Local dynamics identification via a graph-theoretical approach

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1 Introduction

The goal of this work is to recover the local dynamics from the global input-output behavior of a networked system and the knowledge of the network topology.

We consider the identification of a network matrix $G(z)$, where the network is made up of n nodes, with node signals $\{w_1(t),\dots,w_n(t)\}$, and external excitation signals $\{r_1(t),\dots,r_n(t)\}$, related to each other by:

$$\begin{aligned} w(t) &= G(z)w(t) + Br(t) + v(t) \\ y(t) &= Cw(t), \end{aligned} \tag{1}$$

where matrices B and C are binary selections defining which nodes are excited and measured, forming the sets $\mathscr{B}$ and $\mathscr{C}$ respectively. The vector $y(t)$ contains the measured nodes, while $v(t)$ is a noise vector. The nonzero entries of the network matrix $G(z)$ define the topology of the network, and are assumed to be proper and rational.

We assume that the global relation between the excitations r and measures y has been identified, and that the structure of $G(z)$ is known. From this knowledge, we aim at recovering the nonzero entries of $G(z)$.

A first line of work extends the classical closed-loop identification techniques to identify a single module, see e.g. [1]. A recent approach employs graph-theoretical tools to derive identifiability conditions on the graph of the network. Using this approach, [2] address the particular case where all nodes are excited. In the general case of partial measurement *and* excitation, [3] study particular topologies of graphs and provide necessary conditions for identifiability. In this work, we consider partial measurement and excitation for all topologies. We introduce the notion of local generic identifiability, for which we derive conditions and develop an algorithm.

2 Problem reformulation

Starting from the definition of a network system in (1), we first define $T(z) \triangleq (I - G(z))^{-1}$, which is assumed to be proper and stable. The input-output model of network model (1) is then given by

$$y(t) = CT(z)Br(t) + \tilde{v}(t),$$

where $\tilde{v}(t) \triangleq CT(z)v(t)$. We assume that $r(t)$ is sufficiently rich so that, for any B and C, $CT(z)B$ can be consistently estimated from $\{y(t), r(t)\}$ data. From the knowledge of $CT(z)B$, the aim is to re-identify $G(z)$. This motivates the following definition, which restricts the usual generic identifiability from [2] to non-discrete sets of solutions.

Definition 1. *The network matrix $G(z)$ is locally generically identifiable from $\mathscr{B}$ and $\mathscr{C}$ if, for any parametrization $G(P,z)$ consistent with the graph, there exists $\tilde{G}(z)$ consistent with the graph, $\varepsilon > 0$ such that $||\tilde{G}(z) - G(P,z)|| < \varepsilon$ for a suitable notion of norm, and there holds*

$$C(I - G(P,z))^{-1}B = C(I - \tilde{G}(z))^{-1}B \Rightarrow G(P,z) = \tilde{G}(z) \tag{2}$$

for all P except possibly those lying on a zero measure set.

In this definition, a network matrix $G(z)$ is said *consistent with the graph* if $G_{ij}(z)$ is zero when there is no edge (i,j).

3 Results

One can linearize (2) to obtain the following criterion

$$CT(z)\Delta T(z)B = 0 \Rightarrow \Delta = 0, \tag{3}$$

where Δ is consistent with the graph. Through algebraic computations, we can then re-express (3) as a rank condition on a matrix K constructed from C, B and $T(z)$, which incorporates all the global information available under measurements C and excitations B. This generalizes results in [2] that were derived for the full-excitation case.

Following a similar reasoning, we derive a condition on the identifiability of a specific transfer function $G_{ij}(z)$ by inspecting the kernel of that same matrix K. Building on those results, we then develop an algorithm that, provided the measured and excited nodes, computes which edges are identifiable and which are not. It can be run either symbolically, or stochastically *via* Monte Carlo experiments.

References

[1] Van den Hof, P. M., Dankers, A., Heuberger, P. S., Bombois, X. "Identification of dynamic models in complex networks with prediction error methods—Basic methods for consistent module estimates," in *Automatica*, 2013.

[2] Hendrickx, J. M., Gevers, M., and Bazanella, A. S. "Identifiability of dynamical networks with partial node measurements," in *IEEE Trans. on Automatic Control*, 2018.

[3] Bazanella, A. S., Gevers, M., Hendrickx, J. M. "Network identification with partial excitation and measurement," in *Proceedings of the IEEE CDC*, 2019.