in Equation (35). Each thick solid line intersects a set of transmission lines $IG_j$ over which the total amount of power cannot exceed a certain limit $IC_j$. These constraints limit the total flow of power into a load pocket in order to prevent load shedding in the case of generator failure within a load pocket, and also limit the total amount of power flow over combinations of interties that connect the California ISO system to neighboring states. The wind generators of Table 1 are located in the five buses that are depicted as solid black circles. In order of appearance from top to bottom, these wind sites are Solano, Altamont, Tehachapi, Clark and Imperial.

**Acknowledgments**

This research was funded by NSF Grant IIP 0969016, the US Department of Energy through a grant administered by the Consortium for Electric Reliability Technology Solutions (CERTS), the Siemens Corporation under the UC Berkeley CKI initiative and the Federal Energy Regulatory Commission.

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