Local dynamics identification via a graph-theoretical approach

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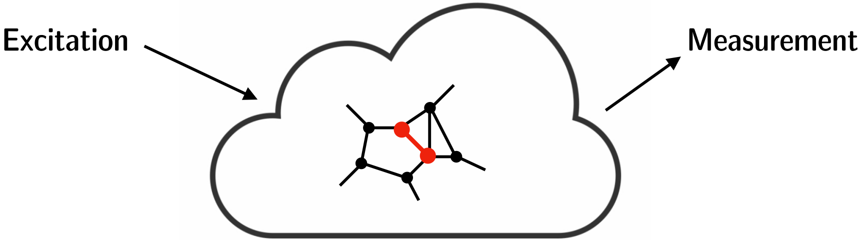
Long-term application

Case study (my Master's thesis)



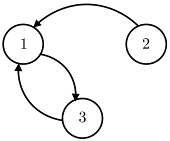
state transfer fct excitation

$$X(z) = G(z)\,X(z) + B\,U(z) + \text{noise}$$
$$Y(z) = C\,X(z)$$

measure binary selections

Which nodes to excite/measure to recover the **transfer functions**?

Theorem: Identifiability is a *generic* property of network topology.

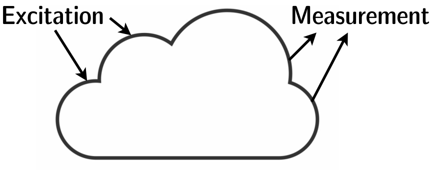


$$G = \begin{bmatrix} 0 & G_{12} & G_{13} \\ 0 & 0 & 0 \\ G_{31} & 0 & 0 \end{bmatrix}$$

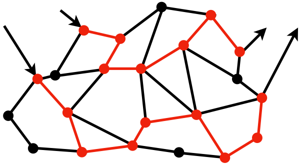
Assumption: Topology is known (often the case).

Approach

- **Global** graph theoretical approach
- Local dynamics from global input-output

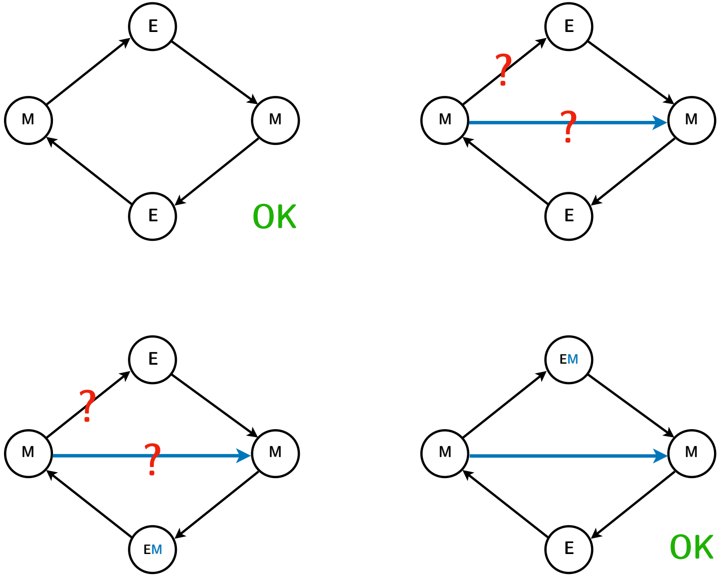


Known global MIMO



Local dynamics to identify

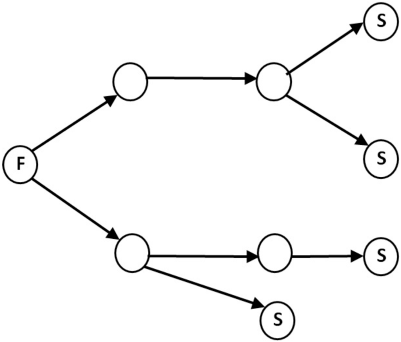
Motivating example



Trees

Theorem (Bazanella et al. 2019)

A tree is generically identifiable IFF all fountains are excited, all sinks are measured and all nodes are either excited or measured.



