Graph-Theoretical Condition for Local Network Identifiability

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1 Introduction

The goal of this work is to recover the local dynamics from the global input-output behavior of a networked system and the knowledge of the network topology.

We consider the identification of a network matrix $G(z)$, where the network is made up of n nodes, with node signals $\{w_1(t), \dots, w_n(t)\}$, and external excitation signals $\{r_1(t), \dots, r_n(t)\}$, related to each other by:

$$\begin{aligned} w(t) &= G(z)w(t) + Br(t) + v_1(t) \\ y(t) &= Cw(t) + v_2(t), \end{aligned} \tag{1}$$

where matrices B and C are binary selections defining which nodes are excited and measured, forming the sets $\mathscr{B}$ and $\mathscr{C}$ respectively. The vector $y(t)$ contains the measured nodes, while $v_1(t)$ and $v_2(t)$ are uncorrelated noise vectors. The nonzero entries of the network matrix $G(z)$ define the topology of the network, and are assumed proper and rational.

We assume that the global relation between the excitations r and measures y has been identified, and that the structure of $G(z)$ is known. From this knowledge, we aim at recovering the nonzero entries of $G(z)$.

A first line of work extends the classical closed-loop identification techniques to identify a single module, see e.g. [1]. A recent approach employs graph-theoretical tools to derive identifiability conditions on the graph of the network. Using this approach, [2] addresses the particular case where all nodes are excited/measured. In the general case of partial measurement *and* excitation, [3] introduces a local version of identifiability and derives algebraic necessary and sufficient conditions. In this work, we consider local identifiability with partial excitation and measurement. From the conditions of [3], we derive a graph-theoretical condition which generalizes the results of [2] when not all nodes are excited/measured.

2 Problem reformulation

Starting from the definition of a network system in (1), we first define $T(z) \triangleq (I - G(z))^{-1}$, which is assumed to be proper and stable. The input-output model of the network model (1) is then given by

$$y(t) = CT(z)Br(t) + v_3(t),$$

where $v_3(t) \triangleq CT(z)v_1(t) + v_2(t)$. We assume that $r(t)$ is sufficiently rich so that, for any B and C, $CT(z)B$ can be consistently estimated from $\{y(t), r(t)\}$ data. From the knowledge of $CT(z)B$, the aim is to identify $G(z)$. This motivates the following definition, which restricts the usual generic identifiability from [2] to non-discrete sets of solutions.

Definition 1. *The network matrix G is* generically locally identifiable *from excitations $\mathscr{B}$ and measurements $\mathscr{C}$ if there exists $\varepsilon > 0$ such that for any $\tilde{G}$ consistent with the graph satisfying $||\tilde{G} - G|| < \varepsilon$, there holds*

$$C(I - G(z))^{-1}B = C(I - \tilde{G}(z))^{-1}B \Rightarrow G(z) = \tilde{G}(z), \tag{2}$$

for all G except possibly those lying on a zero measure set.

In this definition, a network matrix $G(z)$ is said *consistent with the graph* if $G_{ij}(z)$ is zero when there is no edge (i, j).

3 Results

In [3], a linearization of (2) yields a necessary and sufficient condition for generic local identifiability, based on the generic rank of a matrix K constructed from B, C and T.

In this work, we show how the generic rank of K relies on the generic rank of certain particular transfer matrices. Besides, we know from [2] that the generic rank of a transfer matrix between two sets of nodes is equal to the number of vertex disjoint paths[1] that can be routed between those two sets.

Combining those results allows to derive a necessary condition for generic local identifiability in terms of paths in the network. We believe that such path-based characterization will pave the way for further developments in the subject.

References

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[1] A group of paths is said vertex disjoint if no two paths of this group contain the same vertex.