Coupling Renewable Energy Supply with Deferrable Demand

by

Anthony Papavasiliou

A dissertation submitted in partial satisfaction of the requirements for the degree of Doctor of Philosophy in Engineering / Industrial Engineering and Operations Research in the Graduate Division of the University of California, Berkeley

Committee in charge:
Professor Shmuel S. Oren, Chair
Professor Philip Kaminsky
Assistant Professor Duncan Callaway

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Abstract

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The large-scale integration of renewable energy sources such as wind and solar power is advancing rapidly in numerous power systems in Europe and the United States. However, utilizing renewable resources at a bulk scale is hindered by the fact that these resources are neither controllable nor accurately predictable. Our analysis focuses on the cost of balancing power system operations in the presence of renewable resources and on the amount of capital investment in operating reserves that is necessary for ensuring the reliable operation of the system when renewable resources are integrated at a large scale. We also explore the extent to which demand-side flexibility can mitigate these impacts. We specifically focus on a contract that couples the operations of renewable energy resources with deferrable loads that can shift a fixed amount of energy demand over a given time window. Various flexible energy consumption tasks can be characterized in this way, including electric vehicle charging or agricultural pumping.

We use a two-stage stochastic unit commitment model for our analysis. The use of this model is justified by the fact that it is capable of quantifying the operating costs of the system and the amount of required capacity in order to face the increased uncertainty of daily operations. We present a dual decomposition algorithm for solving the model and various scenario selection algorithms for representing uncertainty that are necessary for achieving computational tractability in the stochastic unit commitment formulation.

We present results for a reduced network of the California power system that consists of 124 generators, 225 buses and 375 lines. Our analysis proceeds in two steps. We first validate the stochastic unit commitment policy that we derive from the stochastic optimization model by demonstrating that it outperforms deterministic unit commitment rules commonly used in practice. We demonstrate this superior performance for both a transmission-constrained as well as an unconstrained system for various types of uncertainty including network element failures as well as two levels of wind integration that roughly correspond to the 2012 and 2020 renewable energy integration targets of California. Once we establish the validity of the stochastic unit commitment policy we quantify the impacts of coupling renewable
energy supply with deferrable demand on operating costs and reserve requirements. We also demonstrate the superiority of coupling contracts to demand-side bidding in the day-ahead market which is due to the fact that demand bids fail to account for the inter-temporal dependency of shiftable demand.
In memory of my uncles Niko and Dimitri,

my cousin Ioanna,

and nona Eleni
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After overcoming the hurdle of the preliminary exams in the spring of 2007, I was unaware that I was entering the most challenging phase of my dissertation: selecting a thesis topic. In the process of applying for an NSF fellowship, I articulated the problem: using renewable energy sources to serve the flexible portion of the electricity market. The reasoning was that renewable power suppliers could compete with conventional energy sources in this market segment on the basis of their low fuel costs while exploiting the flexibility of their customers in order to alleviate the problems caused by their unpredictable and uncontrollable supply. The vision of the dissertation was to demonstrate that renewable resources can become economically competitive bulk power suppliers, due to the fact that a large portion of the energy that our society consumes can be deferred with minimal impact on comfort. My very deep gratitude goes to Professors Paul Wright, Andrew Isaacs, and Ikhlal Sidhu, who believed in the project, provided material support by funding it and contributed to its credibility through their support. Special thanks to Professor Wright for his participation in my qualifying exam committee and to Professor Sidhu for our one-year collaboration on the Better Place case study. My deep appreciation also extends to my adviser for his patience during my pursuit of a research identity.

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Chapter 1

Introduction

Over the past decades, society has embraced the goal of curbing carbon emissions and combating global warming. As a result, renewable energy sources have undergone massive growth, to the point that the large-scale integration of resources such as wind and solar power in power systems is technically and economically conceivable.

As numerous power systems in the United States and Europe have come to rely on renewable resources for supplying bulk quantities of power, system operators are faced with the unnerving fact that these resources are available by nature. Consequently, their availability is beyond human control and largely unpredictable. System operators are now faced with exotic challenges from the integration of resources the fluctuations of which are no longer lost as statistical noise in the system. A recent event that exemplifies the force of nature occurred in February 2008, when wind power supply in Texas dropped by 1700 MW within three and a half hours and necessitated the curtailment of large industrial consumers in order to prevent a blackout.

Since the birth of power systems, the prevailing paradigm of power systems operations has been load-following: controllable generation resources have been required to track demand, regardless of its patterns, location, unpredictability and the stress that is imposed on the system. People have become so accustomed to the idea of having power at the flick of a switch that the incredible reliability of electric power supply and the resources that are required to maintain it are often overlooked. The proliferation of renewable energy sources in modern power systems calls for a paradigm shift in our energy consumption philosophy. Since we need to rely on resources that cannot be controlled or accurately forecast, we need to transition to a mode of operation that is supply-following: we need to exploit our flexibility in consuming power in order to accommodate the availability of renewable energy resources.

In this thesis we analyze the impacts of large-scale renewable energy integration. Our analysis focuses on quantifying three metrics:

- power system operating costs,
- renewable energy utilization, and
• generation capacity requirements for reliable power system operations.

We then propose various mechanisms for integrating deferrable electricity demand in power system operations and analyze the impact of deferrable demand on the aforementioned metrics.

1.1 A Brave New World: Renewable Energy and Demand Response Proliferation in the U.S.

The proliferation of renewable energy sources in the United States is taking place at an unprecedented pace. The federal government is coordinating these efforts at a national level. The American Clean Energy and Security Act (2009), also known as the Waxmen-Markey bill, sets a target of sourcing 20% of U.S. electricity consumption from renewable energy by 2020, requires that U.S. emissions be reduced by 17% compared to their 2005 levels and also sets various goals for limiting reliance on non-renewable resources. Twenty four states and the district of Columbia have set renewable portfolio standards, which commit electric utilities to procure at least a certain percentage of their energy from renewable energy sources. California has the second greatest installed wind capacity in the U.S. and covered 11% of state energy needs with renewable energy in 2006. Since 2002, California has enacted the Participating Intermittent Resources Program, which facilitates the integration of renewable energy sources, and the state has set a Renewable Portfolio Standard that requires 20% of the energy mix to be sourced from renewable sources by 2012 and 33% by 2020.

Demand-side integration is also being favored by policy makers both at the federal level and in individual states. The American Clean Energy and Security Act has allocated $3.4 billion in order to spawn the development and deployment of the necessary technology for enabling active management of electricity demand. San Diego Gas and Electric received $28 million from the Act in order to deploy 1.4 million meters and in 2012 Pacific Gas and Electric will have deployed its smart meter program in its entire service area. Anticipating the importance of demand-side flexibility, the California electricity market rules have been adapted in order to accommodate the participation of demand resources through the recent Market Redesign and Technology Upgrade (MRTU) of 2009.

1.2 Limitations to Large-Scale Renewable Energy Integration

The large-scale integration of renewable energy resources is limited by two adverse characteristics of renewable power supply. In contrast to conventional generation, renewable supply cannot be controlled and it cannot be forecast accurately. The requirement of maintaining a continuous balance between the supply and demand of electricity and the fact that the
storage of electricity is not economical greatly complicates the scheduling of power system operations in the presence of renewable resources.

1.2.1 Power Systems Scheduling

To appreciate the complications that renewable power supply disruptions cause, it is useful to understand the process of operating and balancing power system supply. The scheduling of the system is achieved in multiple stages: day-ahead decisions commit high-output, inflexible generation resources such as nuclear generators; hour-ahead scheduling responds to the circumstances that arise as the actual operating hour approaches by rescheduling flexible generators such as gas-fired units; and real-time scheduling and automatic control systems correct supply deviations in real time. In vertically integrated systems the system operator controls generation resources centrally, whereas in deregulated systems most balancing stages are cleared in markets that are operated by the system operator. Each market utilizes the latest information that is available. The scheduling of the California independent system operator (CAISO), which is representative of the operations of deregulated systems, is described in the 2007 CAISO report [23] and briefly summarized here. A more detailed description of the California system operator market operations can be found in the corresponding business practice manual [1].

- **Day-ahead unit commitment.** Load serving entities and power generators submit hourly bids in the day-ahead market. The day-ahead market closes at 10 a.m. the day before actual operation. A unit commitment algorithm determines the optimal dispatch and the system operator publishes instructions by 1 p.m. the same day. The result is an hourly dispatch schedule for generators with 20-minute ramps between hours.

- **Hour-ahead dispatch.** As the actual operating hour approaches, generators and loads adjust their positions to forecast errors or unanticipated events by bidding in the hour-ahead market. These bids are settled by economic dispatch and the results are published by the California system operator 75 minutes before the beginning of the operating hour. Like day-ahead schedules, hour-ahead schedules are also hourly blocks with intra-hour ramps.

- **Real-time dispatch** (also load following or supplemental energy dispatch). Within each operating hour the system operator continues to adjust generator operating points every 5 minutes. 7.5 minutes prior to the beginning of a 5-minute operating interval the system operator uses hour-ahead generation bids and load forecasts to re-adjust the operating point of each generator for that interval.

- **Regulation.** Every one minute the system operator adjusts the output of specific generators and/or loads based on reliability criteria. These generators and loads provide
ancillary services to the system. The ancillary services market is cleared hour-ahead. The actual dispatch of regulation resources is empirical rather than economic.

1.2.2 Reserve Requirements

Uncertainty in power system operations is commonly classified in discrete and continuous disturbances [73]. Discrete disturbances include generation and transmission line outages and require the commitment of contingency reserves. Contingency reserves include spinning reserve, online generators which can respond within a few seconds, and non-spinning, or replacement, reserve, which consists of offline generators that replace spinning reserve a few minutes after the occurrence of a contingency in order to restore the ability of the system to withstand a new contingency. Continuous disturbances most commonly result from stochastic fluctuations in electricity demand. The resulting imbalances require the utilization of operating reserves which, as in the case of contingency reserves, are classified according to their response speed. Regulation reserves are capable of responding within seconds in order to maintain system frequency, and load-following reserves are re-dispatched in the intra-hour time frame in order to balance larger scale disturbances that occur within the hour. Extensive references about reserve requirements and their interplay with large-scale renewable energy integration can be found in Mills and Wiser [56] and Billinton and Allan [10].

Reserve commitment rules have traditionally differentiated between operating and contingency reserves, and have worked effectively in practice for standard system operations. However, the large-scale integration of renewable power supply obscures the differentiation between operating reserves and contingency reserves and necessitates more sophisticated methods for dispatching and operating reserves.

1.2.3 Adverse Impacts of Renewable Energy Supply

The increase in power system operating costs and reserve capacity requirements is the result of various impacts caused by renewable power fluctuations that affect balancing operations in all time scales ranging from real-time control to day-ahead commitment.

- **Hour-ahead re-dispatch.** The unpredictability of renewable power supply may cause imbalances to the system that require expensive deviations from day-ahead dispatch schedules. Starting up units to compensate for a sudden shortage in renewable power supply may take hours, lead to additional air pollution, result in wear and the need for frequent maintenance of startup units, and upset system dispatch due to the minimum generation capacity of startup units. Similar problems are caused by shutting down units to balance an unanticipated increase in power supply.

- **Primary control, secondary control and ramping requirements.** The minute-by-minute variability of renewable resources may also cause system imbalances. This
variability imposes a requirement for primary control, generators that can rapidly adjust their power output in response to an unanticipated event. Moreover, secondary control units are necessary that can back up and dismiss primary control units. The inability to perfectly forecast renewable resources exacerbates this problem. Since renewable supply also tends to vary rapidly and in great magnitude, an additional backup of ramping generators is necessary.

- **Positive correlation with hydropower, negative correlation with load.** Various systems absorb large amounts of hydroelectric power. During the months that snow melts and hydroelectric power supply increases and must be absorbed, the additional generation of renewable energy causes an over-supply problem. In certain systems, including California, renewable supply increases during the night and abates during daytime. The negative correlation with power demand therefore makes it difficult to utilize the resource.

- **Intermittency at high wind speeds.** Wind generators shut down for mechanical protection when winds become very strong. Since wind generators supply a significant amount of power to a system during periods of high winds, there is an increased risk of substantial supply shortage during storms. This problem is exacerbated in large wind parks operating generators with identical cutoff speeds. Similar disruptions can occur in the supply of solar power as a result of clouding.

An additional concern that has been raised regarding the large-scale integration of renewable energy sources such as wind power is the effect on system inertial response. Although the increase in conventional synchronous generation can enhance the inertial response of a system, the increase in wind power generation does not necessarily contribute to system inertia. In scenarios of large-scale wind integration, the displacement of conventional generation may result in more frequent frequency excursions. This issue is not caused by the unpredictable and highly variable fluctuation of renewable power supply and not addressed in this thesis, however it is a major concern regarding the large-scale integration of wind power.

### 1.2.4 Cost, Utilization, Investment

Though the integration of renewable energy is increasing, an integration level beyond 20-30% is hardly perceived as economical (integration levels count 20% in Denmark with a target for 50% integration, 9% in Spain, 7% in Germany, and California is aiming for 33% by 2030). Assuming capital costs for renewable power will continue to decline in the future, one of the major challenges for the large scale integration of renewable energy will be its variability. Currently renewable generators operate under favorable regulations in many markets. A number of system operators in China, Europe (Denmark, Germany, Greece) and the United States (PJM, NYISO, CAISO, Ontario IMO) accept renewable generation on a priority basis
It is clear that this preferential treatment has its limitations. Large-scale renewable power integration cannot rely on regulatory support alone, but will also require technological innovations, such as the utilization of demand-side flexibility. Ultimately, the variable and unpredictable nature of renewable power supply limits large-scale integration for one of the three following reasons, which are the metrics that we focus on quantifying:

- increases in operating costs,
- increases in the amount of renewable energy that is discarded and, most importantly,
- capital investment requirements in generation capacity in order to operate the system reliably.

**Operating costs.** The operating cost impacts of renewable energy variability are captured by market tariffs and may be allocated to the whole market or directly to renewable generators, depending on market regulations. Research and experience indicate that integration costs range between 0 and 7$\text{/MWh}$ \cite{2,41}. Gross et al. \cite{37} place an estimate of no more than 5 British pounds for wind power integration. Another recent study conducted by Enernex for wind power integration in Minnesota \cite{100} concludes that the cost of additional reserves and costs related to variability and day-ahead forecast errors will result in an additional $2.11 (15\% penetration) to $4.41 (25\% penetration) per MWh of delivered wind power. In a similar vein, the renewable integration report of the California Independent System Operator (CAISO) \cite{53} has predicted an expected increase in 10-minute real-time energy prices due to wind forecasting errors that become comparable to load forecasting errors.

**Discarded power.** Renewable energy may be discarded during hours of excess renewable supply if power systems cannot reliably absorb this supply \cite{2,45}. During early spring the California system operator either spills water supplies from hydroelectric dams or discards renewable power \cite{53}. Renewable power is also discarded under normal operating conditions in California whenever forecasting underestimates the amount of renewable power supply to the system and the excess power cannot be sold. In Texas the system operator discards renewable power during load pick-up for reliability reasons \cite{90}.

**Capacity requirements.** Gross et al. \cite{37} have assembled a variety of renewable power integration studies with the objective of estimating the costs and impacts of intermittent generation on the U.K. electricity network. Over 80\% of the studies that the authors examined conclude that for renewable energy penetration levels above 20\% an investment in system backup in the range of 5-10\% of installed renewable capacity is required in order to balance the short term (seconds to tens of minutes) variability of renewable power supply. The authors conclude that additional conventional capacity to maintain system reliability
during demand peaks amounts to 15-22% of installed renewable power capacity. The California Independent System Operator renewable integration report analyzes the integration of 6700 MW of wind power in the California grid. According to the study the 3-hour morning ramp-up will increase by 926 MW to 1529 MW due to the fluctuations of wind generation at the time when morning demand increases, and the evening ramp-down will increase by 427 MW to 984 MW. The regulation capacity requirement will increase by 170 to 250 MW for upward regulation and by 400 to 500 MW for downward regulation. There will be an increase of 15 to 25 MW/min in upward ramping and downward ramping requirements. The upward and downward load-following ramps will increase by 30 to 40 MW/min.

1.3 Coupling Renewable Energy with Deferrable Demand

In the face of the aforementioned challenges that are introduced by the large-scale integration of renewable energy sources, the value of flexibility in electricity consumption becomes immediately apparent. A significant proportion of the energy that we consume across the residential, commercial and industrial sector is dedicated to duties that can be postponed for considerable amounts of time with minimal impact on comfort. Consumptions that naturally fit this description include agricultural pumping, heating and cooling, refrigeration, server farm operations and, most interestingly, electric vehicle charging. In fact, numerous flexible energy consumption tasks can be naturally described as requests for a certain amount of energy within a certain deadline. There is a natural complementarity between coupling renewable resources with deferrable requests, due to the fact that renewable power supply is more predictable over a certain time horizon than in any given moment in time, making it easier to fulfill requests that extend over a time window.

As we discuss in Section 7, an ideal approach for harvesting the benefits of demand flexibility would be the use of real-time pricing at the retail level. However, this is a radical paradigm shift and is unlikely to occur in the immediate future due to political opposition and the status quo of the industry. An alternative approach would be the provision of ancillary services by deferrable loads via an aggregator that would represent these loads in the wholesale market. An aggregator could bid on behalf of a population of loads for providing capacity services to the system operator. The aggregator would then be responsible for coordinating the aggregate consumption of loads by some price-based or direct control method. As ancillary services requirements are expected to increase due to renewable energy integration, this solution could prove lucrative for users who would be willing to respond to the instantaneous needs of power system operators. This approach is certainly appealing, however there are concerns about defining market products that correspond to the types of services that loads can actually offer, which raises the need for reform in existing electricity markets. As a result, it is probable that policy deliberations will delay the process of using
aggregators for managing significant populations of load.

An alternative approach for exploiting load flexibility that could possibly be integrated in existing markets and operational protocols is a direct coupling of renewable energy supply with deferrable demand. According to the proposed coupling, deferrable loads are paired up with large renewable suppliers that dedicate their supply to these loads. Consumers program tasks to be completed within a certain deadline. An aggregator is then responsible for serving these requests primarily from renewable resources, and to a limited extent from the real-time market. In studying this approach, we place a limit on the participation of the aggregator in the market during peak load periods, in order to transfer risk of the unreliability of renewable power supply from the system operator to the aggregator. Coupling closely matches dynamic scheduling, as described by Hirst and Kirby [12], whereby demand and supply resources from different control areas pair their schedules in order to produce a zero net output to the remaining system. Such scheduling is currently implemented in the Electric Reliability Council Of Texas (ERCOT). The proposed coupling also resembles the business model of electric vehicle service provider Better Place. The business plan of Better Place relies on ownership of electric vehicle batteries the charging of which can be managed by the company. Having the flexibility to charge the vehicle battery, electric vehicles can be charged when renewable energy is available, thus undercutting the cost of gasoline and thereby replacing gasoline with renewable power in the transportation sector [74].

The proposed coupling aligns the incentives of both contracting parties. Renewable resources appear "behind the meter", thus relieving the system operator from the obligation of procuring reserves for protecting against intermittent renewable supply. Although the coupled system may resort to the spot market to a limited extent, the coupling contract effectively transfers the risk of renewable power variability from the system operator to deferrable loads, which leads to economic savings by preventing the system operator from over-insuring against renewable power variability. The significant capital savings that stem from avoided investment in backup reserves can be captured through capacity credit, and a share of this credit can be allocated to deferrable loads in order to induce their participation in the contract.

The desired result of the proposed coupling is the minimization of net impact on the power grid. What makes this possible are the two degrees of freedom for aggregators: load flexibility and spot market participation. Aggregators can satisfy load requests upon availability of renewable power. If aggregators risk not meeting schedules they can in advance (after the load has been scheduled, but before due time) purchase power from the spot market without disturbing grid operations significantly. In case of excess renewable power supply, aggregators can also supply the excess resource in the spot market.

The fact that demand flexibility is most naturally characterized as requests for energy over a given time horizon renders traditional economic models for demand flexibility that are based on demand elasticity [90], [12], [48], [49] at a disadvantage. Demand elasticity cannot model the complexity that is entailed in a multi-stage scheduling of electricity demand. This also suggests that the participation of flexible demand in day-ahead markets via decremental
ask bids fails to capture the full potential of shifting demand over time, and that alternative mechanisms such as coupling are needed. We demonstrate the efficiency losses of decremental demand bids relative to coupling contracts in Chapter 7.

