Necessary Graph Condition for Local Network Identifiability

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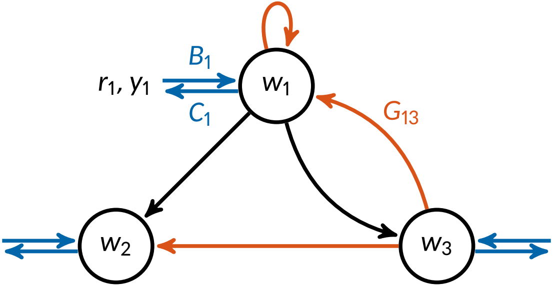
Model

$$\text{state} \longleftarrow w = G\,w + B\,r \longrightarrow \text{excitation}$$

$$\text{measure} \longleftarrow y = C\,w$$

From given exc/meas, can we recover unknown transfer functions?

i.e. From r at B and y at C, can we recover unknown G_{ij}?



Assumption: Network topology is known & **partial** exc/meas

Terminology: Unknown TFs that can be recovered: *identifiable*

From the definition of identifiability...

Definition: G generically identifiable if

$$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$$

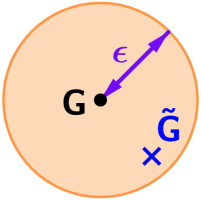
where $T \triangleq (I - G)^{-1}$.

... we introduce *local* identifiability

Definition: G generically *locally* identifiable if *on an* ϵ*—ball,*

$$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$$

- Necessary for generic identifiability
- No counter-example to sufficiency found yet



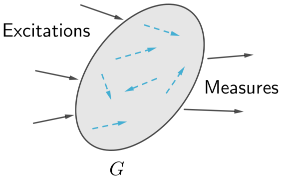
Theorem 1 (Legat; Hendrickx 2020)

G generically locally identif $\iff$ $\boxed{CT\Delta TB = 0 \Rightarrow \Delta = 0}$ $\forall\ \Delta$

almost everywhere

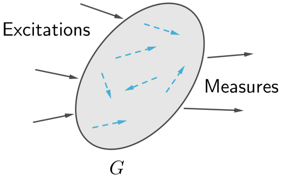
From Theorem 1 ...

Generic ...	Network G	
identifiable	$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$	
	$\Downarrow$	
local identif	$CT\Delta TB = 0 \Rightarrow \Delta = 0$	



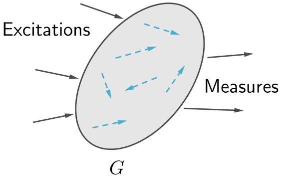
... we introduce decoupled identifiability

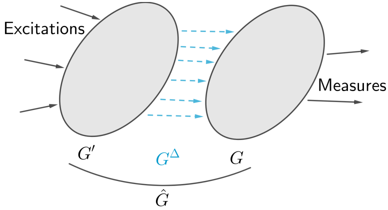
Generic ...	Network G	
identifiable	$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$	
	$\Downarrow$	
local identif	$CT\Delta TB = 0 \Rightarrow \Delta = 0$	
	$\Downarrow$	
decoupled-identif	$CT\Delta T'B = 0 \Rightarrow \Delta = 0$	



… and the decoupled network

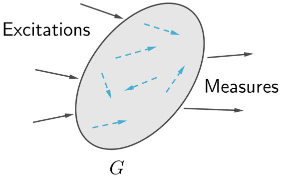
Generic …	Network G	Decoupled network $\hat{G}$
identifiable	$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$	$CT\Delta T'B = 0 \Rightarrow \Delta = 0$
	$\Downarrow$	
local identif	$CT\Delta TB = 0 \Rightarrow \Delta = 0$	
	$\Downarrow$	
decoupled-identif	$CT\Delta T'B = 0 \Rightarrow \Delta = 0$	