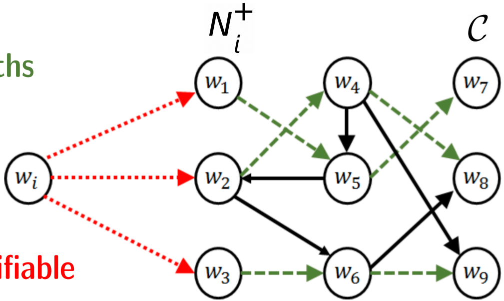
Definition: Fountains: no incoming edge – Sinks: no outgoing edge

Full excitation/measurement

Theorem (Full excitation – Hendrickx et al. 2018)

All transfer functions leaving node i are generically identifiable IFF there are $|N_i^+|$ vertex disjoint paths from N_i^+ to $\mathcal{C}$.

3 v.d. paths



gen. identifiable

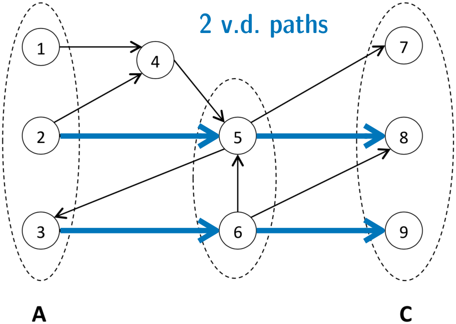
Dual (full measurement): All TFs arriving at node *j* are generically identifiable IFF there are $|N_j^-|$ vertex disjoint paths from N_j^- to $\mathcal{B}$.

Vertex disjoint paths

Theorem (Hendrickx et al. 2018)

- $b_{\mathcal{A}\to\mathcal{C}}$: *max number of vertex disjoint paths*
- $T_{\mathcal{C},\mathcal{A}}(G)$: *restriction of* $T(G) \triangleq (I-G)^{-1}$ *to nodes* $\mathcal{C}$ *and* $\mathcal{A}$

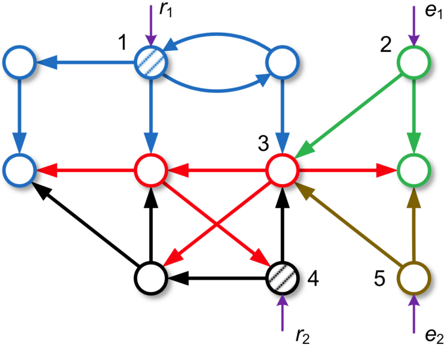
$$b_{\mathcal{A}\to\mathcal{C}} = \textit{generic rank}\,(T_{\mathcal{C},\mathcal{A}}(G))$$



Pseudotrees

Theorem (Full measurement - Cheng, Shi, Van den Hof 2019)

The network is generically identifiable IFF there exists a disjoint pseudotree covering with excited roots (+ dual for full excitation).



Analogy: Vertex disjoint paths - Pseudotree covering

Local generic identifiability

Definition

The transfer function $G_{ji}(z)$ is locally generically identifiable from $\mathcal{B}$ & $\mathcal{C}$ if, for any parametrization $G(P,z)$, there exists $\tilde{G}(z)$ consistent with the graph, $\epsilon > 0$ such that $||\tilde{G}(z) - G(P,z)|| < \epsilon$, and there holds

$$C(I - G(P,z))^{-1}B = C(I - \tilde{G}(z))^{-1}B \Rightarrow G_{ji}(P,z) = \tilde{G}_{ji}(z) \quad (1)$$

$\forall$ parameters P except possibly those lying on a zero measure set.

One can linearize (1) to obtain

$$CT\Delta TB = 0 \Rightarrow \Delta_{ji} = 0,$$

where $T \triangleq (I - G(z))^{-1}$ and Δ is consistent with the graph.

Whole network

For the whole network, it becomes

$$CT\Delta TB = 0 \Rightarrow \Delta_{\cancel{ji}} = 0,$$

which can be reformulated as a linear system

$$K\delta = 0 \Rightarrow \delta = 0,$$

where K is the restriction of $B^T T^T \otimes CT$ to the columns corresponding to actual edges, $\delta \in \mathbb{R}^{|E|}$, and $|E|$ denotes the number of edges.

Theorem

G is locally generically identifiable IFF generic rank $K = |E|$.

