Local Network Identifiability with Partial Excitation and Measurement

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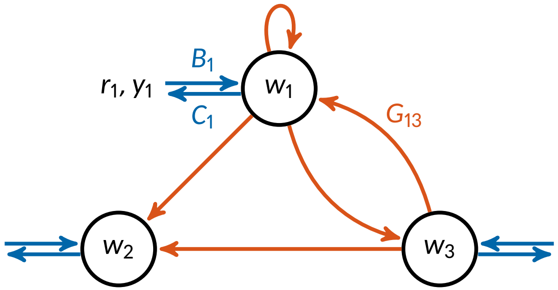
Model

$$\text{state} \longleftarrow w = G\,w + B\,r \longrightarrow \text{excitation}$$

$$\text{measure} \longleftarrow y = C\,w$$

From the given exc/meas, which transfer functions can be recovered?

i.e. From r at B and y at C, which G_{ij} can be recovered?



Assumption: Network topology is known

Terminology: TFs that can be recovered are *identifiable*

Known: exc. & measures B, C, data $T = (I - G)^{-1}$, topology

To find: transfer functions G

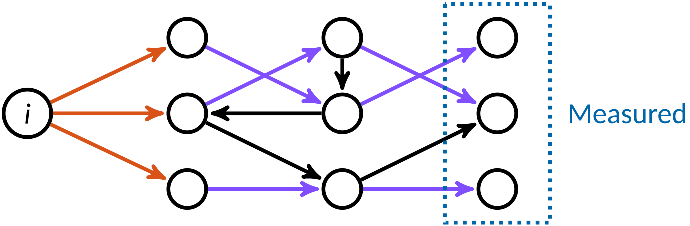
Does $CTB = C(I - G)^{-1}B$ admit a unique solution G?

Full excitation: [Hendrickx, Gevers, Bazanella 2017]

$B = I \quad \rightarrow \quad CT(I - G) = C \quad \rightarrow \quad \text{rank } CT \quad \rightarrow$ paths to Meas.

Theorem

Identifiable IFF # outneighbours = # vertex-disjoint paths to Meas.



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Full excitation:

- Algo allocating measurements [Cheng, Shi, Van den Hof 2019]

Full measurement:

- $C = I \longrightarrow$ Dual results

Partial excitation and measurement:

- Particular topologies [Bazanella, Gevers, Hendrickx CDC 2019]
- *But* arbitrary topology: simplifications KO

$\longrightarrow$ Novel approach needed

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Partial excitation and measurement: **This paper**

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Contribution

1. Definition of local identifiability (necessary)
2. Algebraic necessary and sufficient condition
3. Probability-1 algorithm

From the definition of identifiability...

Known:	exc. & measures B, C, data $T = (I - G)^{-1}$, topology
To find:	transfer functions G

Does $CTB = C(I - G)^{-1}B$ admit a unique solution G?

Definition: G generically identifiable if

$$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$$

... we introduce *local* identifiability

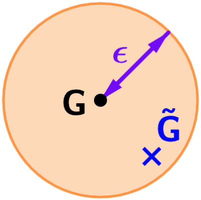
Known: exc. & measures B, C, data $T = (I - G)^{-1}$, topology

To find: transfer functions G

Does $CTB = C(I - G)^{-1}B$ admit a unique solution G?

Definition: G generically *locally* identifiable if *on an ϵ—ball,*

$$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$$

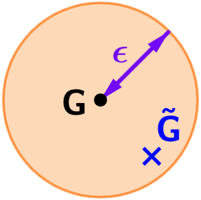


- Necessary for generic identifiability
- No counter-example to sufficiency found yet

Approach: identifiability as injectivity

Definition: G generically *locally* identifiable if *on an ϵ—ball,*

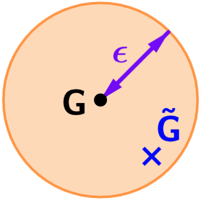
$$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$$



Approach: identifiability as injectivity

Definition: G generically *locally* identifiable if *on an ϵ—ball,*

$$\underbrace{CTB}_{f(G)} = C\tilde{T}B \Rightarrow G = \tilde{G}$$



It can be rewritten as the injectivity of f:

$$f(G) = f(G) \Rightarrow G = \tilde{G}$$

Key intuition: Locally, injectivity should rely on ∇f. Right?

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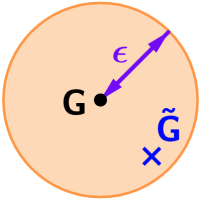

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Key intuition: Locally, injectivity should rely on ∇f. Right?

Correct if ∇f has constant rank

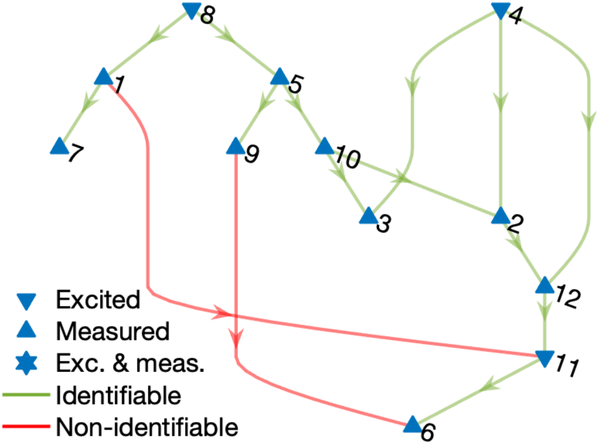
Lemma on injectivity of manifolds (cf paper)

Proof: involving subimmersion theorem and rank theorem

Theorem

G_{ij} generically locally identif $\Longleftrightarrow$ $\boxed{\ker \nabla f \perp \mathbf{e}_{ij}}$ almost[1] everywhere

$\longrightarrow$ Probability-1 algorithm: randomized, proba 0 of inaccuracy

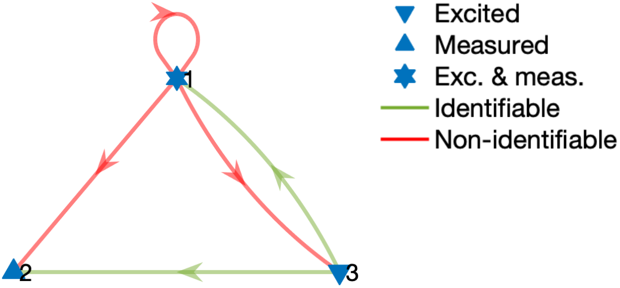


[1]except possibly on a lower-dimensional set

Future work

- Our implementation allows rapidly testing conjectures.
  But a *graph-theoretical* characterization is still to be found.
  Our algebraic condition could pave the way.

- No example identifiable but not locally identifiable found yet.
  The possible equivalence of the two remains an open question.



Take-home message

- First to tackle *partial* excitation/measure for arbitrary topology
- Introduced generic *local* identifiability, i.e. in an ϵ—ball
  - Necessary for generic identifiability
  - No counter-example to sufficiency yet
- Derived a necessary and sufficient algebraic condition using manifold theory
- Designed a probability-1 algorithm for efficient testing: `github.com/alegat/identifiable`

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