1.4 A Modeling Framework for Analyzing Renewable Energy Integration

In order to assess the economic impacts of large-scale renewable energy integration, it is necessary to account for the entire system, including the controllable resources of the system that are used for balancing renewable power supply, by formulating a unit commitment model. As we discuss in Section 1.2, the operation of power systems can be perceived as a multi-stage decision process under uncertainty, where resources are committed in advance of operations and decisions are updated as an operating interval approaches and conditions in the system reveal. In order to make our analysis computationally tractable, we model system operations as a two-stage decision process, following the work of Ruiz et al. [84].

The first stage of decision-making represents the day-ahead commitment of generators based on demand forecasts. Subsequently, uncertainty is revealed and in the second stage the commitment of fast units and the dispatch of all units is updated in order to respond to system conditions. In deregulated power systems, the first and second stage of the model can be thought of as simulating the day-ahead and real-time market respectively.

The randomness that is caused by renewable energy supply fluctuations suggest the use of a stochastic unit commitment model. As we discuss in Section 1.2.4, our analysis focuses on quantifying the impacts of renewable energy and demand response integration on operating costs, renewable energy utilization and generation capacity requirements. The stochastic unit commitment model quantifies these three metrics while accounting for uncertainty. Operating costs are quantified in the objective function of the problem, renewable energy utilization is an explicit decision variable in the problem and capacity requirements are quantified by the fact that the commitment of generators is also an endogenous decision variable in the problem.

The estimation of capacity requirements for operating a power system under uncertainty is an especially challenging aspect of our analysis. In order to commit reserves, system operators and analysts have often resorted to reserve requirements that protect the system against forecast errors and contingencies. These reserve requirements enter the deterministic formulation of the unit commitment problem and require the commitment of excess generation capacity that can protect the system in the face of unanticipated events. In contrast to deterministic unit commitment models, a stochastic unit commitment model endogenously determines the optimal amount of generation capacity that is required for operating the system under uncertainty and therefore yields an accurate assessment of reserve requirements.
1.5 Overview of the Thesis

Chapter 2 introduces the two stages of our analysis, scenario selection validation and the modeling of demand flexibility. The first stage, scenario selection validation, ensures that the stochastic unit commitment model that we use for our analysis can provide meaningful results by ensuring that it outperforms common deterministic reserve rules in the face of uncertainty. Upon validating our scenario selection methodology, we use the resulting stochastic unit commitment algorithm to quantify the economic impacts of renewable energy integration and demand flexibility.

The components of the model that are used for testing our scenario selection methodology are described in Chapters 3 to 6. In order to validate our scenario selection methodology we develop two competing unit commitment models, a deterministic model with exogenous reserve requirements and a stochastic model that explicitly models uncertainty and strives to optimize expected cost. These models, along with the economic dispatch model that is used for simulating their performance, are described in Chapter 3. In Chapters 4 and 5 we address the computational challenges that are introduced by stochastic unit commitment. The problem of scenario selection, the succinct representation of uncertainty in the stochastic model, is addressed in Chapter 4. Even after uncertainty has been reduced to a few representative and appropriately weighted scenarios, the problem at hand remains too large to solve by direct methods. A subgradient algorithm that exploits the decomposable structure of the problem is presented in Chapter 5. The statistical models that are used for the Monte Carlo simulation of the unit commitment policies are presented in Chapter 6.

Upon validating our scenario selection methodology, we proceed to the second stage of our analysis in Chapter 7, the modeling of demand flexibility. The results of our analysis are presented in Chapter 8. The conclusions of our research and the future directions of work that we wish to pursue are summarized in Chapter 9. Reviews of related work are provided at the beginning of each chapter.
Chapter 2

Overview of the Model

In Section 1.4 we motivate the use of stochastic unit commitment as the modeling framework for our analysis. However, in order to accept the results of the model as reasonable it is necessary to ensure that the resulting unit commitment policy outperforms common reserve rules. Ideally, this could be guaranteed by inputting all possible realizations of uncertainty in the model and deriving the unit commitment policy that optimizes the expected performance of the system. However, this approach is not computationally tractable. Instead, it is necessary to select a small number of representative outcomes that succinctly represent the sources of uncertainty in the system while maintaining the computational tractability of the model. The performance of the stochastic unit commitment policy depends crucially on the selection and weighing of these scenarios. Therefore, the first step of our analysis requires developing a methodical approach for selecting scenarios and a test for guaranteeing that the resulting unit commitment policy can outperform common reserve rules used in practice. The test that we employ is described in Section 2.1. Once we guarantee the validity of our scenario selection algorithm, we can utilize the stochastic unit commitment model for quantifying the benefits of coupling renewable energy supply with deferrable resources. This constitutes the second part of our analysis and is discussed in Section 2.2.

2.1 Validating the Stochastic Unit Commitment Policy

We present the scenario selection validation procedure in Figure 2.1. Each component of the model will be described in detail in the subsequent chapters. In this section we focus on the logic of the validation procedure.

We use historical data of wind power production and power demand, as well as typical failure rates of generators and transmission lines, to create a stochastic model that can be used for simulating uncertainty in the system. The stochastic model is used for generating a large number of samples that are filtered by a scenario selection algorithm. The selected scenarios are weighed appropriately by the scenario selection algorithm in order to appropriately
guide the stochastic unit commitment algorithm. The stochastic unit commitment algorithm uses the weighted scenarios to arrive at a schedule that minimizes expected operating costs. The deterministic unit commitment model commits generation according to exogenous reserve requirements that are commonly used in practice. These reserve requirements commit sufficient excess capacity in order to protect the system against unanticipated outcomes.

The deterministic and stochastic unit commitment models are two different approaches for arriving at the same decision: the schedule of slow generators that need to be committed in the day-ahead time frame. The performance of these first-stage decisions is then fixed and the performance of the day-ahead schedule is simulated by running an economic dispatch model against sample outcomes of the stochastic model. In the economic dispatch model, the system can only respond to realized outcomes by adapting the unit commitment schedules of fast resources and the production levels of all resources. The scenario selection algorithm is accepted so long as the average performance of the stochastic unit commitment schedule in terms of minimum load costs, startup costs and fuel costs in the economic dispatch simulations is superior to that of the deterministic unit commitment schedule.

### 2.2 Assessing the Benefits of Coupling

Once we establish a valid scenario selection algorithm, we can use stochastic unit commitment for assessing the three metrics that our analysis focuses on: operating costs, renewable energy
utilization and capacity requirements. We will consider three fundamental approaches for dealing with renewable energy supply variability via demand response:

- **Centralized load dispatch by the system operator:** According to this approach, the system operator centrally coordinates the dispatch of controllable generation resources and flexible loads in the system. This approach is an idealization that yields an estimate of the greatest possible benefits of demand flexibility. In real practice, the system operator cannot operate the system with such granularity and loads are not necessarily willing to surrender the control of their appliances to the system operator.

- **Coupling renewable generation with deferrable demand:** We introduce coupling contracts and the advantages that motivate their consideration in Section 1.3.

- **Price-elastic demand bids:** This is the common approach for treating demand flexibility in power systems operations and the power systems economics literature [90], [12], [48], [49]. The disadvantage of using demand functions for modeling deferrable demand is that demand is represented as a series of bids that are independent across time intervals, whereas in reality the feature that characterizes deferrable demand is dependency across time periods: demand that is satisfied in the current time interval will not recur and, conversely, demand that fails to be satisfied in the current time period will return as additional demand in future time periods.

Figure 2.2 outlines how we carry out our analysis. The decision support module in the upper part of the figure is used for determining the day-ahead schedule of generators while accounting for the flexibility of demand. The problem that is solved for this purpose is described in Section 7.1. The resulting day-ahead unit commitment is used for evaluating the performance of the three aforementioned demand response paradigms in the economic dispatch phase.

In the centralized dispatch model, the problem described in Section 7.1 is solved with the schedule of day-ahead units fixed according to the solution of the centralized stochastic unit commitment problem. Uncertain renewable power production and inflexible demand are random and drive the dispatch of controllable generation and flexible demand. In the case of coupling contracts, loads solve a multi-stage stochastic optimization whereby they seek to satisfy a fixed amount of energy demand within a given time horizon by resorting to freely available renewable energy and, to a limited extent, spot market procurements. The demand schedule of deferrable loads is driven by the available renewable power supply and the real-time price of electricity according to the optimal charging policy discussed in Section 7.3. The net demand of deferrable loads and the firm demand of inflexible loads are the random factors that drive the response of controllable generation in real-time operations. In the case of demand-side bidding, the economic dispatch model of the system operator includes decreasing demand bids in the objective function. The calibration of the demand functions is discussed in Section 7.2. Renewable power production and firm demand drive the dispatch
Figure 2.2: Comparing centralized load dispatch, coupling and demand-side bidding.
of flexible demand and controllable generation. In contrast to the case of centralized load dispatch, however, demand bids are not adequate to capture the inter-temporal complexity of load shifting.
Chapter 3

Unit Commitment and Economic Dispatch

The analysis of the economic impacts of large-scale renewable energy integration requires a detailed representation of the generation commitment and dispatch process. The unit commitment problem is the problem of scheduling generation for the following day in order to meet forecast demand in the system while accounting for numerous operating constraints on generators and the transmission network. It is a mixed integer linear program which is solved by system operators daily in order to both operate the system but also for the purpose of clearing the market. Once the schedules of slow-responding resources are fixed according to their day-ahead schedule, and as the actual operating interval approaches, the economic dispatch problem is solved. The purpose of economic dispatch is to adjust fast-responding resources such that the system responds, at least cost, to the prevailing system conditions which are inevitably different from what was forecast in the day-ahead time frame.

Solution techniques for the unit commitment problem have evolved over time. Simple priority rules were used originally, as discussed by Wood and Wollenberg [98]. Due to the fact that there is a loose coupling of generator operations via the market clearing constraint, one of the earliest approaches for improving the solution of the unit commitment problem was Lagrangian relaxation, which is described by Muckstadt and Koenig [60] and Bertsekas et al. [6]. Lagrangian relaxation was soon thereafter adopted by system operators in actual operations. Recently, advances in mixed integer programming have led to the gradual replacement of Lagrangian decomposition algorithms by branch and bound techniques for solving the unit commitment problem, as discussed by Carrion and Arroyo [23] and Streiffert et al. [92].

Recently, various studies have utilized unit commitment models for assessing the impacts of renewable energy integration. A variety of modeling approaches and solution strategies are employed in these studies. In Table 3.1, we compare the various studies with respect to some of their key modeling and computational features in order to place our work in the context of the existing literature. The renewable integration studies compared in Table 3.1
Table 3.1: A comparative listing of renewable integration studies based on unit commitment.

<table>
<thead>
<tr>
<th>Citation</th>
<th>Model</th>
<th>T.C.</th>
<th>Cont.</th>
<th>S.S.</th>
<th>Algo.</th>
<th>(S,G,T)</th>
<th>Time</th>
<th>Sim.</th>
</tr>
</thead>
<tbody>
<tr>
<td>[83]</td>
<td>2-st. SUC</td>
<td>No</td>
<td>Gen.</td>
<td>No</td>
<td>N/A</td>
<td>(6,60,48)</td>
<td>43</td>
<td>Step</td>
</tr>
<tr>
<td>[90]</td>
<td>Det.</td>
<td>No</td>
<td>No</td>
<td>N/A</td>
<td>N/A</td>
<td>(1,375,24)</td>
<td>N/A</td>
<td>Step</td>
</tr>
<tr>
<td>[96]</td>
<td>2-st. rob.</td>
<td>186</td>
<td>No</td>
<td>N/A</td>
<td>Benders</td>
<td>(10,76,24)</td>
<td>1800</td>
<td>Single</td>
</tr>
<tr>
<td>[25]</td>
<td>2-st. SUC</td>
<td>No</td>
<td>M.C.</td>
<td>N/A</td>
<td>(30,10,24)</td>
<td>540</td>
<td>Step</td>
<td></td>
</tr>
<tr>
<td>[95]</td>
<td>3-st. SUC</td>
<td>No</td>
<td>Gen.</td>
<td>N/A</td>
<td>(6,45,36)</td>
<td>79</td>
<td>Step</td>
<td></td>
</tr>
<tr>
<td>[97]</td>
<td>2-st. SUC</td>
<td>116</td>
<td>No</td>
<td>N/A</td>
<td>(20,9,24)</td>
<td>1600</td>
<td>Single</td>
<td></td>
</tr>
<tr>
<td>[24]</td>
<td>4-st. SUC</td>
<td>No</td>
<td>No</td>
<td>N/A</td>
<td>(2401,3,4)</td>
<td>39</td>
<td>Single</td>
<td></td>
</tr>
<tr>
<td>[73]</td>
<td>2-st. SUC</td>
<td>No</td>
<td>No</td>
<td>L.R.</td>
<td>(11,122,24)</td>
<td>5685</td>
<td>Types</td>
<td></td>
</tr>
<tr>
<td>[71]</td>
<td>2-st. SUC</td>
<td>375</td>
<td>Comp.</td>
<td>L.R.</td>
<td>(42,124,24)</td>
<td>2736</td>
<td>Types</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.1 differs according to the following features:

**Modeling approach (column 1):** Deterministic unit commitment (Det.) is the simplest possible approach for representing uncertainty in the model, where renewable energy supply is replaced by its forecast value for committing resources in the day ahead. Most authors use two- or multi-stage stochastic models (n-st. SUC). Wang et al. [96] use a robust two-stage formulation (2-st. rob.) that requires the system to prevent load shedding given first-stage commitment for all possible renewable supply realizations.

**Transmission constraints (column 2):** The inclusion of transmission constraints is crucial for accurately quantifying the impacts of renewable resource integration, as we demonstrate in Chapter 8. For those studies that include transmission constraints, the number of lines is listed in the second column.

**Contingencies (column 3):** The inclusion of contingencies also improves the accuracy of the analysis. Contingencies include generation capacity and transmission line failure. Most studies that include contingencies are limited to generation capacity loss (Gen.). To the best of our knowledge ours is the only renewable integration study that includes composite generation and transmission line failures (Comp.). Contingencies in this context refer to the explicit modeling of network element failures in the scenario tree.

**Scenario selection (column 4):** Some of the studies that use a stochastic model resort to scenario selection algorithms for reducing the size of the resulting model. Alternative approaches for controlling the run time of the studied models include relaxing the integrality of the commitment variables of fast-start units [83], grouping generators by fuel type [59] or forcing all generators to fix their schedules in the day-ahead commitment [25]. As we discuss in Chapter 4 the use of the scenario generation and scenario reduction algorithms of
Kuska et al. [38] and their extensions by Heitsch and Römisch [39] are commonly used in the stochastic unit commitment literature. An alternative approach employed by Constantinescu et al. [25] is the generation of Monte Carlo (M.C.) outcomes of wind power that are used for the estimation of lower and upper bound estimates on the optimal solution.

**Decomposition algorithms (column 5):** Stochastic formulations exhibit a decomposable structure that makes them amenable to decomposition algorithms. In our analysis we use a Lagrangian relaxation (L.R.) algorithm that is described in detail in Chapter 5. The only other study that utilizes a special-purpose algorithm is the work by Wang et al. [96] who use Benders’ decomposition [61, 5].

**Problem size (column 6):** The size of the problem \((S, G, T)\) is determined by the number of scenarios \(S\), the number of generators \(G\) and the horizon \(T\) of the problem. The number of binary decision variables in the model is \(S \cdot G \cdot T\) and therefore these three dimensions determine the computational requirements of the model. For the deterministic model of Sioshansi and Short [90] the scenario size is trivially equal to 1.

**Running time (column 7, in seconds):** Running times are reported for most studies. These running times depend both on the size of the problems at hand as well as on the special-purpose algorithms that are used for solving the problems.

**Simulation method (column 8):** A common approach for simulating the impact of renewable power integration is a full-year simulation with one-day steps (Step), where the unit commitment problem is solved for each day and then economic dispatch is performed against a Monte Carlo sample of the uncertain parameters. An alternative approach that we adopt in our study in order to reduce the computational demand of the model is to focus on representative day types (Types), and weigh the results of each day type according to the frequency of occurrence of that day type. In the analysis of Chapter 8 we consider eight day types, one for each season and further differentiating between weekdays and weekends. Some studies consider a single hour (Single) of system operations.

### 3.1 Notation

Before presenting the unit commitment and economic dispatch models, we introduce the notation that is used in the models.

**Sets**

- \(G\): set of all generators
- \(G_s\): subset of slow generators
- \(G_f\): subset of fast generators
$S$: set of scenarios
$T$: set of time periods
$L$: set of lines
$N$: set of nodes
$G_n$: set of generators that are located in bus $n$

$$LI_n = \{l \in L : l = (k, n), k \in N\}$$

$$LO_n = \{l \in L : l = (n, k), k \in N\}$$

$IG$: set of import groups

$IG_j$: set of lines in import group $j$

**Decision variables**

- $u_{gst}$: commitment of generator $g$ in scenario $s$, period $t$
- $v_{gst}$: startup of generator $g$ in scenario $s$, period $t$
- $p_{gst}$: production of generator $g$ in scenario $s$, period $t$
- $\theta_{nst}$: phase angle at bus $n$ in scenario $s$, period $t$
- $w_{gt}$: commitment of slow generator $g$ in period $t$
- $z_{gt}$: startup of slow generator $g$ in period $t$
- $s_{gt}$: slow reserve provided by generator $g$ in period $t$
- $f_{gt}$: fast reserve provided by generator $g$ in period $t$
- $e_{lst}$: power flow on line $l$ in scenario $s$, period $t$

**Parameters**

- $\pi_s$: probability of scenario $s$
- $K_g$: minimum load cost of generator $g$
- $S_g$: startup cost of generator $g$
- $C_g$: marginal cost of generator $g$
- $D_{nst}$: demand in bus $n$, scenario $s$, period $t$
- $P_{gst}^+, P_{gst}^-$: minimum and maximum capacity of generator $g$ in scenario $s$
- $R_g^+, R_g^-$: minimum and maximum ramping of generator $g$
- $UT_g$: minimum up time of generator $g$
- $DT_g$: minimum down time of generator $g$
- $N$: number of periods in horizon
- $T_{req}$: total reserve requirement in period $t$
- $F_{req}$: fast reserve requirement in period $t$
- $B_{ls}$: susceptance of line $l$ in scenario $s$
- $TC_l$: maximum capacity of line $l$
- $FR_g$: fast reserve limit of generator $g$
- $IC_j$: maximum capacity of import group $j$
- $\gamma_{jl}$: polarity of line $l$ in import group $j$

The binary variables $w_{gt}$ indicate whether a generator is turned on or off during a specific time period over the planning horizon of the unit commitment problem. If a generator is committed, the unit commitment problem specifies the level of output $p_{gt}$ of the generator. The set of generators is partitioned into a set of slow resources $G_s$ and fast resources $G_f$. Each generator $g$ is associated with a set of time periods $T$ and a set of scenarios $S$. The decision variables include the commitment $u_{gst}$, startup $v_{gst}$, production $p_{gst}$, and phase angle $\theta_{nst}$ of the generator. The parameters include the probability of scenario $\pi_s$, minimum load cost $K_g$, startup cost $S_g$, marginal cost $C_g$, demand $D_{nst}$, minimum and maximum capacity $P_{gst}^+, P_{gst}^-$, minimum and maximum ramping $R_g^+, R_g^-$, minimum up time $UT_g$, minimum down time $DT_g$, total reserve requirement $T_{req}$, and fast reserve requirement $F_{req}$. The susceptance $B_{ls}$, maximum capacity $TC_l$, fast reserve limit $FR_g$, and maximum capacity of import group $IC_j$ are also included. The polarity $\gamma_{jl}$ of each line $l$ is determined by the import group $j$.
$G_f$. In the deterministic formulation, both types of generators can provide slow reserve $s_{gt}$, whereas only fast resources can provide fast reserve $f_{gt}$. In the stochastic unit commitment formulation, slow resources are committed in the first stage, whereas fast resources are sufficiently flexible to be committed in the second stage.

As we discuss in Section 1.2.2, uncertainty in power system operations is caused either by errors in the forecast of renewable power or demand in the system, or by the failure of transmission lines and generators. There are two fundamental approaches in order to ensure the commitment of sufficient resources in the day ahead that can protect the system against uncertainty. In the deterministic unit commitment formulation, the approach is to impose redundancy by forcing the commitment of excess capacity beyond what is required for meeting forecast load, as well as to limit certain power flows in order to ensure that the dispatch of the system can withstand the failure of critical elements. Alternatively, the stochastic unit commitment model represents uncertainty explicitly and produces a unit commitment policy that can attain minimum expected cost over a wide range of outcomes.