Specific edge

Otherwise, $\ker K \neq \emptyset \longrightarrow$ inspect $\ker K$ to find problematic edges

The linearization

$$CT\Delta TB = 0 \Rightarrow \Delta_{ji} = 0$$

can be reformulated as

$$K\delta = 0 \Rightarrow \delta_e = 0,$$

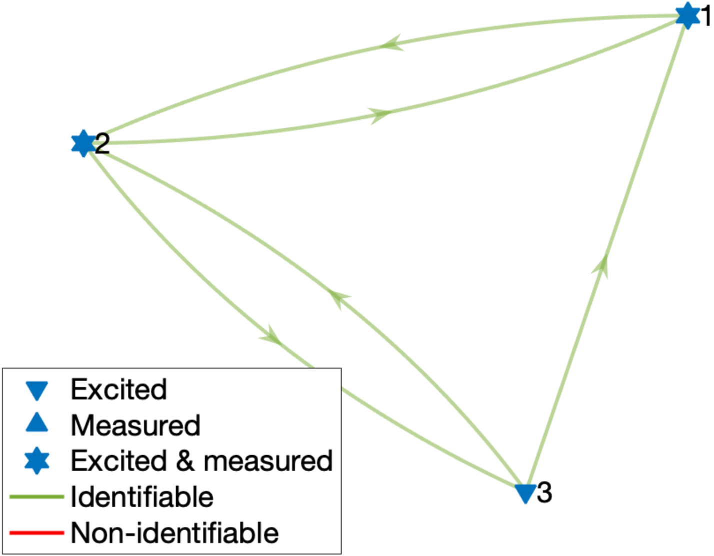
where e corresponds to edge (ji).

Theorem

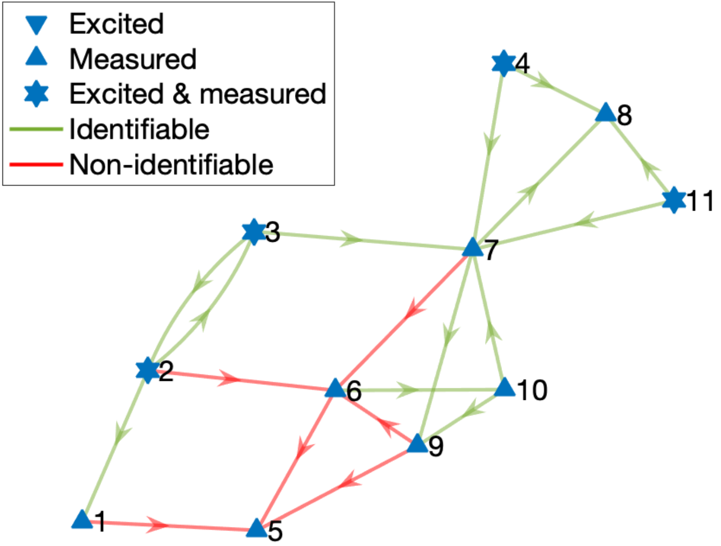
The transfer function e is locally generically identifiable IFF for each $\delta \in \ker K$, there holds $\delta_e = 0$.

1. Introduction
2. Previous results
3. General case
4. Algorithm
5. Conclusion

Symbolic run



Stochastic run (probability 1)



Conclusion

- Introduced local generic identifiability
- By a linearization, obtained necessary and sufficient conditions for recovering (i) the whole network (ii) a specific edge
- Algorithm: symbolic and exact, or stochastic and efficient
- Future work
  - Characterize our results in graph-theoretical terms
  - Extend our results to nonlocal generic identifiability

• Hendrickx, J.M., Gevers, M., and Bazanella, A.S. (2018). Identifiability of dynamical networks with partial node measurements. *IEEE Transactions on Automatic Control*.
• Bazanella, A. S., Gevers, M., and Hendrickx, J. (2019). Network Identification with Partial Excitation and Measurement. In *IEEE Conference on Decision and Control. Proceedings*.

• Cheng, X., Shi, S., and Van den Hof, P. M. (2019). Allocation of excitation signals for generic identifiability of dynamic networks. In *IEEE Conference on Decision and Control*. *Proceedings*.