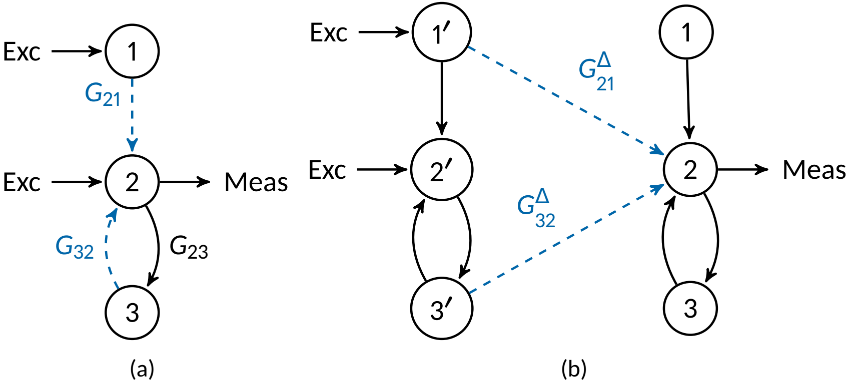


Necessary... and sufficient?

Generic ...	Network G	
identifiable	$CTB = C\tilde{T}B \Rightarrow G = \tilde{G}$	
	$\Downarrow \quad \Uparrow?$	No counter-ex so far
local identif	$CT\Delta TB = 0 \Rightarrow \Delta = 0$	
	$\Downarrow \quad \Uparrow?$	No counter-ex so far
decoupled-identif	$CT\Delta T'B = 0 \Rightarrow \Delta = 0$	



Basic example



(a): Unknowns in dashed blue

(b): Decoupled network: unknowns in the middle

Theorem

If a network is generically decoupled-identifiable, then there is at least one assignation σ : edge $\longrightarrow$ (excitation, measure) s.t.:

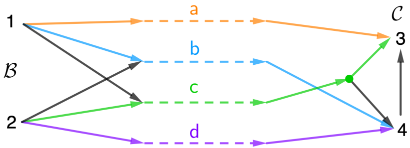
(a) $|\mathcal{C}|$ (= 2 here) unknown edges are assigned to each excitation

(b) $|\mathcal{B}|$ (= 2 here) unknown edges are assigned to each measure

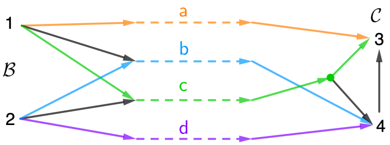
(c) σ is connected (e.g. edge a must be assigned to (1,3))

(d) for each excitation b, there are $|\mathcal{C}|$ vertex-disjoint paths between the edges assigned to b and the measures $\mathcal{C}$.

(e) for each measure c, there are $|\mathcal{B}|$ vertex-disjoint paths between the edges assigned to c and the measures $\mathcal{B}$.



σ OK



σ KO

A necessary condition and a sufficient one

Theorem

If a network is generically decoupled-identifiable, then there is at least one assignation σ such that:

(a) $|\mathcal{C}|$ unknown edges are assigned to each excitation

(b) $|\mathcal{B}|$ unknown edges are assigned to each measure

(c) σ is connected

(d) for each excitation b, there are $|\mathcal{C}|$ vertex-disjoint paths between the edges assigned to b and the measures $\mathcal{C}$.

(e) for each measure c, there are $|\mathcal{B}|$ vertex-disjoint paths between the edges assigned to c and the measures $\mathcal{B}$.

If there is only one such assignation, then this condition is also sufficient for generic decoupled identifiability.

Take-home message

- Introduced generic *decoupled-*identifiability,
  - Necessary for (generic) (local) identifiability
  - New: larger graph which decouples excitations and measures
- Derived a *graph-theoretical necessary condition* which applies to (generic) (local) identifiability
- Whether the sufficient condition extends as well remains an open question
- There *could be a stronger version* of our theorem, extending previous results of full excitation/measurement
- **Further work:** when not all edges are identifiable, obtain a graph-theoretical condition for the recovery of some edges