### 3.2 Deterministic Unit Commitment

The formulation of the deterministic unit commitment problem follows Sioshansi and Short [90].

\[
\text{(DUC)} : \min \sum_{g \in G} \sum_{t \in T} (K_g w_{gt} + S_g z_{gt} + C_g p_{gt})
\]

s.t.
\[
\sum_{l \in L} e_{lt} + \sum_{g \in G} p_{gt} = D_{nt} + \sum_{l \in L} e_{lt}, n \in N, t \in T
\]

\[
e_{lt} \leq TC_l, l \in L, t \in T
\]

\[
-TC_l \leq e_{lt}, l \in L, t \in T
\]

\[
e_{lt} = B_l(\theta_{nt} - \theta_{mt}), l = (m, n) \in L, t \in T
\]

\[
p_{gt} + f_{gt} \leq P_g^+ w_{gt}, g \in G, t \in T
\]

\[
p_{gt} + s_{gt} + f_{gt} \leq P_g^+ g \in G, t \in T
\]

\[
p_{gt} \geq P_g^- w_{gt}, g \in G, t \in T
\]

\[
p_{gt} - p_{g,t-1} + s_{gt} \leq R_g^+, g \in G, t \in T
\]

\[
p_{g,t-1} - p_{gt} \leq R_g^-, g \in G, t \in T
\]

\[
\sum_{g \in G_f} f_{gt} + \sum_{g \in G} s_{gt} \geq T_t^{req}, t \in T
\]

\[
f_{gt} \leq FR_g, g \in G, t \in T
\]
The objective of the unit commitment problem is to minimize system costs, which consist of fuel costs, minimum load costs that are incurred whenever a generator is online and startup costs that are incurred whenever a generator is turned on. Load shedding is modeled as a generator with marginal cost equal to the value of lost load. Imports and controllable renewable resources can be used at no cost by the system operator, meaning that they can be used at zero marginal cost but also discarded in case of excess supply. The market clearing constraint of Equation (3.2) requires that the amount of power that is injected and produced in each bus equals the amount of power consumed and exported from the bus. The transmission capacity constraints of Equations (3.3) and (3.4) limit the amount of power that can flow over a line, either due to thermal constraints on the lines or for maintaining the angle difference between buses in a stable region. Equation (3.5) is a linearization of Kirchoff’s voltage and current laws. Due to the fact that our model includes line contingencies, we use line susceptances instead of power transfer distribution factors (PTDFs) for modeling the direct current (DC) power flow equations since the former are independent of topology and only depend on the electrical characteristics of the lines. The generation capacity constraints of Equations (3.6) and (3.7) limit the amount of power and reserves that can be supplied by a generator in both the on and off state. The production of a generator is limited by the amount of capacity that is not committed for providing reserve. If a generator is off it can only provide fast reserve. Constraint (3.12) limits the amount of fast reserves that can be provided by generators, since only fast resources can provide fast reserves. The minimum run constraint of Equation (3.8) imposes a minimum output level for each generator that is operational. The total reserve requirement of Equation (3.11) commits a minimum amount of excess capacity that is readily available to the system operator from both fast and slow resources. At least a certain amount of this total reserve capacity needs to be available from fast.
resources according to the fast reserve requirement in Equation (3.13). In the same spirit of imposing redundancy in the system in order to ensure that it can withstand deviations from forecasts, the import constraints of Equation (3.14) impose a limit on the amount of power that can flow on certain groups of lines in order to protect against generator and line contingencies. Import constraints can be categorized in two types. "Bubble" constraints limit the total amount of power that can flow into a load pocket in order to ensure that the unit commitment schedule reserves sufficient transfer capability on the lines in order to protect against the possibility of generation capacity failure within the load pocket. On the other hand, inter-tie constraints limit the amount of power that can flow over inter-ties in order to protect the system against the failure of major corridors that bring significant amounts of power from out-of-state. Both reserve requirements as well as import constraints are imposed in an ad hoc fashion by system operators, based on experience rather than some methodical approach.

The ramping constraints of Equations (3.9) and (3.10) limit the rate at which the output of a generator can change. These ramping constraints are especially relevant to our analysis due to the fact that renewable resource production tends to vary rapidly and therefore imposes excessive ramping requirements on the system.

The minimum up time constraint of Equation (3.15) requires that once a generator has been turned on it needs to stay on for at least a certain number of hours. An analogous constraint for minimum down time is imposed by Equation (3.16). Minimum up and down times are modeled following O’Neill et al. [63]. Note that the integrality of the startup variables $z_{gst}$ can be relaxed in order to reduce the size of the resulting branch and bound tree, which reduces computation time. Equation (3.17) is necessary for relaxing the integrality of the startup variables. Equation (3.18) models the state transition of the startup variables.

### 3.3 Stochastic Unit Commitment

The stochastic unit commitment problem can be stated as follows:

$$ (SUC) : \min \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) $$

s.t.

$$ \sum_{l \in LI_n} e_{lst} + \sum_{g \in G_n} p_{gst} = D_{nst} + \sum_{l \in LO_n} e_{lst}, n \in N, s \in S, t \in T $$

$$ e_{lst} = B_{ls}(\theta_{nst} - \theta_{mst}), l = (m, n) \in L, s \in S, t \in T $$

$$ e_{lst} \leq TC_{l}, l \in L, s \in S, t \in T $$

$$ -TC_{l} \leq e_{lst}, l \in L, s \in S, t \in T $$

$$ p_{gst} \leq P_{gs}^+ u_{gst}, g \in G, s \in S, t \in T $$
\[ P_{gst} u_{gst} \leq p_{gst}, g \in G, s \in S, t \in T \]  
\[ p_{gst} - p_{gst-1} \leq R_g^+, g \in G, s \in S, t \in T \]  
\[ p_{gst-1} - p_{gst} \leq R_g^-, g \in G, s \in S, t \in T \]  
\[ \sum_{q=t-UT_g+1}^{t+DT_g} z_{gq} \leq w_{gt}, g \in G, t \geq UT_g \]  
\[ \sum_{q=t+1}^{t+DT_g} z_{gq} \leq 1 - w_{gt}, g \in G, t \leq N - DT_g \]  
\[ \sum_{q=t-UT_s+1}^{t+DT_s} v_{gsq} \leq u_{gst}, g \in G, s \in S, t \geq UT_g \]  
\[ \sum_{q=t+1}^{t+DT_g} v_{gsq} \leq 1 - u_{gst}, g \in G, s \in S, t \leq N - DT_g \]  
\[ z_{gt} \leq 1, g \in G, s \in S, t \in T \]  
\[ v_{gst} \leq 1, g \in G, s \in S, t \in T \]  
\[ z_{gt} \geq w_{gt} - w_{g,t-1}, g \in G, s \in S, t \in T \]  
\[ v_{gst} \geq u_{gst} - u_{gst,t-1}, g \in G, s \in S, t \in T \]  
\[ \pi_s u_{gst} = \pi_s w_{gt}, g \in G, s \in S, t \in T \]  
\[ \pi_s v_{gst} = \pi_s z_{gt}, g \in G, s \in S, t \in T \]  
\[ p_{gst}, v_{gst} \geq 0, u_{gst} \in \{0, 1\}, g \in G, s \in S, t \in T \]  
\[ z_{gt} \geq 0, w_{gt} \in \{0, 1\}, g \in G, s \in S, t \in T. \]  

The additional feature of the model compared to the deterministic unit commitment formulation is the explicit consideration of a set of scenarios, each occurring with a probability \( \pi_s \). The objective of the problem now becomes the minimization of expected operating costs across all scenarios. Slow resources are committed in the day ahead, with \( w_{gt} \) representing their commitment. This is the first-stage decision of the problem. In contrast, fast resources can be committed in the second stage of the problem according to the realized uncertainty, hence the fast resource commitment variables are indexed by scenario, \( u_{gst} \). Additionally, the production schedule \( p_{gst} \) of all resources can be revisited in the second stage and is therefore also indexed by scenario.

Uncertainty enters the model through net load uncertainty and contingencies. Net load uncertainty, that results from uncertainty in demand and the production of renewable resources, appears in the market-clearing constraint (3.21) by indexing net demand \( D_{nst} \) by scenario. Transmission line contingencies are modeled by indexing the the susceptance of each line by scenario, where \( B_{ls} = 0 \) for the scenarios \( s \in S \) in which line \( l \) is out of order, thereby forcing the flow in line \( l \) to equal zero. Generator contingencies are modeled in the
minimum and maximum generator capacity limits of Equations (3.25) and (3.26). Similarly to the case of transmission line failures, $P_{gs}^+ = 0$ and $P_{gs}^- = 0$ holds for those scenarios $s \in S$ in which generator $g$ fails, thereby forcing power production for generator $g$ to equal zero.

The non-anticipativity constraints of Equations (3.37) and (3.38) force the second-stage commitment of slow generators for each scenario to be consistent with the first-stage commitment. Note that we introduce redundant second-stage unit commitment and startup decisions $u_{gst}, v_{gst}$ for slow generators. As we explain in Chapter 5, this is necessary for the derivation of the decomposition algorithm.

### 3.4 Economic Dispatch

The deterministic and stochastic unit commitment models in the previous two sections represent two different methods for committing slow generation resources in the day-ahead scheduling timeframe. Once slow resources are committed, the performance of the system is tested by performing Monte Carlo simulations of its response to net demand and contingency outcomes, given the unit commitment schedule of slow generators. The economic dispatch of units for each outcome $c$ requires solving the following problem:

$$
(ED_c) : \min \sum_{g \in G} \sum_{t \in T} (K_g w_{gt} + S_g z_{gt} + C_g p_{gt}) \tag{3.41}
$$

s.t.

$$
\sum_{l \in L_{I_n}} e_{lt} + \sum_{g \in G_n} p_{gt} = D_{nct} + \sum_{l \in L_{O_n}} e_{lt}, \, n \in N, \, t \in T \tag{3.42}
$$

$$
e_{lt} = B_{lt} (\theta_{nt} - \theta_{mt}), \, l = (m, n) \in L, \, t \in T \tag{3.43}
$$

$$
p_{gt} \leq P_{g+}^{u_{gt}}, \, g \in G, \, t \in T \tag{3.44}
$$

$$
p_{gt} \geq P_{g-}^{u_{gt}}, \, g \in G, \, t \in T \tag{3.45}
$$

$$
p_{gt} - p_{g,t-1} \leq R_g^+, \, g \in G, \, t \in T \tag{3.46}
$$

$$
w_{gt} = w_{gt}^{*}, \, g \in G_s, \, t \in T \tag{3.47}
$$

$$
z_{gt} = z_{gt}^{*}, \, g \in G_s, \, t \in T \tag{3.48}
$$

$$
p_{gt}, z_{gt}, \geq 0, \, w_{gt} \in \{0, 1\}, \, g \in G, \, t \in T \tag{3.49}
$$

The economic dispatch problem closely follows the formulation of $(DUC)$ in Equations (3.1) - (3.19). In contrast to $(DUC)$, reserve decision variables $s_{gt}, f_{gt}$ and reserve requirement constraints do not enter the formulation and the commitment for slow generators is fixed in Equations (3.47) and (3.48).
Chapter 4

Scenario Selection

The challenge of selecting scenarios for the stochastic unit commitment problem is to discover a small number of representative daily net load time series and contingencies that properly guide the stochastic program to produce a unit commitment schedule that improves average costs, as compared to a unit commitment schedule determined by solving a deterministic model.

The basic tradeoff that needs to be balanced in dispatching fast reserves is the flexibility that fast units offer in utilizing renewable generation versus their higher operating costs. Fast generators are fueled by gas, which has a relatively high marginal cost. In addition, the startup and minimum load costs of these units are similar to those of slow units, however their capacity is smaller; hence, their startup and minimum load cost per unit of capacity is greater than that of slow generators. The advantage of largely relying on fast units is that the system is capable of reducing discarded renewable power, which results in significant savings in fuel costs. Unlike fast generators which can shut down with short notice in the case of increased renewable power generation, slow generators cannot back down from their minimum generation levels and therefore result in the waste of excess renewable energy.

The introduction of transmission constraints complicates scenario selection considerably due to the fact that fast reserves are not readily accessible when certain transmission lines are congested and the availability of renewable resources may be limited due to transmission constraints. The further introduction of composite outages introduces the complication of protecting the system against very low probability outcomes that can severely impact system reliability. There have been numerous publications that demonstrate the complex influence of transmission constraints on locational reserve requirements. Arroyo and Galiana [8] demonstrate that an ad hoc allocation of reserves in various locations of a transmission-constrained model results in suboptimal system performance. The complex influence of transmission constraints on locational reserve requirements is also demonstrated by Galiana et al. [33] as well as Bouffard et al. [15].

The stochastic unit commitment literature has relied extensively on the scenario selection and scenario reduction algorithms proposed by Dupacova et al. [30] and their faster variants.
that were proposed by Heitsch et al. [39]. The effectiveness of these algorithms in the stochastic unit commitment problem was first demonstrated by Gröwe-Kuska et al. [38] who apply the algorithms for scheduling hydro and thermal units in a German utility.

The original scenario selection and scenario reduction algorithms proposed by Dupacova et al. [30] focus on two- and multi-stage stochastic programming with convex feasible regions for the first-stage decisions. The authors are motivated by a stability result on the optimal value of a stochastic program with respect to perturbations in the probability measure of the underlying probability space. The authors bound the distance among any two probability measures by the Kantorovitch functional on the two measures, and use the stability result to select scenarios by minimizing the following Kantorovich functional:

$$\hat{\mu}(P, Q) = \inf \left\{ \int_{\Omega \times \Omega} \| \omega - \tilde{\omega} \|^2 \eta(d(\omega, \tilde{\omega})) : \eta(B \times \Omega) = P(B), \eta(\Omega \times B) = Q(B), \forall B \in \mathcal{B} \right\}$$

where $(\Omega, \mathcal{B}, P)$ is the probability space of the stochastic programming formulation and the minimization is over $\eta$. For discrete probability measures $P$, $Q$, calculating the distance $\hat{\mu}(P, Q)$ requires solving a linear program. However, for a given original measure $P$ the search for a measure $Q$ that minimizes the Kantorovich functional becomes a combinatorial problem. The authors provide efficient scenario reduction and scenario selection algorithms for approximate solutions to the problem. The scenario reduction algorithm proceeds by removing scenarios from the set $\Omega$ and redistributing mass in order to arrive at a pre-specified number of desired scenarios. The scenario selection algorithm proceeds by adding scenarios to an originally empty set of scenarios in order to arrive at a desired number of selected scenarios.

In the scenario reduction algorithm, the scenario to be removed is the one closest to all scenarios remaining in the scenario set. The mass of the removed scenario is allocated to its nearest neighbor that remains in the scenario set. Morales et al. [59] propose a variant of this scenario reduction algorithm that removes the scenario that causes the least change in the second stage costs of the optimization problem.

We proceed by presenting a scenario selection algorithm for systems exposed to net load uncertainty that can be caused by renewable production uncertainty or demand uncertainty. We then consider the more complex situation of a transmission-constrained system with transmission line and generator failures, and propose a scenario selection algorithm that formalizes the intuition that is used in the scenario selection algorithm of Section 4.1.

1Note that stochastic unit commitment does not belong to this family of problems since it is a mixed integer linear program. Nevertheless, the proposed scenario selection algorithms are shown to attain satisfactory performance in the case study of Gröwe-Kuska et al. [38].
4.1 Scenario Selection in the Absence of Contingencies

In Section 8.2 we study a unit commitment model without transmission constraints and contingencies, where wind power production is the only stochastic input. Despite the theoretical justification of the scenario reduction algorithm presented by Dupacova et al. [30] and Heitsch et al. [39], the algorithm is not guaranteed to preserve the moments of hourly wind generation. Due to the predominant role of fuel costs in the operation of the system, the accurate representation of average wind supply in the case of large-scale wind integration is crucial for properly guiding the weighing of scenarios. Moreover, the modeler cannot specify certain scenarios which are deemed crucial. For example, in the case of wind integration, the realization of minimum possible wind output throughout the entire day needs to be considered explicitly as a scenario. Otherwise, there is the possibility of under-committing resources and incurring overwhelming costs from load shedding in economic dispatch.

In order to overcome the drawbacks that arise from implementing the scenario reduction algorithm of Dupacova et al. [30] and Heitsch et al. [39], we generate a large number of samples from the statistical model of the underlying process and select a subset of samples based on a set of prescribed criteria that are deemed important. We then assign weights to each scenario such that the first moments of hourly wind output are matched as closely as possible. Scenario selection algorithms that match certain statistical properties of the underlying process have been proposed in previous literature, e.g. by Hoyland and Wallace [44]. Our proposed algorithm proceeds as follows:

**Step (a).** Define the size $N$ of the reduced scenario set $\hat{\Omega} = \{\omega_1, \ldots, \omega_N\}$, where $\Omega \subset \mathbb{R}^T$ is the sample space and $T$ is the planning horizon of the unit commitment problem.

**Step (b).** Generate a sample set $\Omega_S \subset \Omega$ that is adequately large. The appropriate sample size can be chosen such that $\sum_{\omega \in \Omega_S} \frac{C_D(\omega)}{|\Omega_S|}$ converges, where $C_D(\cdot)$ is the cost of a commonly used deterministic unit commitment policy that can be derived from the solution of $(DUC)$, Equations (3.1) - (3.19).

**Step (c).** Define a set of criteria that are deemed important for the scenario set. Select the set of scenarios $\hat{\Omega} \subset \Omega_S$ that best satisfy these criteria.

**Step (d).** Weigh the selected scenarios such that hourly moments are matched as closely as possible:

$$\min_{\pi_s, s \in \{1, \ldots, N\}} \sum_{t \in T} \left( \sum_{s \in S} \pi_s \omega_t^s - \mu_t \right)^2$$

s.t.

$$\sum_{s \in S} \pi_s = 1$$

$$\pi_s \geq 0.01,$$
where $\mu_t$ is the mean of the process for period $t$. The lower bound for the probabilities is included in order to ensure that all scenarios are considered, albeit with a small weight.

In the results that we present in Section 8.2 we use the following eleven criteria in step (c) for selecting wind power production scenarios: the scenario closest to the sampled mean; the scenario resulting in net load with the greatest variance; the scenario resulting in net load with the least variance; the scenario resulting in net load with the greatest morning up-ramp; the scenario resulting in net load with the greatest evening up-ramp; the scenario resulting in net load with the greatest sum of hourly absolute differences; the scenario resulting in net load with the greatest min-to-max within the day; the scenario with the least aggregate wind output throughout the day; the scenario with the greatest aggregate wind output throughout the day; the scenario resulting in the greatest observed net load peak; and the scenario resulting in net load with the greatest observed change within one hour.

4.2 Scenario Selection in the Presence of Contingencies

The introduction of transmission constraints and contingencies to the model complicates the task of scenario selection significantly. Transmission constraints necessitate the consideration of locational renewable energy production in each renewable energy site of the network. The scenario selection algorithms of Dupacova et al. [30] and the scenario selection algorithm of Section 4.1 have been applied to systems that are not constrained by transmission, with the sample space $\Omega$ typically representing a single stochastic process rather than correlated processes in multiple regions or contingencies.

Assuming independence among net load outcomes and contingencies, which is a very reasonable assumption, one natural approach for generating scenarios would be to decouple the selection of contingencies from the selection of net load scenarios. The previously discussed algorithms could then be adapted for selecting multi-area net load scenarios, and these scenarios could be multiplexed with a set of significant contingencies. One natural question that this approach raises is which contingencies to select and how to weigh them relative to each other. For example, the failure of any given generator has a likelihood of 1%, however in a network with 124 generators the chances of a single-generator failure are approximately 36.0%. The question arises, then, if a scenario includes the failure of a single generator, how should that scenario be weighed against other scenarios? Which generator failures should we include in the scenario set? Should we consider composite failures, for example the failure of multiple generators, multiple lines, or generators and lines in the same scenario? It quickly becomes clear that a methodical approach for scenario selection is needed.
4.2.1 Importance Sampling

The scenario selection algorithm that we propose in this section is inspired by importance sampling. Importance sampling is a statistical technique for reducing the number of Monte Carlo simulations that are required for estimating the expected value of a random variable within a certain accuracy. For an exposition see Mazumdar [55] and Infanger [46]. As Pereira and Balu [73] report, this technique has been used in reliability analysis in power systems with composite generation and transmission line failures, where the estimated random variable is a reliability metric (e.g. loss of load probability or expected load not served).

Given a sample space Ω and a measure p on this space, importance sampling defines a measure q on the space that reduces the variance of the observed samples of the random variable C, and weighs each simulated outcome ω by \( p(\omega)/q(\omega) \) in order to un-bias the simulation results. The measure q is ideally chosen such that it represents the contribution of a certain outcome to the expected value that is being computed, i.e.

\[
q^*(\omega) = \frac{p(\omega)C(\omega)}{\mathbb{E}_pC} \tag{4.4}
\]

Of course, it is not possible to determine this measure since \( \mathbb{E}_pC \) is the quantity we wish to compute. Nevertheless, the intuition of selecting samples according to their contribution to the expected value can be carried over to scenario selection. For example, in the algorithm of Section 4.1 we include the wind power production outcome with the lowest aggregate production over the entire day. Although the likelihood of this outcome is very low, its impact on system costs can be extremely high, making its contribution to expected cost \( p(\omega)C(\omega) \) significant.

4.2.2 Proposed Algorithms

The extension of the intuition of importance sampling to the case of scenario selection is straightforward: if the ideal measure \( q^* \) of Equation (4.4) were closely approximated by a measure q, then selecting a small number of outcomes according to this measure and weighing them according to \( p(\omega)/q(\omega) \) would provide an accurate estimate of the expected cost. Therefore, samples selected according to q can be interpreted as representative scenarios that need to be weighted according to \( p(\omega)/q(\omega) \) relative to each other in the stochastic unit commitment formulation in order not to bias the result.

As in the case of the scenario selection algorithm of Section 4.1, we proceed by generating an adequately large subset of the sample space \( \Omega_s = \{\omega^1, \ldots, \omega^M\} \) and we calculate the cost of each sample against a deterministic unit commitment policy \( C_D(\cdot) \). Assuming \( \bar{C} = \sum_{i=1}^M \frac{C_D(\omega_i)}{M} \) provides an accurate estimate of expected cost, we interpret the sample space of the system as \( \Omega_s \) and the measure as the uniform distribution over \( \Omega_s \), hence \( p(\omega) = M^{-1} \) for all \( \omega \in \Omega_s \). We then obtain \( q(\omega_i) = C_D(\omega_i)/\bar{C}, i = 1, \ldots, M \) and each selected scenario
is weighed according to $\pi_s = p(\omega)/q(\omega)$, hence $\pi_s/\pi_{s'} = C_D(\omega^s)/C_D(\omega^s)$ for each pair of selected scenarios $\omega^s, \omega^{s'} \in \Omega$. Hence, the proposed algorithm selects scenarios with a likelihood that is proportional to their cost impact, and discounts these scenarios in the stochastic unit commitment formulation in proportion to their cost impact in order not to bias the stochastic unit commitment policy. We therefore propose the following algorithm:

1. **Step (a).** Define the size $N$ of the reduced scenario set $\hat{\Omega} = \{\omega^1, \ldots, \omega^N\}$.
2. **Step (b).** Generate a sample set $\Omega_S \subset \Omega$, where $M = |\Omega_S|$ is adequately large. Calculate the cost $C_D(\omega)$ of each sample $\omega \in \Omega_S$ against a deterministic unit commitment policy and compute the average cost $\bar{C} = \frac{1}{M} \sum_{i=1}^{M} C_D(\omega_i)$.
3. **Step (c).** Choose $N$ scenarios from $\Omega_S$, where the probability of picking a scenario $\omega$ is $C_D(\omega)/\bar{C}$.
4. **Step (d).** Set $\pi_s = C_D(\omega)^{-1}$ for all $\omega^s \in \hat{\Omega}$.

Motivated by the fact that the cost for each scenario is largely dependent on the type of contingency that occurs (e.g. single-generator failure, 2-generator failure and so on) and, to a lesser extent dependent on the net load realization, we can first select the types of contingencies that are deemed most important and subsequently nest the previously proposed method for selecting net load realizations for each type of contingency.

### 4.2.3 Discussion

The proposed algorithm ensures that, as long as the cost impacts of all selected scenarios are of the same order of magnitude, which is commonly the case, then so are the weights in the stochastic unit commitment formulation. As a result, each scenario influences the first-stage decisions. This should be contrasted to the case where the probabilities of certain scenarios are very small compared to the probabilities of other scenarios. In that case, as we can see in Equation (5.9), the influence of scenarios with very small probability is minimal in the objective function of (P2), and consequently these scenarios will not tend to influence the optimal solution of (P2) and therefore the first-stage decision, i.e. the commitment of slow generators. In addition, from Equations (5.11) and (5.12) we see that scenarios with small probability exhibit very small changes in the values of their dual multipliers, which further supports the argument that these scenarios do not influence (P2). In fact, we observed that the stochastic unit commitment policy remained completely unaffected by scenarios that were 100 times less likely to occur than their competing scenarios in the stochastic unit commitment formulation. Therefore, the inclusion of these scenarios introduced superfluous computational load. This is guaranteed not to happen with the proposed scenario selection algorithm.

The rationale of the proposed scenario selection method suggests a heuristic for deciding
on the appropriate number of scenarios to include in the stochastic unit commitment formu-
lation. Namely, the modeler should select a certain confidence interval and select $N$ to be
such that the moving average $\bar{C}_n = \frac{\sum_{i=1}^{n} C_D(\omega_i)}{n}$ lies within the interval.

An additional appealing feature of the proposed algorithm is that it selects a rich set
of multi-area net load outcomes. This should be contrasted to the case where we would
multiplex net load scenarios selected according to the previously discussed algorithms [30],
[38], [73] with contingency scenarios. Moreover, in contrast to the scenario selection method
proposed in Section 4.1 the proposed algorithm does not depend on the judgement of the
modeler for specifying criteria that are deemed important. The proposed procedure can be
applied to a broad setting of problems under uncertainty in a straightforward fashion. In
Section 8.3 it is shown that the resulting stochastic unit commitment policy outperforms
common deterministic rules for four case studies of uncertainty.
Chapter 5

Decomposition of the Stochastic Unit Commitment Problem

The stochastic formulation of the unit commitment problem was pioneered by Takriti et al. in 1996 [93]. The authors formulated a multi-stage stochastic unit commitment problem for generating a robust unit commitment schedule that can withstand load forecast errors and generator failures at an acceptable cost. Numerous publications adopted Takriti’s formulation for studying the operation of power systems under uncertainty. As we discuss in Chapter 3, interest in the stochastic unit commitment problem has recently been revived due to its relevance to renewable energy integration studies.

The majority of researchers that consider stochastic unit commitment problems of considerable size cannot attack the problem directly by the branch and bound algorithm, but instead exploit the decomposable structure of the problem across scenarios, generators and time periods. The literature presents a variety of decomposition schemes and algorithms for optimizing the dual function, nevertheless the key idea of formulating and decomposing the dual is common across all works. We summarize a variety of publications in this area.

Takriti et al. [93] use the progressive hedging algorithm of Rockafellar and Wets [82] to decompose the problem across time periods. The authors apply the model to the Michigan Electric Power Coordinating Center. The model includes one hydro facility and more than one hundred thermal units. The authors solve the problem for 22 scenarios.

The work of Takriti et al. was soon followed by the multi-stage model of Carpentier et al. [22]. The authors use the augmented Lagrangian algorithm to decompose the problem across generators by relaxing the market clearing constraint at each node of the scenario tree and introducing a quadratic penalty term for deviating from this constraint. They study a system with 50 generators over a 48 hour horizon with approximately 100 scenarios and report duality gaps that do not exceed 2%.

Nowak and Römisch [62] study a stochastic unit commitment model with load uncertainty. The authors decompose the problem across generators and hydro facilities by relaxing the market clearing and reserve requirement constraints and use a proximal bundle algorithm
for updating dual multipliers. The authors test their algorithm on a German utility system with 25 thermal generators and 7 hydro reservoirs. They are able to solve problems with 20 scenarios within 2 minutes, and problems with 200 scenarios within 20 minutes.

Shiina and Birge [88] develop a column generation algorithm for decomposing a multi-stage stochastic program across generators. Their solution times range between 79 seconds and 8962 seconds for problems consisting of 10-20 units, 24-48 periods and 4-8 scenarios.

Recently, alternative formulations of the unit commitment problem have been proposed in order to address uncertainty. These formulations are motivated by the fact that system operators tend to operate the system in order to prevent worst-case outcomes, and also by the fact that the stochastic programming formulation requires an excessive amount of information about the underlying uncertainty. Ozturk et al. [70] formulate a chance-constrained optimization of the unit commitment problem where reserve requirements are replaced by a constraint that bounds the probability of not meeting load for some hour of the day. The authors use Lagrangian relaxation on the market clearing constraint and use the subgradient algorithm for updating dual multipliers. The authors solve various systems with up to 100 generators and a planning horizon of up to 96 hours within 188 minutes. Robust optimization has also been considered recently as an appealing formulation. Jiang et al. [47] use a robust formulation of the unit commitment problem and assume polyhedral and cardinal uncertainty [8] on system demand. The authors use Benders’ decomposition for solving the problem and demonstrate that the inclusion of transmission and ramping constraints complicates the problem considerably. Similar work has been done recently by Bertsimas et al. [7]. However, these models do not extend to account for contingencies in a straightforward fashion.

We proceed with a brief introduction of Lagrangian relaxation and then present a dual decomposition algorithm for solving the stochastic unit commitment model. We also discuss the possibility of parallelizing the algorithm.

5.1 Lagrangian Decomposition and the Subgradient Algorithm

Problems with decomposable structure are often characterized by two types of constraints: a set of constraints that partition decision variables and strongly couple decision variables within each subset of the partition, and a set of complicating constraints that loosely couple decision variables across the subsets of the partition. An approach for exploiting the structure of these problems, then, is to relax the complicating constraints and optimize the dual function. Since the complicating constraints are loosely coupled, there is a manageable number of dual variables, but most importantly the dual function can be decomposed to subproblems containing only subsets of the decision variables of the original problem.

The dual function is guaranteed to be convex, but not differentiable, which suggest
the use of the subgradient algorithm for optimizing the dual function. The subgradient algorithm is a first-order algorithm which is an analog of the steepest descent algorithm for nondifferentiable optimization. The following is drawn from Section II.5.4 of Nemhauser and Wolsey \[61\].

Given a function \( z \), which need not be convex, \( s \) is defined as a subgradient of \( z \) at \( x^0 \) if

\[
z(x) \geq z(x^0) + s(x - x^0) \quad \text{for all } y.
\]

Consider a mixed integer program \( MIP \) with a set of complicating constraints and a second set of constraints \( Q \):

\[
z_{MIP} = \max \{cx : Ax \leq b, x \in Q\}
\]

By relaxing the complicating constraints, we obtain the following dual function \( z_{LR}(\lambda) \) in the dual space:

\[
z_{LR}(\lambda) = \max_{x \in Q} \{cx + \lambda(b - Ax)\}.
\]

The Lagrangian is an upper bound to the optimal solution of the original problem. The best upper bound \( z_{LD} \) to the optimal solution of the original problem is obtained by solving for the dual function.

\[
z_{LD} = \min_{\lambda \geq 0} z_{LR}(\lambda).
\]

What makes the subgradient algorithm appropriate for optimizing the dual function is that the subgradient of a dual function can be readily computed once we compute the dual function at a certain point \( \lambda^0 \). This follows from Proposition 4.1 of Nemhauser and Wolsey \[61\]:

**Proposition 5.1.1** If \( x^0 \) is an optimal solution to \( LR(\lambda^0) \) in Equation (5.3), then \( s^0 = b - Ax^0 \) is a subgradient of \( z_{LR}(\lambda) \) at \( \lambda^0 \).

The subgradient method is based on the following iteration:

\[
x^{(k+1)} = x^{(k)} - \alpha_k s^{(k)},
\]

where \( x^{(k)} \) is the \( k \)-th iterate, \( s^{(k)} \) is any subgradient of \( z \) at \( x^{(k)} \) and \( \alpha_k > 0 \) is the \( k \)-th step size. Since this is not a descent method, we keep track of the best solution so far, \( z_{\text{best}}^{(k)} = \min_{i=1, \ldots, k} z(x^{(i)}) \). In theory, for non-summable diminishing stepsizes, which is a sequence of stepsizes satisfying \( \lim_{k \to \infty} \alpha_k = 0 \) and \( \sum_{k=1}^{\infty} \alpha_k = \infty \), the algorithm converges to the optimal solution: \( \bar{z} = \lim_{k \to \infty} z_{\text{best}}^{(k)} \). In practice, non-summable diminishing stepsize results in very slow convergence. An alternative stepsize is presented in the next section that works better in practice.
5.2 Subgradient Algorithm for Stochastic Unit Commitment

By dualizing the constraints of Equations (3.37) and (3.38) we get the following Lagrangian:

$$
L = \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
+ \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (\mu_{gst} u_{gst} - w_{gt}) + \nu_{gst} (v_{gst} - z_{gt})).
$$

(5.6)

The first subproblem is, for each scenario,

$$
(P1_s) : \min \sum_{g \in G} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \\
+ \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (\mu_{gst} u_{gst} + \nu_{gst} v_{gst})
$$

s.t.

$$
(3.21), (3.22), (3.23), (3.24), (3.25), (3.26), (3.27), (3.28), (3.31), (3.32), (3.34), (3.36)
$$

$$
p_{gst} \geq 0, v_{gst} \geq 0, u_{gst} \in \{0, 1\}, g \in G, t \in T.
$$

(5.7)

Note that if we did not have $v_{gst} \leq 1$ for $g \in G_s$, the problem could have been unbounded. Therefore, although the constraint in Equation (3.34) is superfluous for the original formulation, it is necessary for the proposed relaxation otherwise we get a dual function that is unbounded and cannot be used for a decomposition algorithm. The second subproblem becomes:

$$
(P2) : \min - \sum_{g \in G_s} \sum_{s \in S} \sum_{t \in T} \pi_s (\mu_{gst} w_{gt} + \nu_{gst} z_{gt})
$$

s.t.

$$
(3.29), (3.30), (3.33), (3.35)
$$

$$
w_{gt} \in \{0, 1\}, z_{gt} \geq 0, g \in G_s, t \in T.
$$

(5.8)

The updating of the dual variables is as follows:

$$
\mu_{gst}^{k+1} = \mu_{gst}^k + \alpha_k \pi_s (w_{gt}^k - u_{gst}^k), g \in G_s, s \in S, t \in T
$$

(5.11)

$$
\nu_{gst}^{k+1} = \nu_{gst}^k + \alpha_k \pi_s (z_{gt}^k - v_{gst}^k), g \in G_s, s \in S, t \in T,
$$

(5.12)

where $w_{gst}^k, z_{gst}^k$ is the optimal solution of $(P2)$ at iteration $k$ and $u_{gst}^k, v_{gst}^k$ is the optimal solution of $(P1_s)$ at iteration $k$. We could have relaxed only the non-anticipativity constraint on the
commitment variables. The advantage of also relaxing the non-anticipativity constraint on
the startup variables is that \((P_1_s), s \in S\), is a smaller problem, since the constraints on the
unit commitment of the slow generators are a part of \((P2)\). An additional advantage of this
choice of decomposition is that, at each step, the slow generator unit commitment solutions of
the first subproblem can be used for generating a feasible solution to the original problem by
solving an economic dispatch problem \((ED_s)\), Equations \((3.41)-(3.49)\), for scenario \(s \in S\).
As a result, at each step of the algorithm we get an upper bound on the optimal solution,
as well as a feasible schedule. This should be contrasted to the case where we would have
chosen to relax only the non-anticipative constraints on the unit commitment variables, and
not the startup variables.

The step size rule follows Fisher \cite{32} and Held et al. \cite{40} and is given by

\[
\alpha_k = \frac{\lambda(\hat{L} - L^k) \sum_{g \in G_s} \sum_{s \in S} \sum_{t \in T} \left( \pi_s(u_{gst}^k - w_{gt}^k)^2 + \pi_s(v_{gst}^k - z_{gt}^k)^2 \right)}{
\sum_{g \in G_s} \sum_{s \in S} \sum_{t \in T} \left( \pi_s(u_{gst}^k - w_{gt}^k)^2 + \pi_s(v_{gst}^k - z_{gt}^k)^2 \right)}, \tag{5.13}
\]

where \(\lambda\) is a constant parameter, \(L^k\) is the value of Equation \((5.6)\) at the optimal solution
and \(\hat{L}\) is an upper bound on the optimal solution.

In Figure 5.1 we present a schematic of the decomposition algorithm implemented in
parallel solvers. The second-stage subproblems \((P1_s)\), and the economic dispatch problems
\((ED_s)\) that are solved for obtaining a feasible solution can be solved in parallel. Additionally,
the Monte Carlo simulations that are performed for evaluating the performance of the first-
stage decisions can be implemented in parallel. We have implemented the algorithm on
a cluster of 16 machines using the Parallel Virtual Machine (PVM) library and the Java
callable library of CPLEX.
Figure 5.1: Parallelization of the decomposition algorithm for the stochastic unit commitment model.
Chapter 6

Stochastic Models

Our model uses three stochastic inputs, as we show in Figures 2.1 and 2.2: renewable power production, firm (inflexible) demand and real-time prices. The case study in Chapter 8 focuses on wind power integration. We will therefore discuss the calibration and simulation of a multi-area wind production model. The demand model can be developed as a special case. The real-time price process is modeled as a recombinant lattice and is used for driving the response of deferrable demand coupled with renewable production. The derivation of the real-time price model is described in Section A of the appendix.

The power output of wind generators is a nonlinear function of wind speed. Wind generators produce no output whenever wind speed is below a cut-in threshold. Once wind speed exceeds the cut-in threshold the power output of a wind generator increases as a cubic function of wind speed, until a saturation point. Beyond the saturation point, wind power production remains constant at the nominal power output of the generator. When wind speed exceeds a certain cut-off threshold, generators shut down in order to prevent mechanical damage. Due to the highly nonlinear relationship of wind power production to wind speed, the statistical modeling of wind power production is very challenging. It is therefore common in the wind power modeling literature to model wind speed and use a static power curve to calculate the corresponding wind power production.

The task of modeling wind speed consists of fitting wind speed data to a parametric or non-parametric distribution, removing seasonal and daily patterns and fitting an appropriate time series model to the underlying noise in order to capture the strong temporal correlation of wind speed time series. Early work on wind power modeling performed by Brown et al. [18] follows this approach. The authors list various parametric distributions for fitting wind speed data, such as the Weibull, inverse Gaussian and exponential distribution. The authors use an exponential function to transform their data to an approximately Gaussian data set. They remove hourly means and estimate the order of an appropriate autoregressive model and they use the Yule-Walker equations [17] to estimate the parameters of the autoregressive model. Torres et al. [94] follow the same methodology as Brown et al. [18]. The authors use autoregressive moving average models and find that these more general models provide
a more satisfactory fit than simpler autoregressive models.

Due to the introduction of transmission constraints, it is not sufficient to describe aggregate wind power production in the network, but instead it is necessary to specify the production of wind power in each location of the network. This necessitates the development of a multi-area wind power production model, which needs to faithfully reproduce both the temporal as well as spatial correlations of wind power production. In recent work, Morales et al. [58] develop a multi-area wind speed model by using a noise vector that drives a vector autoregressive process. In order to simplify the calibration of the model, the authors assume a diagonal matrix of autoregressive coefficients, which implies that spatial correlations among wind speed in various locations are captured fully by the underlying noise vector. The calibration and simulation model that we use follows the approach of Morales et al. [58].

6.1 Calibration

Given a multi-area data set $y_{kt}$, where $k$ indexes location and $t$ indexes time, the first step of the calibration procedure is to filter the data set in order to obtain an approximately Gaussian data set $y_{Gkt}$. Brown et al. [18], Torres et al. [94] and Morales et al. [58] use this approach for transforming Weibull-distributed wind speed data to Gaussian data, and Callaway [21] uses a non-parametric transformation. In the single-area wind integration study of Section 8.2 we find that the inverse Gaussian distribution provides a satisfactory fit for the data set. For the multi-area wind integration study of Section 8.3, no single parametric distribution provides a close fit for the observed data in all locations, therefore we fit an empirical distribution $\hat{F}_k(\cdot)$ to the data of each location $k$.

We next follow the methodology that is suggested in Brown et al. [18], Torres et al. [94] and Callaway [21] for removing diurnal and seasonal patterns. We normalize the data by subtracting the hourly mean and dividing by the hourly standard deviation in order to obtain a Gaussian stationary data set $y_{GSkt}$ for each location. Systematic patterns can be monthly, seasonal, or may even vary between weekdays and weekends as is the case for load data. In each case, the appropriate portion of the data set should be chosen for estimating the mean and variance. The resulting time series $y_{GSkt}$ can be modeled by an autoregressive model:

$$y_{GSk,t+1} = \sum_{j=0}^{p} \hat{\phi}_{kj} y_{GSk,t-j} + \hat{\omega}_{kt}, \quad (6.1)$$

where $\hat{\omega}_{kt}$ is the estimated noise and $\hat{\phi}_{kj}$, $j \in \{1, \ldots, p\}$, are the estimated coefficients of the autoregressive model. The calibration process is summarized in the following steps:
Step (a). Transform the data in order to obtain a data set that follows a Gaussian distribution:

\[ y_{kt}^G = N^{-1}(\hat{F}_k(y_{kt})) \]

where \( y_{kt} \) is the data, \( y_{kt}^G \) is the transformed data that follows a Gaussian distribution, \( N^{-1}(\cdot) \) is the inverse of the cumulative distribution function of the normal distribution and \( \hat{F}_k \) is the cumulative function of the (parametric or non-parametric) fit for the data in location \( k \).

Step (b). Remove systematic seasonal and diurnal effects:

\[ y_{kt}^{GS} = y_{kt}^G - \hat{\mu}_{kmt} \hat{\sigma}_{kmt}, \]

where \( y_{kt}^{GS} \) is the transformed data that is Gaussian distributed and stationary, and \( \hat{\mu}_{kmt} \) and \( \hat{\sigma}_{kmt} \) are the sample mean and standard deviation respectively for location \( k \), epoch (e.g. month or season) \( m \) and hour \( t \).

Step (c). Use the Yule-Walker equations [17] to estimate the autoregressive parameters \( \hat{\phi}_{kj} \) and covariance matrix \( \hat{\Sigma} \) of the residual noise obtained from Equation (6.1).

Load time series are typically simpler than wind speed time series since the distribution of noise about the mean load is already Gaussian, making it possible to skip step (a) of the above procedure. It is, however, necessary to differentiate between weekdays and weekends for the estimation of the mean and variance in step (b), in contrast to the wind speed calibration procedure where this differentiation is not necessary.

### 6.2 Simulation

In order to simulate multi-area wind power production, we assume that the process is driven by an autoregressive ‘noise’ vector. For \( K \) locations and \( p \) periods of lag the model is:

\[ Y_{kt} = \sum_{j=1}^{p} \phi_{kj} Y_{k,t-j} + \omega_{kt}, \]

where \( \Phi = (\phi_{kj}), k \in \{1, \ldots, K\}, j \in \{1, \ldots, p\} \), is the matrix of autoregressive parameters and \( (\omega_{kt}), k \in \{1, \ldots, K\} \), are independent, identically distributed, multivariate Gaussian random variables with mean 0 and covariance matrix \( \Sigma \). The simulation of the multi-area process can then be summarized in the following steps:

Step (a). Generate autoregressive noise of order \( p \) by using the estimated autoregressive
parameters and variance.

\[ Y_{kt}^{GS} = \sum_{j=1}^{p} \phi_{kj} Y_{k,t-j}^{GS} + \omega_{kt}, \]  

where \( \omega_{kt} = (\hat{L}\omega)_k \), \( \omega \) are independent standard normal random vectors with \( K \) entries, \( \hat{L} \) is the Cholesky factorization of \( \Sigma \) and \( Y_{kt}^{GS} \) is the Gaussian stationary autoregressive process for location \( k \).

*Step (b).* Transform \( Y_{kt}^{GS} \) by its seasonal and hourly mean and variance:

\[ Y_{kt}^{G} = \hat{\sigma}_{kmt} Y_{kt}^{GS} + \hat{\mu}_{kmt}, \]  

where \( Y_{kt}^{G} \) is the resulting process that is non-stationary, Gaussian distributed.

*Step (c).* Transform the resulting process such that it obeys the non-Gaussian distribution of the original data:

\[ Y_{kt} = \hat{F}_{k}^{-1}(N(Y_{kt}^{G})) \]  

where \( Y_{kt} \) is the non-stationary process with the same distribution as the original data set for each location, \( N(\cdot) \) is the cumulative distribution function of the normal distribution and \( \hat{F}_{k}^{-1} \) is the inverse of the cumulative function of the data for each location.

*Step (d).* Use an approximation \( \hat{P}_k(\cdot) \) of the aggregate power curve for each location to simulate wind power production:

\[ P_{kt} = \hat{P}_k(Y_{kt}), \]  

where \( P_{kt} \) is the simulated wind power production process for each location.

For the load time series, steps (c) and (d) of the simulation procedure are not necessary.

### 6.3 Fit of the Models to Observed Data

For the case studies of Chapter 8 we use wind speed and wind power production data from the 2006 data set of the National Renewable Energy Laboratory (NREL) Western Wind and Solar Integration Study (WWSIS), described by Potter et al. [79]. We study two wind integration cases. The first represents a moderate energy integration level for wind power corresponding to the 2012 integration target of California, and the second case represents a deep integration level corresponding to the 2020 integration target. Ex post we have estimated that the moderate integration case corresponds to approximately 7% wind energy penetration, while the deep integration case corresponds to approximately 14% wind energy penetration. In the subsequent analysis we will refer to these cases as moderate and deep integration respectively.
Table 6.1: Current and projected capacity of wind power installations (MW).

<table>
<thead>
<tr>
<th>County</th>
<th>Existing</th>
<th>Moderate</th>
<th>Deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Altamont</td>
<td>954</td>
<td>954</td>
<td>1,086</td>
</tr>
<tr>
<td>Clark</td>
<td>-</td>
<td>-</td>
<td>1,500</td>
</tr>
<tr>
<td>Imperial</td>
<td>-</td>
<td>-</td>
<td>2,075</td>
</tr>
<tr>
<td>Solano</td>
<td>348</td>
<td>848</td>
<td>1,149</td>
</tr>
<tr>
<td>Tehachapi</td>
<td>1,346</td>
<td>4,886</td>
<td>8,333</td>
</tr>
<tr>
<td>Total</td>
<td>2,766</td>
<td>6,688</td>
<td>14,143</td>
</tr>
</tbody>
</table>

In order to collect data for each case, we examined the interconnection queue of the California ISO until 2020 (see [19]), and placed individual wind generators in our model by matching the geographical locations of planned wind power installations with the corresponding wind park data in the WWSIS data set. In Table 6.1 we present the location of existing wind generation capacity, as well as capacity for the moderate and deep integration cases.

**Single-area wind integration study (Section 8.2).** In Section 8.2 we present a study of integrating wind power in a network without transmission constraints, where load is assumed to follow a deterministic pattern and the unique source of uncertainty is wind power production. We isolate wind power production uncertainty in order to carefully analyze its impacts on the system, before proceeding to a more complicated analysis in Section 8.3. As we discuss in the beginning of this chapter, we model wind speed with a time series model and use a piecewise linear approximation of wind speed to wind power production. The piecewise linear approximation of wind speed to wind power for the Tehachapi area is shown in the lower right frame of Figure 6.2. The fit of the wind speed and wind power production model to the corresponding data for the deep integration case is shown in the left and right frame of Figure 6.1 respectively. We use an inverse Gaussian distribution as a parametric fit to the wind speed data. The deviations in the fit arise from the fact that the wind speed distribution is not exactly inverse Gaussian and also due to the fact that the aggregate power curve cannot exactly reproduce the behavior of the scatter plot in Figure 6.2, which is produced by aggregating data from hundreds of locations.

**Multi-area wind integration study with contingencies (Section 8.3).** In Section 8.3 we present an extension of our study to the case of a transmission-constrained network, which necessitates the development of a multi-area wind power production model. Demand is assumed to be deterministic. The fit of the wind power production model to the data set for each location is shown in Figure 6.2. In the lower left frame of the figure we observe a discrepancy between the model and the data for peak wind production levels in the area of Tehachapi. This can be attributed to the piecewise linear approximation of the power curve in the lower right frame of the figure. In order to alleviate this problem, we experimented
with further partitioning the Tehachapi area in smaller regions. However, this introduced greater inaccuracy to the model due to the higher dimensions of the correlation matrix $\Sigma$. As a result, we chose to model five areas as the best compromise between capturing locational dependencies and retrieving marginal wind speed distributions at each location.

### Demand response integration study (Section 8.4)

In Section 8.4 we study the impact of integrating demand flexibility to the system. In this study we assume that inflexible (firm) demand is stochastic. As in the case of wind power production, we assume a second-order autoregressive model. The fit of the model to the data is shown in Figure 6.3.

As we discuss in Chapter 7, in order to model the optimal response of deferrable demand across time periods we use a stochastic dynamic programming formulation where both real-time electricity prices and wind power production are stochastic. In order to control the growth of the state space of the dynamic program, we use a lattice model for real-time prices and wind power production. The fit of the price and wind power production lattice models are shown in the left and right frame of Figure 6.4 respectively.

---

1 The stochastic model used in this part of the study reverses the order of steps (b) and (c) in the calibration and simulation phases of Sections 6.1 6.2. In order for the procedure of Sections 6.1 to be valid, the distribution of the transformed data after step (b) of the calibration has to be Gaussian for each hour of the day. This is a stringent assumption that we relax in the third part of our study on demand flexibility by reversing the order of steps (b) and (c). By reversing the order of steps (b) and (c), we are implicitly assuming that the observed signal is produced by overlapping noise of an arbitrary distribution over a systematic average pattern. The Kantorovich distance of the estimated measure from the measure obtained from the data set is 39.7 for the price data and 4824 for the wind data using the original methodology presented in Sections 6.1 6.2. The respective distances when steps (b) and (c) are reversed are evaluated to 37.2 and 4975 for the price and wind data respectively, indicating a small difference among the two procedures.
Figure 6.2: In reading order: load duration curves for Altamont, Clark County, Imperial, Solano and Tehachapi, and power curve at the Tehachapi area for deep integration for the multi-area wind integration case study.
Figure 6.3: Probability distribution function of inflexible demand for the deferrable demand case study.

Figure 6.4: Probability distribution function of real-time electricity prices (left) and wind power production (right) for deep integration for the deferrable demand case study.
Chapter 7

Demand Flexibility

The large-scale integration of renewable resources in power systems has revived the interest of researchers on schemes for incorporating demand-side flexibility in power systems operations and power markets. In this Chapter we describe three fundamental approaches for modeling demand-side flexibility: centralized load dispatch by the system operator, demand-side bidding and coupling renewable power supply with deferrable demand. We discuss the issues that arise with centralized load dispatch and demand-side bidding that motivate us to consider coupling renewable supply with deferrable demand. We then present the mathematical model of each demand-response approach and explain how it can be integrated in the stochastic unit commitment model in order to quantify the benefits of integrating flexible demand in power system operations.

The most efficient approach for exploiting demand-side flexibility would be for the system operator to centrally co-optimize the dispatch of demand-side resources and generators. This is unrealistic in practice as the system operator operates the system at a bulk scale and cannot enforce control on the system down to individual household loads. In addition, the optimization problem at hand is too complex to solve. Nevertheless, this ideal model provides a limit on the potential benefits of demand flexibility. Sioshansi [89] considers this model in a deterministic setting. We extend this approach to account for the uncertainty introduced by renewable energy supply and inflexible demand and present the formulation in Section 7.1.

An alternative approach for exploiting demand-side flexibility that we explore in Section 7.2 is to establish real-time pricing at the retail level. This possibility was introduced by Schewppe et al. [87] and is discussed by Borenstein et al. [13]. The common approach of reasoning about real-time pricing in the power system economics literature is the use of decremental demand bids. Sioshansi and Short [90] use this approach in a study of the impact of real-time pricing on wind power integration. Borenstein and Holland [12] and Joskow and Tirole [48], [49] also use this approach for analyzing retail pricing. However, there is strong political opposition to this approach as it exposes retail consumers to the volatility of electricity prices. In addition, real-time prices often fail to convey the economic
value of demand response due to the non-convex operating costs of system operations. This effect has been reported by Sioshansi, who notes that the failure of real-time prices to capture non-convexities induces a dispatch of deferrable resources that results in excessive startup and minimum load costs. Moreover, demand-side bidding fails to capture the cross-elasticity of deferrable demand over time. Demand flexibility is often characterized as a request for a certain amount of energy over a given time interval. Demand bids fail to capture this effect by making demand appear independent across time periods.

In Section 7.3 we present a contractual agreement for coupling the operations of renewable resources with deferrable demand that attempts to override the difficulties that arise from centralized load dispatch and demand-side bidding. The motivation of coupling renewable generation with deferrable demand is to create a net resource that appears “behind the meter” from the point of view of the system operator. By coupling their consumption with renewable resources, deferrable consumers can largely absorb the uncertainty of renewable energy fluctuation. The control of the deferrable resources is assigned to a demand-side aggregator rather than the system operator. The aggregator adapts the inter-temporal demand of power from deferrable loads in order to utilize the available renewable resource while minimizing its reliance on the backup generators of the system.

In an alternative approach, which is described by Hirst and Kirby and Kirby, flexible loads can participate in the ancillary services market. An aggregator could bid on behalf of a population of loads for providing ancillary services to the system operator. The aggregator would then be responsible for coordinating the aggregate consumption of loads by some price-based or direct control method. The technical feasibility of demand-side aggregation for the provision of spinning reserve has been studied in practice by Eto. As ancillary services requirements are expected to increase due to renewable energy integration, this solution could prove lucrative for users who would be willing to respond to the instantaneous needs of power system operators. However, there are concerns about defining market products that correspond to the types of services that loads can actually offer, which raises the need for reform in existing electricity markets. As a result, it is probable that policy deliberations will delay the process of using aggregators for managing significant populations of load.

### 7.1 Centralized Load Control

In the centralized load control approach we assume that the system operator co-optimizes the dispatch of flexible loads and generation resources both in the day-ahead scheduling as well as in the economic dispatch phase. The system operator solves the following problem in the day-ahead phase:
\[(CSUC) : \min \sum_{g \in G} \sum_{s \in S} \sum_{t \in T} \pi_s (K_g u_{gst} + S_g v_{gst} + C_g p_{gst}) \tag{7.1}\]

s.t.
\[\sum_{g \in G} p_{gst} = D_{st}, \ s \in S, \ t \in T \tag{7.2}\]
\[\sum_{t \in T} E_{st} = R, \ s \in S \tag{7.3}\]
\[0 \leq E_{st} \leq C, \ s \in S, \ t \in T \tag{7.4}\]

Note that, in contrast to problem \((SUC)\) of Equations (3.20) - (3.40), we do not account for transmission constraints. Total inflexible demand in the system is represented by \(D_{st}\), while \(E_{st}\) represents deferrable demand. The total amount of energy consumed by deferrable consumers is \(R\) and \(C\) represents the consumption rate constraint of deferrable demand.

Despite the fact that the centralized model is not realistic in practice, it is useful in estimating the capacity savings of demand flexibility. As we discuss in Figure 2.2, we use this model to determine a day-ahead schedule for slow resources while accounting for demand flexibility. We then compare the performance of centralized load dispatch, demand-side bidding and coupling in the economic dispatch stage, after slow resources have been committed according to \((CSUC)\). Thus, we use \((CSUC)\) as the stochastic unit commitment model that allows us to quantify the capacity savings of integrating deferrable demand in power system operations, as if the system operator had full control of load dispatch.

In order to evaluate the performance of centralized load dispatch in real time, we solve \((CSUC)\) for each stochastic realization of wind power supply and firm demand after fixing the schedule of slow generators according to the optimal solution of \((CSUC)\).

### 7.2 Demand Bids

The demand model that we present in this section is based on Borenstein and Holland \[12\] and Joskow and Tirole \[48, 49\]. We assume a linear demand function that consists of a fraction \(\alpha\) of inflexible consumers who face a fixed retail price \(\lambda^R\), and a fraction \(1 - \alpha\) of price-responsive consumers who face the real-time price of electricity \(\lambda_t\). The demand function \(D_t(\cdot)\) for each period can therefore be expressed as:

\[D_t(\lambda_t; \omega) = a_t(\omega) - \alpha b \lambda^R - (1 - \alpha) b \lambda_t, \tag{7.5}\]

where \(\omega\) represents an element of the sample space that determines the realized inflexible demand, \(a_t(\omega)\) is the intercept and \(b\) is the slope of the demand function. Note that we
assume a common slope for all time periods and a time-varying stochastic intercept that depends on the realization of inflexible demand.

We calibrate the demand functions such that they satisfy the following two prerequisites: the demand functions have to yield a total daily demand of $R$ subject to the charging rate constraint $C$, and the demand functions have to be consistent with the observed inflexible demand in the system. The calibration process can be summarized in the following steps:

*Step (a).* Select the fraction of inflexible demand $\alpha$ such that $R$ represents a fraction $1 - \alpha$ of total daily demand for each day type.

*Step (b).* Set the slope $b$ such that the supply to price-responsive consumers equals $R$ in the economic dispatch formulation (with slow generator schedules fixed according to the optimal solution of ($CSUC$)).

*Step (c).* For each realization $\omega$ resulting in inflexible demand $\alpha D_t(\lambda; \omega)$, set $a_t(\omega) = D_t(\lambda; \omega) + b\lambda$ in order to be consistent with the observed inflexible demand.

*Step (d).* The inverse demand function for deferrable demand is given by $P_t(q_t; \omega) = \frac{1}{b}(a_t(\omega) - \frac{q_t}{1 - \alpha})$, $q_t \leq C$. We can then discretize the inverse demand function and include it in the objective function of ($ED_c$) in Equation 3.41. Demand can be represented in this general model as a resource with negative marginal cost, and cost minimization can be interpreted as welfare maximization.

7.3 Coupling

In this section we present a contractual agreement for coupling renewable resources with deferrable demand. We discuss the implementation of the contract in a fashion that is compatible with existing power system operations and power market mechanisms, and we describe the optimal control problem faced by an aggregator that is responsible for serving deferrable loads with renewable resources. Such load may include heating, cooling and air conditioning, agricultural pumping, electric vehicles/plug-in hybrid electric vehicles etc.

7.3.1 Implementation

Consider an aggregator that contractually owns the output from a large group of renewable generation assets. The aggregator enters into a contractual agreement to supply deferrable loads. Loads specify their energy demand in the form of requests for a certain amount of energy over a fixed time window. The aggregator can control the loads directly and uses renewable power from its assets as the primary energy source for satisfying deferrable demand. In the case of renewable supply shortage, the aggregator can resort (to a limited extent) to the real-time market for procuring power at the prevailing real-time price. The aggregator compensates deferrable loads at a rate $\rho$ for each unit of unserved energy. Any excess renewable power is supplied to the system. The setup is similar to dynamic scheduling
[42], whereby demand and supply resources from different control areas pair their schedules in order to produce a zero net output to the remaining system. Such scheduling is currently implemented in the ERCOT market. In order to ensure that deferrable loads gain top priority to the renewable resource, aggregators are not compensated for residual renewable supply to the system on the basis of real-time prices, but instead receive the average real-time price for their total supply over a fixed time interval, e.g. a month.

As Schweppe et al. [87] discuss, the operating cost benefits of incorporating demand flexibility in power systems are expected to be outweighed by the savings in capital investment on balancing generation capacity. Such savings can be ensured, in the context of coupling contracts, by limiting the participation of aggregators in the real-time market or imposing a demand charge on such participation that will incentivize aggregators to self-impose such limits. It is therefore necessary to provide financial incentives to deferrable loads for limiting their consumption to an efficient level that ensures the satisfaction of their demand while not imposing excessive capacity requirements on the system. Priority pricing introduced by Chao, Wilson, Oren and Smith [24], [69] and the derivative idea of callable forward contracts introduced by Gedra and Varaiya [34], and Oren [67] can be used for limiting the participation of deferrable loads in the real-time market, while compensating loads for the capacity savings they enable. Callable forward contracts bundle a forward contract on power supply with a call option that can be exercised by the system operator in real time in order to limit the consumption of deferrable loads whenever real-time price exceeds a strike price $k$. Callable forward contracts therefore enable flexible consumers to enter the merit order stack of the system operator at the price $k$, which translates to capacity savings for the system operator.

It is important to ensure that callable forward contracts, or other mechanisms for inducing deferrable loads to limit the degree of their participation in real-time markets, induce loads to self-select the degree of their participation in the real-time market efficiently. In particular, it is desirable to provide strong financial incentives for loads to limit their participation in the real-time market to the greatest possible extent, without however making these financial incentives so strong that loads compromise their welfare. In the context of callable forward contracts, this translates to inducing loads to self-select the lowest strike price $k$ that still provides sufficient flexibility for deferrable loads to participate in the real-time market in order to satisfy their entire demand. Self-selection has been addressed by Gedra and Varaiya [34] in a static context, however deferrable demand introduces inter-temporal dependencies in the valuation for power across time periods.

7.3.2 Problem Formulation

The coupling contract that we introduced in the previous section can be formulated as a stochastic optimal control problem. The aggregator solves the following:
\[
\min_{\mu_t(x_t)} \mathbb{E}\left[\sum_{t=1}^{N} \lambda_t (\mu_t(x_t) - w_t)^+ \Delta t + \rho r_N\right],
\]  
\[(7.6)\]

where \(\mu_t(x)\) represents the rate at which power is supplied to deferrable loads and \(N\) is the horizon of the optimal control problem. The state vector \(x_t = (\lambda_t, w_t, r_t)\) consists of the real-time price \(\lambda_t\), the available renewable power supply \(w_t\) and the remaining energy demand of the deferrable consumer \(r_t\). The initial condition for the residual demand is \(r_1 = R\), where \(R\) is the amount of energy demand to be satisfied. The control \(u_t\) is constrained by the rate of supply \(C\) and by the amount of energy that can be procured in the real-time electricity market \(M_t\), which is a random variable. Hence, we obtain \(u_t \leq CM_t\). Unsatisfied energy incurs a penalty \(\rho\). The limit on real-time market participation depends on the choice of strike price. The optimal control problem stated above is solved by backward dynamic programming, with a lattice representing the state space of the stochastic processes. The lattice model of renewable power supply and real-time prices is presented in Chapter A of the appendix.

The use of callable forward contracts does not need to be implemented strictly in the context of coupling, but can also apply for deferrable loads that seek to satisfy their energy demand exclusively in the real-time market. In that case the optimal control of deferrable loads is formulated as follows:

\[
\min_{\mu_t(x_t)} \mathbb{E}\left[\sum_{t=1}^{N} \lambda_t (\mu_t(x_t)) \Delta t + \rho r_N\right],
\]  
\[(7.7)\]

with \(u_t \leq CM_t\).

### 7.3.3 Incorporating the Coupling Model in Economic Dispatch

The integration of the coupling model with the economic dispatch model, presented in Figure 2.2, requires that the prices loads respond to be consistent with the market-clearing prices generated from the solution of the economic dispatch model. Sioshansi [89] raises this issue in a detailed model of electric vehicle charging in conjunction with unit commitment. In order to achieve the desired equilibrium, Sioshansi iterates between the solution of the dynamic optimization problem of electric vehicles and the economic dispatch problem faced by the system operator. We adopt the same approach in our model. In particular, we generate an outcome of firm demand and wind power production. We then generate a multi-period load response profile based on the lookup table that we obtain from the dynamic programming algorithm of Section 7.3.2. The induced load response is then used to generate market clearing prices in the economic dispatch model. The resulting prices are discretized and smoothed such that the load response moves closer to equilibrium. In the results presented in Section 8.4, this process is iterated five times for each day type. For certain realizations
of wind power production and firm load, the system reaches equilibrium in the sense that the lattice values of real-time prices that loads are responding to are consistent with the discretized value of the market-clearing prices obtained from economic dispatch. In other cases, however, load exhibits an oscillatory behavior across iterations. For these realizations, we adopt the convention that the equilibrium response of loads corresponds to that iteration for which the norm of the difference between the prices loads are responding to and market clearing prices is minimized. It is left as a topic of future research to consider alternative equilibrium definitions that are guaranteed to exist and that can be computed efficiently.
Chapter 8

Case Study

In this chapter we present results for three case studies. The first two case studies ignore firm demand uncertainty and demand flexibility in order to focus on the impacts of large-scale renewable power integration. The first wind integration case study considers a network without transmission constraints and contingencies. These features are included in the second case study. These case studies validate the scenario selection algorithms that we propose in Chapter 4.

Once we establish the superior performance of the proposed stochastic unit commitment algorithm, we use the stochastic model for assessing operating costs and capacity requirements in the presence of deferrable demand in the third case study. In order to gain an initial understanding on the impacts of demand flexibility we ignore transmission constraints. The integration of deferrable demand in a transmission-constrained network will be a topic for future research.

8.1 Preliminaries

We first present the model of the Western Electricity Coordinating Council (WECC) that we use for our case study. We also discuss two deterministic unit commitment rules against which we compare the stochastic unit commitment policy.

8.1.1 The WECC Model

We use a model of the California ISO with imports from the Western Electricity Coordinating Council (WECC) that is also used by Yu et al. [99]. We do not use the wind production data from [99] since our wind production model is more detailed. Since the model in [99] reflects import, hydroelectric, geothermal and biomass production data for a six-month period from May 1, 2004, to October 1, 2004, we replicate the data for the remaining six months of the year in order to produce an entire year of data. This extrapolation is justified by the fact
that the average production profiles of all these resources are almost identical for the three seasons that are covered by the data set. Since we are using 2004 import data, we also use load data from the same year, which is publicly available at the CAISO Oasis database [20].

As we discuss in Chapter 3, in order to reduce the computational requirements of the model we focus on eight representative day types instead of simulating an entire year of operations for the system. This should be contrasted to the approach of Ruiz et al. [83], Sioshansi and Short [90], Constantinescu et al. [25] and Tuohy et al. [95], who simulate an entire year of operations. We consider one day type for each season and in addition we differentiate between weekdays and weekends. The results of our analysis are weighed by the frequency of occurrence of each day type.

The average load in the system is 27298 MW, with a minimum of 18412 MW and a peak of 45562 MW. The net load profile for each type of day, that needs to be served by thermal generators and wind power, is shown in Figure 8.1. The generation mix of the system is presented in Table 8.1.

We use a more general model for thermal generators within CAISO, with 124 generators, compared to the model of Yu et al. [99], who use 23 aggregated thermal generators. The value of lost load is set to 5000 $/MW-h. The number of generators and the capacity for each fuel type are shown in Table 8.1. The last two rows of Table 8.1 describe how the fossil fuel generation mix is partitioned into fast and slow generators. Most fast generators have a capacity no greater than 250 MW. The entire thermal generation capacity of the system is 28381.5 MW. The merit order curve of the thermal resources is shown in Figure 8.2.

In Figure 8.3 we present a schematic diagram of the WECC model. The dashed boxes represent load and generation pockets. The thick solid lines represent the import constraints.
Table 8.1: Generation mix of the WECC model

<table>
<thead>
<tr>
<th>Type</th>
<th>No. of units</th>
<th>Capacity (MW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nuclear</td>
<td>2</td>
<td>4,499</td>
</tr>
<tr>
<td>Gas</td>
<td>86</td>
<td>18,745.6</td>
</tr>
<tr>
<td>Coal</td>
<td>6</td>
<td>285.9</td>
</tr>
<tr>
<td>Oil</td>
<td>5</td>
<td>252</td>
</tr>
<tr>
<td>Dual fuel</td>
<td>23</td>
<td>4,599</td>
</tr>
<tr>
<td>Import</td>
<td>22</td>
<td>12,691</td>
</tr>
<tr>
<td>Hydro</td>
<td>6</td>
<td>10,842</td>
</tr>
<tr>
<td>Biomass</td>
<td>3</td>
<td>558</td>
</tr>
<tr>
<td>Geothermal</td>
<td>2</td>
<td>1,193</td>
</tr>
<tr>
<td>Wind (7.1% pen.)</td>
<td>5</td>
<td>6,688</td>
</tr>
<tr>
<td>Wind (14% pen.)</td>
<td>10</td>
<td>14,143</td>
</tr>
<tr>
<td>Fast thermal</td>
<td>82</td>
<td>9,156.1</td>
</tr>
<tr>
<td>Slow thermal</td>
<td>40</td>
<td>19,225.4</td>
</tr>
</tbody>
</table>

that are defined in Equation (3.14). Each thick solid line intersects a set of transmission lines $IG_j$ over which the total amount of power cannot exceed a certain limit $IC_j$. These constraints limit the total flow of power into a load pocket in order to prevent load shedding in the case of generator failure within a load pocket, and also limit the total amount of power flow over combinations of inter-ties that connect the California ISO system to neighboring states. The wind generators of Table 6.1 are located in the five buses that are depicted as solid black circles. In order of appearance from top to bottom, these wind sites are Solano, Altamont, Tehachapi, Clark and Imperial.

8.1.2 Deterministic Reserve Commitment Rules

In Section 3.2 we present the deterministic unit commitment model. There are various ad hoc approaches for determining the total reserve and fast reserve requirements in the deterministic formulation. In the following case studies we compare stochastic unit commitment against two rules for setting reserve requirements in order to validate the scenario selection algorithms that we propose in Chapter 4.

The first deterministic unit commitment policy sets a total reserve requirement for all hours of the day which is a certain fraction of the forecast peak load for the day. We perform a sensitivity analysis on the fraction of forecast peak load in order to recover the policy that performs best. The fast reserve requirement is then set equal to half the total reserve requirement.

The other deterministic policy that we consider is a variant of a reserve commitment policy that was recently discussed by NREL in Piwko et al. [78]. The authors propose a heuristic approach for committing spinning reserves, the '3+5 rule', that requires the system...
to carry hourly spinning reserve no less than 3% of hourly forecast load plus 5% of hourly forecast wind power. We set this as the fast reserve requirement in our model, with the total reserve requirement at twice the level of the fast reserve requirement. In future work we intend to compare the performance of the stochastic unit commitment model against commitment rules based on probabilistic forecasting [4], [5].

8.2 Wind Integration in a System without Transmission Constraints

We first analyze the system for the case where there are no transmission constraints or contingencies in the network. As we discuss in Chapter 6, we consider two wind integration cases, one that corresponds to the 2012 wind integration targets for California, and one that corresponds to the 2020 targets. We refer to these as the moderate and deep integration cases respectively. We assume that wind power variability is the unique source of uncertainty in the model.

8.2.1 Relative Performance of Policies

We begin by assessing the relative performance of the stochastic unit commitment policy against a perfect forecast policy, as well as against the deterministic reserve rules discussed in Section 8.1.2. The perfect foresight policy commits reserves with advance knowledge of wind production for each day.
Figure 8.3: A schematic of the WECC model.

Figure 8.4: Cost as a function of total reserve requirements for deep integration (left) and moderate integration (right) for the single-area case study.
Table 8.2: Daily cost of operations for each day type for the single-area case study - moderate integration

<table>
<thead>
<tr>
<th></th>
<th>Cost ($)</th>
<th>∆ Cost ($)</th>
<th>∆ Cost ($)</th>
<th>∆ Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stoch</td>
<td>Foresight</td>
<td>20% peak</td>
<td>3+5</td>
</tr>
<tr>
<td>WinterWD</td>
<td>5,970,040</td>
<td>-21,816</td>
<td>20,484</td>
<td>39,747</td>
</tr>
<tr>
<td>SpringWD</td>
<td>6,003,520</td>
<td>-37,218</td>
<td>-2,147</td>
<td>14,870</td>
</tr>
<tr>
<td>SummerWD</td>
<td>11,272,575</td>
<td>-62,634</td>
<td>102,183</td>
<td>110,793</td>
</tr>
<tr>
<td>FallWD</td>
<td>8,081,245</td>
<td>-39,921</td>
<td>15,751</td>
<td>21,618</td>
</tr>
<tr>
<td>WinterWE</td>
<td>3,166,890</td>
<td>-32,214</td>
<td>1,346</td>
<td>-3,587</td>
</tr>
<tr>
<td>SpringWE</td>
<td>2,642,864</td>
<td>-28,857</td>
<td>-12,622</td>
<td>14,463</td>
</tr>
<tr>
<td>SummerWE</td>
<td>7,595,842</td>
<td>-42,179</td>
<td>46,661</td>
<td>46,892</td>
</tr>
<tr>
<td>FallWE</td>
<td>5,106,143</td>
<td>-25,350</td>
<td>-1,806</td>
<td>1,002</td>
</tr>
<tr>
<td>Total</td>
<td>6,916,442</td>
<td>-38,041</td>
<td>26,733</td>
<td>37,596</td>
</tr>
<tr>
<td>improv. (%)</td>
<td>-0.55</td>
<td>0.39</td>
<td>0.54</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.3: Daily cost of operations for each day type for the single-area case study - deep integration

<table>
<thead>
<tr>
<th></th>
<th>Cost ($)</th>
<th>∆ Cost ($)</th>
<th>∆ Cost ($)</th>
<th>∆ Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stoch</td>
<td>Foresight</td>
<td>30% peak</td>
<td>3+5</td>
</tr>
<tr>
<td>WinterWD</td>
<td>4,121,453</td>
<td>-169,553</td>
<td>27,642</td>
<td>55,358</td>
</tr>
<tr>
<td>SpringWD</td>
<td>3,906,408</td>
<td>-120,016</td>
<td>71,468</td>
<td>103,306</td>
</tr>
<tr>
<td>SummerWD</td>
<td>9,773,670</td>
<td>-111,811</td>
<td>147,861</td>
<td>67,553</td>
</tr>
<tr>
<td>FallWD</td>
<td>6,125,650</td>
<td>-89,470</td>
<td>27,721</td>
<td>34,900</td>
</tr>
<tr>
<td>WinterWE</td>
<td>1,967,672</td>
<td>-75,346</td>
<td>92,732</td>
<td>3,619</td>
</tr>
<tr>
<td>SpringWE</td>
<td>1,482,317</td>
<td>-57,696</td>
<td>113,434</td>
<td>96,514</td>
</tr>
<tr>
<td>SummerWE</td>
<td>6,309,549</td>
<td>-78,993</td>
<td>79,931</td>
<td>39,757</td>
</tr>
<tr>
<td>FallWE</td>
<td>3,524,599</td>
<td>-78,288</td>
<td>-2,508</td>
<td>2,443</td>
</tr>
<tr>
<td>Total</td>
<td>5,551,907</td>
<td>-108,389</td>
<td>69,309</td>
<td>56,795</td>
</tr>
<tr>
<td>improv. (%)</td>
<td>-2.08</td>
<td>1.33</td>
<td>1.09</td>
<td></td>
</tr>
</tbody>
</table>

In Figure 8.4 we present the average cost of the peak-load-based unit commitment rule discussed in Section 8.1.2 for various levels of total reserve requirements. We see that the optimal reserve requirement for the moderate wind integration level is at 20% of maximum load, whereas for the deep integration level it is at 30% of maximum load, and slightly outperforms the policy that commits 40% of maximum load. Reserve requirements that are exceedingly low result in significant load shedding, whereas exceedingly high reserve requirements result in high fuel costs due to the excessive rejection of wind power.

The perfect foresight policy sets a lower limit on the cost of any policy. In Tables 8.2 and 8.3 we compare the cost performance of the perfect foresight policy, the stochastic policy, the 3+5 rule and the best peak-load-based policy for the two different wind integration cases.
The column with bold figures, that corresponds to the stochastic policy, contains absolute cost values. Cost figures corresponding to the other policies are relative to the stochastic policy costs. The row with total costs weighs the cost of each day type with its relative frequency in the year in order to yield annual results. The last row shows the improvement of the stochastic policy over each other policy, normalized by the cost of the stochastic policy.

The stochastic policy indeed improves on the deterministic policies. The relative savings are greater for the case of deep wind integration. This indicates that the benefits of stochastic unit commitment are larger as uncertainty increases in the system. The perfect foresight policy has a significant advantage over the stochastic policy in the deep integration case, versus the moderate case, because greater wind integration exacerbates the level of uncertainty in the system. The 3+5 rule performs better in the deep integration case versus the moderate integration case, compared to the peak-load-based policy. The stochastic policy yields 41% of the potential benefits of having perfect knowledge of the future compared to the best deterministic policy for the moderate integration integration case, and 34% of the benefits for the deep integration case.

We present the total fossil fuel capacity and the average slow generator capacity that is committed for each day type and each policy in Tables 8.4 and 8.5. For the stochastic unit commitment formulation, this table includes the capacity of those slow generators that are committed for at least one hour of the day, or those fast generators that are committed for at least one hour for at least one scenario. For the deterministic unit commitment formulation, this table includes those generators which are required to supply power, slow reserves or fast reserves for at least one hour of the day. The last line of these tables presents total capacity, which is calculated by weighing the results of each day type by the frequency of occurrence of the respective day type. We note that in the moderate integration case, the stochastic unit commitment policy tends to commit less total capacity, and less slow capacity. In contrast, in the deep integration case, the stochastic policy commits more slow capacity and less total capacity. It is interesting to note that the stochastic policy achieves savings with respect to the deterministic policies both in the case where it commits more, as well as less capacity. In the cases where the stochastic policy commits less slow capacity (e.g. summer weekends), the savings result from peak load periods during which the deterministic policies incur large startup costs by committing an excessive amount of slow reserves in order to satisfy reserve requirement constraints. In the cases where the stochastic policy commits more slow capacity (e.g. spring weekdays) the deterministic policies commit less capacity because they underestimate the potential fuel and minimum run savings. This is due to the fact that the deterministic policies optimize for expected wind supply, instead of averaging the cost savings of insuring against fast capacity dispatch for various wind supply outcomes. Due to the fact that fuel and minimum run costs are convex for the system under consideration (see Figure 8.2), deterministic policies underestimate savings from committing slow reserves.

We also present the daily amount of wind that is spilled in Tables 8.6 and 8.7. In contrast to the results presented in Ruiz et al. 83, the average wind that is spilled by the perfect
Table 8.4: Slow and total capacity commitment for each policy for the single-area case study (MW) - moderate integration

<table>
<thead>
<tr>
<th>Day Type</th>
<th>Stochastic Slow</th>
<th>Stochastic Total</th>
<th>3+5 Slow</th>
<th>3+5 Total</th>
<th>20% of peak Slow</th>
<th>20% of peak Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>8,128</td>
<td>14,811</td>
<td>8,220</td>
<td>17,358</td>
<td>7,946</td>
<td>17,401</td>
</tr>
<tr>
<td>SpringWD</td>
<td>8,041</td>
<td>14,910</td>
<td>7,989</td>
<td>17,258</td>
<td>7,858</td>
<td>16,855</td>
</tr>
<tr>
<td>SummWD</td>
<td>11,261</td>
<td>20,969</td>
<td>11,646</td>
<td>25,069</td>
<td>11,999</td>
<td>25,254</td>
</tr>
<tr>
<td>FallWD</td>
<td>9,173</td>
<td>15,693</td>
<td>9,296</td>
<td>18,531</td>
<td>9,377</td>
<td>18,396</td>
</tr>
<tr>
<td>WinterWE</td>
<td>6,044</td>
<td>11,503</td>
<td>6,167</td>
<td>14,702</td>
<td>6,131</td>
<td>13,626</td>
</tr>
<tr>
<td>SpringWE</td>
<td>5,804</td>
<td>11,183</td>
<td>6,276</td>
<td>13,991</td>
<td>6,135</td>
<td>14,020</td>
</tr>
<tr>
<td>SummWE</td>
<td>9,018</td>
<td>16,647</td>
<td>9,401</td>
<td>21,141</td>
<td>9,443</td>
<td>21,076</td>
</tr>
<tr>
<td>FallWE</td>
<td>7,187</td>
<td>12,842</td>
<td>7,028</td>
<td>16,840</td>
<td>6,918</td>
<td>15,907</td>
</tr>
<tr>
<td>Total</td>
<td>8,540</td>
<td>15,580</td>
<td>8,696</td>
<td>18,729</td>
<td>8,648</td>
<td>18,528</td>
</tr>
</tbody>
</table>

Table 8.5: Slow and total capacity commitment for each policy for the single-area case study (MW) - deep integration

<table>
<thead>
<tr>
<th>Day Type</th>
<th>Stochastic Slow</th>
<th>Stochastic Total</th>
<th>3+5 Slow</th>
<th>3+5 Total</th>
<th>30% of peak Slow</th>
<th>30% of peak Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>7,012</td>
<td>15,160</td>
<td>7,014</td>
<td>14,856</td>
<td>6,779</td>
<td>14,837</td>
</tr>
<tr>
<td>SpringWD</td>
<td>7,818</td>
<td>14,845</td>
<td>6,810</td>
<td>15,889</td>
<td>6,643</td>
<td>15,506</td>
</tr>
<tr>
<td>SummWD</td>
<td>10,858</td>
<td>20,766</td>
<td>11,033</td>
<td>24,809</td>
<td>11,555</td>
<td>25,591</td>
</tr>
<tr>
<td>FallWD</td>
<td>8,608</td>
<td>16,476</td>
<td>8,417</td>
<td>18,557</td>
<td>8,493</td>
<td>19,143</td>
</tr>
<tr>
<td>WinterWE</td>
<td>5,630</td>
<td>11,746</td>
<td>5,569</td>
<td>14,815</td>
<td>5,353</td>
<td>12,010</td>
</tr>
<tr>
<td>SpringWE</td>
<td>5,553</td>
<td>11,639</td>
<td>5,670</td>
<td>11,637</td>
<td>5,239</td>
<td>12,151</td>
</tr>
<tr>
<td>SummWE</td>
<td>8,759</td>
<td>17,799</td>
<td>8,873</td>
<td>20,956</td>
<td>8,804</td>
<td>21,686</td>
</tr>
<tr>
<td>FallWE</td>
<td>6,904</td>
<td>12,823</td>
<td>6,632</td>
<td>15,349</td>
<td>6,687</td>
<td>16,044</td>
</tr>
<tr>
<td>Total</td>
<td>8,041</td>
<td>15,866</td>
<td>7,848</td>
<td>17,716</td>
<td>7,840</td>
<td>17,827</td>
</tr>
</tbody>
</table>

The foresight policy is less. The spillage in the moderate integration case is negligible, and the stochastic policy spills less wind compared to the deterministic policies, which is consistent with the observations in Ruiz et al. [83]. On the contrary, losses from the stochastic unit commitment policy in the deep integration case are slightly greater due to the fact that the average slow capacity that is committed in the stochastic policy is greater than the average slow capacity committed in the deterministic policies. Hence we observe that, in order to reduce fuel and startup costs in the deep integration case, the stochastic policy commits more reserves and sheds slightly more wind power.
Table 8.6: Daily wind spillage (MWh) for each policy for the single-area case study - moderate integration

<table>
<thead>
<tr>
<th></th>
<th>Stoch</th>
<th>Clair</th>
<th>3+5</th>
<th>20% peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>5</td>
<td>7</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>SpringWD</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>SummerWD</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FallWD</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>WinterWE</td>
<td>3,034</td>
<td>3,000</td>
<td>3,004</td>
<td>3,033</td>
</tr>
<tr>
<td>SpringWE</td>
<td>1,641</td>
<td>1,648</td>
<td>2,145</td>
<td>2,136</td>
</tr>
<tr>
<td>SummerWE</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>FallWE</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.2</td>
</tr>
<tr>
<td>Total</td>
<td>335</td>
<td>333</td>
<td>369</td>
<td>371</td>
</tr>
</tbody>
</table>

Table 8.7: Daily wind spillage (MWh) for each policy for the single-area case study - deep integration

<table>
<thead>
<tr>
<th></th>
<th>Stoch</th>
<th>Clair</th>
<th>3+5</th>
<th>30% peak</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>8,970</td>
<td>7,460</td>
<td>9,446</td>
<td>9,429</td>
</tr>
<tr>
<td>SpringWD</td>
<td>8,641</td>
<td>5,438</td>
<td>8,240</td>
<td>8,205</td>
</tr>
<tr>
<td>SummerWD</td>
<td>542</td>
<td>453</td>
<td>463</td>
<td>486</td>
</tr>
<tr>
<td>FallWD</td>
<td>1,746</td>
<td>1,248</td>
<td>1,697</td>
<td>1,696</td>
</tr>
<tr>
<td>WinterWE</td>
<td>28,920</td>
<td>19,721</td>
<td>28,901</td>
<td>28,870</td>
</tr>
<tr>
<td>SpringWE</td>
<td>32,261</td>
<td>19,330</td>
<td>32,344</td>
<td>32,040</td>
</tr>
<tr>
<td>SummerWE</td>
<td>3,886</td>
<td>3,324</td>
<td>3,731</td>
<td>3,705</td>
</tr>
<tr>
<td>FallWE</td>
<td>8,427</td>
<td>5,654</td>
<td>8,376</td>
<td>8,389</td>
</tr>
<tr>
<td>Total</td>
<td>8,803</td>
<td>6,038</td>
<td>8,783</td>
<td>8,753</td>
</tr>
</tbody>
</table>

8.2.2 Computational Performance

The stochastic unit commitment algorithm was implemented in AMPL. The mixed integer programs were solved with CPLEX 11.0.0 on a DELL Poweredge 1850 server (Intel Xeon 3.4 GHz, 1GB RAM). The first and second subproblem were run for 200 iterations. For the last 100 iterations ($ED_s$) for all $s \in S$ was solved in order to obtain an upper bound and a feasible solution. The average elapsed time for this entire process was 5685 seconds. The MIP gap for the first and second subproblem was set to $\epsilon_1 = 1\%$, and the MIP gap for obtaining a feasible schedule was set to $\epsilon_2 = 0.1\%$. The sum of the optimal solutions of the first and second subproblem yield a lower bound $LB$ on the optimal cost, whereas the optimal solution of the feasibility run results in an upper bound $UB$. The average gap, $\frac{UB-LB}{LB}$, that we obtained was 0.80%. However, to estimate an upper bound on the optimality gap we also need to account for the MIP gap $\epsilon_1$ that we introduce in the solution of the first and second subproblem. The average upper bound on the optimality gap, $\frac{UB-(1-\epsilon_1)LB}{(1-\epsilon_1)LB}$, is
Table 8.8: Deterministic policy cost comparison for the multi-area case study.

<table>
<thead>
<tr>
<th>Case</th>
<th>10%</th>
<th>20%</th>
<th>30%</th>
<th>40%</th>
<th>50%</th>
<th>3+5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deep-Simple</td>
<td>8,115,833</td>
<td>5,073,691</td>
<td>5,075,194</td>
<td>5,119,032</td>
<td>5,213,208</td>
<td>5,050,497</td>
</tr>
<tr>
<td>No wind</td>
<td>11,549,985</td>
<td>11,487,036</td>
<td>11,663,405</td>
<td>N/A</td>
<td>11,508,482</td>
<td></td>
</tr>
<tr>
<td>Moderate</td>
<td>10,073,080</td>
<td>9,598,033</td>
<td>9,615,189</td>
<td>9,593,490</td>
<td>9,593,490</td>
<td></td>
</tr>
<tr>
<td>Deep</td>
<td>N/A</td>
<td>7,742,715</td>
<td>7,669,950</td>
<td>7,780,604</td>
<td>7,671,088</td>
<td></td>
</tr>
</tbody>
</table>

1.75%.

8.3 Wind Integration in a System with Transmission Constraints and Contingencies

We now study an enhanced model of the previous network that includes transmission constraints and contingencies. As we will demonstrate, it is crucial to model these features in order to assess the cost impacts and capacity savings of wind integration accurately. We study three levels of wind integration, the zero wind integration case as well as a case of moderate and deep wind integration. We also perform the deep integration case study for the case where transmission constraints and contingencies are not accounted for, in order to quantify the impact of these effects on the analysis. The latter case is denoted as Deep-Simple.

8.3.1 Relative Performance of Policies

As in the previous case study, we first discuss the relative performance of the stochastic and deterministic unit commitment policies. The results are obtained by running the economic dispatch model against 1000 Monte Carlo outcomes of wind power production and contingencies, with a probability of generator failure of 1% \cite{regulators} and a probability of transmission line failure of 0.1% \cite{transmission}. Wind production outcomes, generator failures and transmission line failures are assumed to be independent. The results are shown in Table 8.8, where the best peak-load-based policy is highlighted in italic font and the best deterministic policy is highlighted in bold font. N/A denotes that the specific policy was not evaluated in order to avoid superfluous computation. We note that the best peak-load policy outperforms the 3+5 rule for all but the deep integration case study without transmission constraints and contingencies.

The relative performance of stochastic unit commitment with respect to the deterministic policies and the perfect foresight policy for the four case studies are presented in Figure 8.5. The results are presented in terms of the relative cost of each policy compared to

\footnote{This corresponds to the case study of Section 8.2 however the results of the previous study cannot be used directly since in this case study we permit the spillage of imports and non-wind renewable supply.}
the stochastic unit commitment policy for each of the eight day types. In the last two rows of Table 8.9 we present the absolute cost of the stochastic unit commitment policy as well as its gains over the best deterministic policy relative to the perfect foresight policy. We note that the stochastic policy benefits range between 32.4% to 46.7% of the potential benefits of perfect forecasting, with higher wind integration resulting in higher benefits due to the increased uncertainty in the system. Moreover, the introduction of transmission constraints affects both the relative as well as absolute gains of stochastic unit commitment, which supports the argument that stochastic unit commitment is especially valuable for the determination of locational capacity requirements.

### 8.3.2 Renewables Utilization, Operating Costs and Capacity Requirements

In Table 8.9 we present summary results for renewable energy spillage, operating costs and capacity requirements for the four case studies under consideration. Renewable energy losses are negligible relative to total renewable energy production, although accounting for transmission constraints and contingencies results in a twentyfold increase in the estimated loss of renewable power production in the case of deep integration. Operating costs decline steeply
Table 8.9: Renewable energy spillage, operating costs and capacity requirements for the multi-area case study.

<table>
<thead>
<tr>
<th></th>
<th>Deep-Simple</th>
<th>No Wind</th>
<th>Moderate</th>
<th>Deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>RE daily waste (MWh)</td>
<td>100</td>
<td>0</td>
<td>890</td>
<td>2,186</td>
</tr>
<tr>
<td>Cost ($M)</td>
<td>5.012</td>
<td>11.508</td>
<td>9.363</td>
<td>7.481</td>
</tr>
<tr>
<td>Capacity (MW)</td>
<td>20,744</td>
<td>26,377</td>
<td>26,068</td>
<td>26,068</td>
</tr>
<tr>
<td>Daily savings ($)</td>
<td>38,628</td>
<td>104,321</td>
<td>198,199</td>
<td>188,735</td>
</tr>
<tr>
<td>Forecast gains (%)</td>
<td>32.4</td>
<td>35.4</td>
<td>41.9</td>
<td>46.7</td>
</tr>
</tbody>
</table>

As the level of renewable power penetration increases, due to the decrease in fuel costs, which is the predominant cost in the system. Failing to account for transmission constraints and contingencies results in an underestimation of operating costs by 33.0% relative to the costs when accounting for these factors. The significant cost increase resulting from transmission constraints can be attributed to increased spillage of freely available energy but also to the reduced flexibility of dispatching units in the system.

Capacity requirements, which are the most important factor in analyzing the economics of renewable energy integration, present the most interesting results. We note that moderate wind integration reduces capacity requirements by a mere 1.2% of the installed wind capacity, whereas the capacity requirements for the deep integration case are the same as for the moderate integration scenario, indicating that the excess wind capacity cannot contribute to capacity savings. Most importantly, we note that failing to account for transmission constraints results in an overestimation of the capacity credit of wind power production by 39.8% relative to the 1.2% capacity credit when these features are accounted for. This strongly supports the argument that the inclusion of transmission constraints and contingencies is crucial for accurately assessing the impact of large-scale renewable energy integration.

8.3.3 Computational Performance

The stochastic unit commitment algorithm was implemented in the Java callable library of CPLEX 11.0.0, and parallelized using the Parallel Virtual Machine (PVM) on a network of 16 DELL Poweredge 1850 servers (Intel Xeon 3.4 GHz, 1GB RAM). \((P1_s), s \in S\) and \((P2)\) were run for 120 iterations. For the last 40 iterations, \((ED_s)\) was run for each \(s \in S\) in order to obtain a feasible solution and an upper bound for the stochastic unit commitment problem. The average elapsed time on a single machine was 43,776 seconds. The MIP gap for \((P1_s), s \in S\) and \((P2)\) was set to \(\epsilon_1 = 1\%\), and the MIP gap for obtaining a feasible schedule from \((ED_s)\) was set to \(\epsilon_2 = 0.1\%\). The sum of the optimal solutions of the first and second subproblem yield a lower bound \(LB\) on the optimal cost, whereas the optimal solution of the feasibility run results in an upper bound \(UB\). The average gap, \(\frac{UB-LB}{LB}\), that we obtained is 1.39%. However, to estimate an upper bound on the optimality gap it is also necessary to account for the MIP gap \(\epsilon_1\) that is introduced in the solution of \((P1)\) and
Table 8.10: Key parameters of the demand response case study.

<table>
<thead>
<tr>
<th></th>
<th>No Wind</th>
<th>Moderate</th>
<th>Deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wind capacity (MW)</td>
<td>0</td>
<td>6688</td>
<td>14143</td>
</tr>
<tr>
<td>DR Capacity C (MW)</td>
<td>0</td>
<td>5000</td>
<td>10000</td>
</tr>
<tr>
<td>Daily wind energy (MWh)</td>
<td>0</td>
<td>46485</td>
<td>95414</td>
</tr>
<tr>
<td>Daily DR energy R (MWh)</td>
<td>0</td>
<td>40000</td>
<td>80000</td>
</tr>
<tr>
<td>Flexible/firm demand (%)</td>
<td>0</td>
<td>6.1</td>
<td>12.2</td>
</tr>
</tbody>
</table>

\((P_{2_s}), s \in S\). The average upper bound on the optimality gap, \(\frac{UB-(1-\epsilon_1)LB}{(1-\epsilon_1)LB}\), is 2.41%.

### 8.4 Demand Response Integration

In this section we consider the simultaneous integration of renewable resources and deferrable demand. We consider three integration cases that are summarized in Table 8.10. The wind integration levels correspond exactly to the zero wind, moderate wind and deep wind integration studies presented in the previous sections. For each level of wind integration, we assume a demand response integration level that is approximately one-for-one in terms of energy demand and capacity. We assume that deferrable requests span over 24 hours. To put these values in perspective, if we assume that a typical electric vehicle has a power rating of 3.6 kW and a mileage of 0.25 kWh per mile, the deep integration case with \(R = 80000\) MWh and \(C = 10000\) MW roughly represents the electricity demand of 4.2 million electric vehicles that travel 96 miles per vehicle per day. We consider 6 levels of charge for the control problem. The penalty of unserved energy is \(\rho = 5000\) $/MWh. In addition to wind power production uncertainty, we also model firm (non-deferrable) demand as a second-order autoregressive process. As we discuss in Chapter 7, the commitment of slow generators in the day ahead is determined from the solution of \((CSUC)\) in Section 7.1. We use 12 scenarios for the formulation of the stochastic unit commitment model.

As we discuss in Section 7.3.1, deferrable demand can produce great economic value by limiting the requirements for balancing capacity, and this can be achieved by limiting the extent to which deferrable loads participate in the real-time market. Callable forward contracts can be used for this purpose. The strike price of the callable forward contracts determines the extent to which loads can participate in the market. As the strike price of the contracts decreases, the participation of loads in the real-time market is increasingly limited. Below a certain threshold, deferrable loads cannot be satisfied against all possible realizations of wind power production and prices. In Table 8.11 we present this threshold for each of the day types for each integration study. In order to simplify the analysis, we assume a common strike price for each hour of the day.

In table 8.12 we present the operating costs and daily load losses for the case with no wind and no demand response in the system. The operating costs do not include the cost of
Table 8.11: Strike price threshold for deferrable load callable forward contracts ($/MWh).

<table>
<thead>
<tr>
<th></th>
<th>Moderate</th>
<th>Deep</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>45</td>
<td>47</td>
</tr>
<tr>
<td>SpringWD</td>
<td>45</td>
<td>49</td>
</tr>
<tr>
<td>SummerWD</td>
<td>49</td>
<td>53</td>
</tr>
<tr>
<td>FallWD</td>
<td>49</td>
<td>54</td>
</tr>
<tr>
<td>WinterWE</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>SpringWE</td>
<td>45</td>
<td>47</td>
</tr>
<tr>
<td>SummerWE</td>
<td>48</td>
<td>51</td>
</tr>
<tr>
<td>FallWE</td>
<td>49</td>
<td>52</td>
</tr>
</tbody>
</table>

lost load. Note that for the average demand of the system under consideration, the 1-day-in-10-years reliability criterion requires daily load shed of no more than 179 MWh. This can be used as a benchmark against which we can compare the extent to which each demand response mechanism is acceptable from a reliability perspective.

In Tables 8.13 and 8.15 we present the daily operating cost of each policy for the moderate and deep integration cases respectively. The column with bold figures, that corresponds to centralized load dispatch by the system operator, contains absolute cost values. Cost figures corresponding to the other policies are relative to the centralized operating costs. The row with total costs weighs the cost of each day type with its relative frequency in the year in order to yield annual results. The last row shows the relative performance of centralized control with respect to the other policies, normalized by the cost of centralized control. Note that the operating costs of the decoupled demand response mechanism outperform those of the coupling mechanism. This can be attributed to the diversification effect of including flexible demand in the market.

The diversification benefits of demand-side bidding can be elucidated by the following example. If, in a certain hour of operations, firm demand is excessively high and necessitates the shedding of load, then demand-side bids will result in an efficient dispatch of demand down to the point where load shedding is prevented. Instead, coupling contracts will result in deferrable consumers increasing their consumption up to the level of available wind power supply, as they are not exposed to the real-time price up to this level of consumption. This results in efficiency losses as the available wind power supply is, at the given hour, more valuable to other consumers than it is to the consumers that have coupled their operations with renewable supply.

The "cost of anarchy" that results from using price signals in order to control load response, rather than centralized control, ranges from 2.43% - 6.88% for the case of demand-side bidding and 3.06% - 8.38% in the case of coupling. We note that the rate of increase in costs relative to decentralized control increases as uncertainty in the system increases.

Although demand bids result in lower operating costs, demand-side bidding results in load shedding that is 3.4 times greater than the 1-day-in-10-years criterion for the moderate
Table 8.12: Daily cost of operations and load shedding for each day type for the demand response study - no wind.

<table>
<thead>
<tr>
<th></th>
<th>Daily Cost ($)</th>
<th>Shed (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>7,390,206</td>
<td>0.001</td>
</tr>
<tr>
<td>SpringWD</td>
<td>7,145,737</td>
<td>4.317</td>
</tr>
<tr>
<td>SummerWD</td>
<td>13,684,880</td>
<td>30.869</td>
</tr>
<tr>
<td>FallWD</td>
<td>9,589,506</td>
<td>0</td>
</tr>
<tr>
<td>WinterWE</td>
<td>6,079,003</td>
<td>0.001</td>
</tr>
<tr>
<td>SpringWE</td>
<td>5,855,883</td>
<td>0</td>
</tr>
<tr>
<td>SummerWE</td>
<td>11,839,573</td>
<td>0</td>
</tr>
<tr>
<td>FallWE</td>
<td>7,868,146</td>
<td>154.285</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>9,012,031</strong></td>
<td><strong>17.301</strong></td>
</tr>
</tbody>
</table>

Table 8.13: Daily cost of operations for each day type for the demand response study - moderate integration.

<table>
<thead>
<tr>
<th></th>
<th>Cost ($)</th>
<th>∆ Cost ($)</th>
<th>∆ Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Centralized</td>
<td>Coupled</td>
<td>Decoupled</td>
</tr>
<tr>
<td>WinterWD</td>
<td>7,320,620</td>
<td>256,740</td>
<td>300,051</td>
</tr>
<tr>
<td>SpringWD</td>
<td>6,408,355</td>
<td>172,006</td>
<td>139,589</td>
</tr>
<tr>
<td>SummerWD</td>
<td>13,625,136</td>
<td>155,096</td>
<td>219,124</td>
</tr>
<tr>
<td>FallWD</td>
<td>9,640,017</td>
<td>316,089</td>
<td>157,159</td>
</tr>
<tr>
<td>WinterWE</td>
<td>5,890,755</td>
<td>300,701</td>
<td>246,408</td>
</tr>
<tr>
<td>SpringWE</td>
<td>3,637,240</td>
<td>707,223</td>
<td>244,353</td>
</tr>
<tr>
<td>SummerWE</td>
<td>11,739,177</td>
<td>176,230</td>
<td>234,101</td>
</tr>
<tr>
<td>FallWE</td>
<td>7,735,502</td>
<td>277,817</td>
<td>189,465</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>8,677,857</strong></td>
<td>265,128</td>
<td>211,010</td>
</tr>
<tr>
<td>relative (%)</td>
<td></td>
<td>3.06</td>
<td>2.43</td>
</tr>
</tbody>
</table>

integration case and 6.8 times greater for the deep integration case. The use of coupling contracts results in the operation of the system within reliability limits as we note in Tables 8.14 and 8.16.

In Table 8.17 we present a breakdown of operating costs by type for each of the four policies that we consider for each integration level. We note that the demand function and coupling models result in cost increases in all cost categories. As Sioshansi [89] argues, the marginal cost signal itself does not necessarily induce efficient load response due to the fact that it fails to capture the non-convex operating costs of the system. The observation of Sioshansi is also supported by our results.

In Table 8.18 we present the amount of capacity that is committed by each policy as well as the amount of renewable supply spillage. Capacity requirements do not change significantly for each integration study, which suggests that the additional deferrable demand
Table 8.14: Daily load loss for each day type for the demand response study - moderate integration.

<table>
<thead>
<tr>
<th></th>
<th>Shed (MWh)</th>
<th>Shed (MWh)</th>
<th>Shed (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Centralized</td>
<td>Coupled</td>
<td>Decoupled</td>
</tr>
<tr>
<td>WinterWD</td>
<td>0</td>
<td>0</td>
<td>177.257</td>
</tr>
<tr>
<td>SpringWD</td>
<td>1.532</td>
<td>1.869</td>
<td>701.828</td>
</tr>
<tr>
<td>SummerWD</td>
<td>3.617</td>
<td>4.346</td>
<td>821.719</td>
</tr>
<tr>
<td>FallWD</td>
<td>1.661</td>
<td>1.661</td>
<td>799.323</td>
</tr>
<tr>
<td>WinterWE</td>
<td>0</td>
<td>0</td>
<td>642.105</td>
</tr>
<tr>
<td>SpringWE</td>
<td>0</td>
<td>0.249</td>
<td>453.791</td>
</tr>
<tr>
<td>SummerWE</td>
<td>0.059</td>
<td>1.100</td>
<td>215.816</td>
</tr>
<tr>
<td>FallWE</td>
<td>6.792</td>
<td>10.005</td>
<td>976.766</td>
</tr>
<tr>
<td>Total</td>
<td>1.705</td>
<td>2.217</td>
<td>609.914</td>
</tr>
</tbody>
</table>

Table 8.15: Daily cost of operations for each day type for the demand response study - deep integration.

<table>
<thead>
<tr>
<th></th>
<th>Cost ($) Centralized</th>
<th>Δ Cost ($) Coupled</th>
<th>Δ Cost ($) Decoupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>WinterWD</td>
<td>6,656,665</td>
<td>633,164</td>
<td>556,775</td>
</tr>
<tr>
<td>SpringWD</td>
<td>5,692,860</td>
<td>978,182</td>
<td>572,465</td>
</tr>
<tr>
<td>SummerWD</td>
<td>13,661,862</td>
<td>505,869</td>
<td>835,609</td>
</tr>
<tr>
<td>FallWD</td>
<td>9,321,281</td>
<td>772,659</td>
<td>404,523</td>
</tr>
<tr>
<td>WinterWE</td>
<td>5,220,109</td>
<td>711,882</td>
<td>616,931</td>
</tr>
<tr>
<td>SpringWE</td>
<td>4,251,600</td>
<td>910,253</td>
<td>576,010</td>
</tr>
<tr>
<td>SummerWE</td>
<td>12,136,223</td>
<td>329,929</td>
<td>472,930</td>
</tr>
<tr>
<td>FallWE</td>
<td>7,930,823</td>
<td>700,205</td>
<td>515,431</td>
</tr>
<tr>
<td>Total</td>
<td>8,419,322</td>
<td>705,497</td>
<td>578,909</td>
</tr>
<tr>
<td>relative (%)</td>
<td></td>
<td>8.38</td>
<td>6.88</td>
</tr>
</tbody>
</table>

can be fully absorbed by the installed renewable capacity. Wind spillage is negligible across all cases.
Table 8.16: Daily load loss for each day type for the demand response study - deep integration.

<table>
<thead>
<tr>
<th>Day Type</th>
<th>Shed (MWh)</th>
<th>Centralized</th>
<th>Coupled</th>
<th>Decoupled</th>
</tr>
</thead>
<tbody>
<tr>
<td>Winter WD</td>
<td>0.001</td>
<td>8.290</td>
<td>552.769</td>
<td></td>
</tr>
<tr>
<td>Spring WD</td>
<td>0</td>
<td>351.782</td>
<td>1382.459</td>
<td></td>
</tr>
<tr>
<td>Summer WD</td>
<td>0.001</td>
<td>36.643</td>
<td>1952.332</td>
<td></td>
</tr>
<tr>
<td>Fall WD</td>
<td>33.660</td>
<td>143.629</td>
<td>1210.443</td>
<td></td>
</tr>
<tr>
<td>Winter WE</td>
<td>0</td>
<td>0</td>
<td>929.960</td>
<td></td>
</tr>
<tr>
<td>Spring WE</td>
<td>0</td>
<td>32.601</td>
<td>1008.222</td>
<td></td>
</tr>
<tr>
<td>Summer WE</td>
<td>2.081</td>
<td>58.725</td>
<td>1157.565</td>
<td></td>
</tr>
<tr>
<td>Fall WE</td>
<td>57.005</td>
<td>132.134</td>
<td>1260.137</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>10.231</td>
<td>112.452</td>
<td>1221.492</td>
<td></td>
</tr>
</tbody>
</table>

Table 8.17: Breakdown of daily operating costs for each demand response policy for each integration level ($).

<table>
<thead>
<tr>
<th>Policy</th>
<th>Min load</th>
<th>Fuel</th>
<th>Startup</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No wind</td>
<td>1,382,156</td>
<td>7,549,491</td>
<td>80,384</td>
<td>9,098,537</td>
</tr>
<tr>
<td>Centralized Moderate</td>
<td>1,246,552</td>
<td>7,364,815</td>
<td>66,489</td>
<td>8,677,857</td>
</tr>
<tr>
<td>Bids Moderate</td>
<td>1,317,383</td>
<td>7,471,363</td>
<td>100,123</td>
<td>8,888,866</td>
</tr>
<tr>
<td>Coupled Moderate</td>
<td>1,330,130</td>
<td>7,532,898</td>
<td>79,958</td>
<td>8,942,958</td>
</tr>
<tr>
<td>Centralized Deep</td>
<td>1,194,606</td>
<td>7,174,611</td>
<td>50,105</td>
<td>8,419,322</td>
</tr>
<tr>
<td>Bids Deep</td>
<td>1,360,543</td>
<td>7,494,472</td>
<td>143,217</td>
<td>8,998,232</td>
</tr>
<tr>
<td>Coupled Deep</td>
<td>1,432,948</td>
<td>7,592,595</td>
<td>99,276</td>
<td>9,124,819</td>
</tr>
</tbody>
</table>

Table 8.18: Capacity requirements and wind power spillage for each demand response policy.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Capacity (MW)</th>
<th>Spillage (MWh)</th>
</tr>
</thead>
<tbody>
<tr>
<td>No wind</td>
<td>26,123</td>
<td>N/A</td>
</tr>
<tr>
<td>Moderate</td>
<td>26,254</td>
<td>0</td>
</tr>
<tr>
<td>Deep</td>
<td>26,789</td>
<td>2</td>
</tr>
</tbody>
</table>
Chapter 9

Conclusions and Perspectives

In this thesis we present a two-stage stochastic unit commitment model that can be used for assessing the impact of integrating renewable power and deferrable demand on renewable energy utilization, operating costs and generation capacity requirements. We present a subgradient algorithm that can be used for solving the problem, and we validate our scenario selection methods by demonstrating the superior performance of the stochastic unit commitment policy relative to common deterministic reserve rules. We then use the stochastic unit commitment model for studying the impact of deferrable demand on mitigating the disturbances of large-scale renewable energy integration. We first present a summary of our conclusions and then discuss a list of areas for future work that have been inspired by the present research.

9.1 A Summary of Conclusions

9.1.1 Single-Area Case Study

(a) The benefits of stochastic unit commitment: Stochastic unit commitment benefits the system more in the case of deeper renewable energy integration, due to the fact that it copes better with the increased uncertainty in the system. The stochastic unit commitment policy yields 34.4% - 41.5% of the potential benefits of the perfect foresight policy compared to the best deterministic policy.

(b) Using more wind power is not always better: In the deep integration case, the stochastic unit commitment policy commits more slow capacity than the best deterministic policy. This is attributed to the fact that, due to the convexity of the marginal cost curve of the fast generation stack, the deterministic policy underestimates the value of insuring against the utilisation of fast reserves. The fact that the stochastic unit commitment policy commits more slow resources results in increased shedding of wind power due to the minimum
loading requirement of the slow units, but results in better performance of the stochastic unit commitment policy.

9.1.2 Multi-Area Case Study

(a) The benefits of stochastic unit commitment: The stochastic unit commitment policy yields 46.7% of the potential benefits of the perfect foresight policy relative to the best deterministic policy. Note that the inclusion of transmission constraints enhances the benefits of stochastic unit commitment.

(b) Transmission constraints matter for capacity requirement assessments: Ignoring transmission constraints can lead to highly misleading results regarding the capacity savings of renewable energy integration. Failing to account for transmission constraints and contingencies features results in an overestimation of wind capacity credit by 39.8%, relative to 1.2% when these features are accounted for.

9.1.3 Demand Response Case Study

(a) The cost of anarchy: Centralized load dispatch represents a limit on the potential benefits of demand flexibility. The "cost of anarchy" incurred by decentralizing demand response ranges between 3.06% - 8.38% for the case of coupling. Demand-side bidding outperforms coupling with respect to operating costs, resulting in a cost increase ranging between 2.43% - 6.88% of the cost resulting from centralized load dispatch. However, demand-side bidding fails to capture the cross-elasticity of demand across time periods, resulting in excessive load failures that violate the 1-day-in-10-years by 3.4 to 6.8 times. Instead, centralized load dispatch and coupling maintain the operation of the system within acceptable reliability criteria.

(b) Capacity requirements and renewable supply utilization: For the case studies that we consider, the additional integration of deferrable demand imposes no additional capacity requirements to the system. Renewable supply capacity is adequate for satisfying the added demand, which represents 6.1% - 12.2% of firm power demand for the 2012 and 2020 renewable integration targets respectively. The waste of available renewable power supply is negligible for the demand response integration study, where transmission constraints are ignored.

9.2 Future Areas of Research

Solar power modeling: Our analysis in Chapter 8 has focused exclusively on the impacts of wind power integration. The same tools can also be used for analyzing the impact of solar
power integration. The extension of our analysis in this direction is interesting for various reasons: firstly, solar power integration is expected to increase dramatically in California in the coming years [53], solar power supply complements wind power supply very well and may improve the economics of large-scale renewable energy integration, but additionally solar power supply introduces dramatic ramping requirements. We expect the modeling of solar power supply to present interesting challenges. Mills and Wiser [56] present an extensive literature review on solar power integration studies that have been conducted since 1981.

**Price and Market Impacts of Demand Response and Renewable Energy Integration:** Our model so far has focused on renewable energy utilization, cost of operations and capacity requirements. However, we have made little mention of the impact of renewables and demand response on market prices. This is particularly interesting since the increased integration of renewables is expected to depress energy prices while making them at the same time more volatile, and this is certain to result in certain technologies coming out as winners and others as losers. We wish to examine the profitability of individual generating technologies; whether certain areas in the network are more profitable to invest than others and how the profitability of flexible consumers varies across the various demand response integration approaches. One of the outputs of the economic dispatch model is the market clearing prices for energy, therefore the information that is required for this analysis is readily available from our model.

A major issue that can be addressed from this economic analysis is the extent to which uplift payments become necessary for operating the system. As Scarf [86] points out, in markets with non-convexities, of which power markets are a prime example due to their startup and minimum load costs, efficient market clearing prices do not necessarily exist, which raises the need for uplift payments from the system operator to generators [65]. Renewable energy integration is expected to increase uplift payments in the network due to the fact that backup reserves will need to be maintained online during off-peak night hours, during which hours these units operate at a loss. The system operator will be called to cover these losses and it is interesting to quantify the extent of these uplift payments in scenarios of large-scale renewable energy integration and the extent to which demand flexibility can mitigate these payments.

**Improved representation of demand response:** The lattice model that we use for modeling deferrable demand can be improved by formulating a multi-stage stochastic linear program and using stochastic dual dynamic programming [76], [35] for solving the problem. The advantage of this approach against a lattice model is the fact that the stochastic real-time price and wind power production processes can be simulated as continuous processes.

**Integrating demand response in a transmission-constrained model:** As we demonstrate in Section 8.3 the inclusion of transmission constraints and contingencies is essential
for properly assessing the cost impacts and capacity savings of renewable energy integration. We expect our analysis on demand response integration to be analogously illuminated by the inclusion of transmission constraints.

**Parallelization:** The decomposition algorithm presented in Section 5.2 is particularly well suited for parallelization. This greatly expands the size of problems that we can address using our model.

**Contracts for deferrable consumers:** It is interesting to investigate business models for enrolling flexible consumers in demand response programs. A particular problem that we wish to address are mileage programs for enrolling drivers in electric vehicle aggregation services. The challenge in designing these contracts is to elicit the true information about the charging preferences of consumers. This research seeks to exploit insights from priority service pricing, see for example Oren et al. [68], Oren et al. [69], Wilson [97], Chao and Wilson [24].

**Transmission expansion planning:** The expansion of the transmission network in order to increase the penetration of renewable resource supply from remote areas is becoming an increasingly interesting topic to researchers [29]. The formulation of the problem resembles the stochastic unit commitment formulation of Section 3.3 with the added complication of deciding on transmission line buildup.

**Improvement of stochastic models:** The stochastic models presented in Section 4 can be extended to autoregressive moving average models [17]. We also wish to perform diagnostic checking of the performance of our models against reserved historical data. We further wish to explore kernel density estimation techniques for modeling wind power supply data [50], [9].

**Co-optimization of hydro and non-controllable renewables:** In the current model, hydro supply is treated as a parameter that is exogenously supplied. We are interested in exploring the simultaneous co-optimization of hydroelectric supply as well as other renewable energy sources characterized by stochastic availability, such as wind power and solar power, as in Gröwe-Kuska et al. [38]. For this purpose, we are interested in exploiting stochastic dual dynamic programming, which has been used in the hydro scheduling literature [76].
Appendix A

Recombinant Lattice Model of Renewable Supply and Load

Recombinant lattices are used for controlling the rate of growth of the dynamic programming lattice. Due to the fact that the state space of the optimal control problem of Equation (7.6) includes residual energy demand, we need to limit the size of the state space for the stochastic state variables, in order to solve the problem using the dynamic programming algorithm. Therefore, although it is well known that wind power production and load (and therefore real-time prices) exhibit significant autocorrelation [18], [94], [58], [21], we will simplify the stochastic models of wind power and real-time prices by assuming first-order autoregressive processes in order to control the size of the state space.

We assume that wind speed and real-time prices are driven by two correlated mean-reverting processes:

\[
X_{t+1} = X_t + \kappa_{\lambda} (\theta_{\lambda} - X_t) \Delta t + \sigma_{\lambda} \sqrt{\Delta t} \omega_1
\]

\[
Y_{t+1} = Y_t + \kappa_{w} (\theta_{w} - Y_t) \Delta t + \rho \sigma_{w} \sqrt{\Delta t} \omega_1 + \sqrt{(1 - \rho^2)} \sigma_{w} \sqrt{\Delta t} \omega_2,
\]

where \(X_t\) and \(Y_t\) are the noise terms of the price and wind models respectively, \(B_{1t}\) and \(B_{2t}\) are independent standard Brownian motion processes, \(\theta_{\lambda}\) and \(\theta_{w}\) represent the average trends of the price and wind noise respectively, the variance terms \(\sigma_{\lambda}\) and \(\sigma_{w}\) capture the effect of random shocks, \(\kappa_{\lambda}\) and \(\kappa_{w}\) model the rate at which the processes return to their mean value and \(\rho\) is a correlation coefficient that couples the evolution of the two processes.

In our study we employ a discrete model that approximates the model of Equations (A.1), (A.2). The model is presented in Deng and Oren [28]. The dynamics of the process are given by:
\[ X_{t+1}^j = \begin{cases} 
X_t + \sigma_\lambda \sqrt{\frac{3}{2}} \sqrt{\Delta t}, & j = 1 \\
X_t, & j = 2 \\
X_t - \sigma_\lambda \sqrt{\frac{3}{2}} \sqrt{\Delta t}, & j = 3 
\end{cases} \]

\[ Y_{t+1}^j = \begin{cases} 
Y_t + (\sqrt{3}\rho + \sqrt{1 - \rho^2})\sigma_w \sqrt{\frac{\Delta t}{2}}, & j = 1 \\
Y_t - \sigma_w \sqrt{1 - \rho^2} \frac{\Delta t}{\sqrt{2}}, & j = 2 \\
Y_t - (\sqrt{3}\rho - \sqrt{1 - \rho^2})\sigma_w \sqrt{\frac{\Delta t}{2}}, & j = 3
\end{cases} \]

where \( X_t^j \) and \( Y_t^j \) are the noise terms of the discrete price and wind models respectively and \( \Delta t \) is the discretization interval. Each state \( j \) is visited with a probability \( p_j \) that depends on the current state. The transition probabilities are defined in [28]. In order to describe the transition probabilities, denote

\[
A_1 = \frac{\kappa_\lambda (\theta_\lambda - X_t) + \kappa_w (\theta_w - Y_t)}{\sqrt{6}\sigma_\lambda} \sqrt{\Delta t} \\
A_2 = \frac{\kappa_w (\theta_w - Y_t)}{\sqrt{6}\sigma_w} \sqrt{\Delta t} \\
A_3 = \frac{\kappa_\lambda (\theta_\lambda - X_t) - \kappa_s (\theta_w - Y_t)}{\sqrt{6}\sigma_\lambda = \lambda} \sqrt{\Delta t}
\]

The transition probabilities are then given by the following 7 cases:

\[-\frac{1}{3} < A_1 < \frac{2}{3}, -\frac{2}{3} < A_2 < \frac{1}{3}, -\frac{2}{3} < A_3 < \frac{1}{3} \implies
p_1 = \frac{1}{3} + A_1, p_2 = \frac{1}{3} - A_2, p_3 = \frac{1}{3} - A_3\]

\[A_2 \geq \frac{1}{3} \implies p_1 = \frac{1}{2}, p_2 = 0, p_3 = \frac{1}{2}\]

\[A_2 \leq -\frac{2}{3} \implies p_1 = 0, p_2 = 1, p_3 = 0\]

\[A_1 < -\frac{1}{3}, -\frac{2}{3} < A_2 < \frac{1}{3} \implies p_1 = 0, p_2 = \frac{1}{3} - A_2, p_3 = \frac{2}{3} + A_2\]
\[ A_1 \geq \frac{2}{3}, -\frac{2}{3} < A_2 < \frac{1}{3} \Rightarrow p_1 = \frac{2}{3} + A_2, p_2 = \frac{1}{3} - A_2, p_3 = 0 \]

\[ -\frac{1}{3} < A_1 < \frac{2}{3}, -\frac{2}{3} < A_2 < \frac{1}{3}, A_3 \geq -\frac{2}{3} \Rightarrow p_1 = 0, p_2 = \frac{1}{3} - A_2, p_3 = \frac{2}{3} - A_2 \]

\[ -\frac{1}{3} < A_1 < \frac{2}{3}, -\frac{2}{3} < A_2 < \frac{1}{3}, A_3 \leq -\frac{2}{3} \Rightarrow p_1 = \frac{2}{3} + A_2, p_2 = \frac{1}{3} - A_2, p_3 = 0 \]

The state space of the trinomial lattice can be derived from Equation (A.3) [27]:

\[ A_n = \bigcup_{i=0}^{n} \{(X_{i,j}, Y_{i,j}) : X_{i,j} = X_0 - (i - 2j)\Delta X, \]
\[ Y_{i,j} = Y_0 - \left(\frac{2n - 3i}{\sqrt{2}}\sqrt{1 - \rho^2} + \frac{(i - 2j)\sqrt{3}}{\sqrt{2}}\rho\right)\Delta Y, \]
\[ j = 0, 1, \ldots, i\} \]

where \( \Delta X = \sigma_\lambda \sqrt{\frac{3}{2}} \sqrt{\Delta t} \) and \( \Delta Y = \sigma_w \sqrt{\Delta t} \).

It is therefore clear that the lattice grows as \( O(n^2) \), which enables us to control the growth rate and therefore the running time of the dynamic programming algorithm. In Figure A.1 we present a one-dimensional and two-dimensional lattice evolving in time and the cross-section of a two-dimensional lattice for a given time period. Arrows denote state transitions that are permissible from one time period to the next. Note that states in a given time period can transition to the same state in the next time period. As a result, the two-dimensional lattice grows within a two-dimensional cone, where the number of states at evolves as a quadratic function of time.
Figure A.1: From left to right: a one-dimensional recombinant lattice, a two-dimensional recombinant lattice and a cross-section of a two-dimensional recombinant lattice for a given time period.
Bibliography


